LABORATORY PHYSICS

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A STUDENT'S MANUAL FOR COLLEGES
AND SCIENTIFIC SCHOOLS

BY

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PREFACE

THIS Manual is designed to be a student's handbook for the performance of experimental problems in physics. The selection of problems and their treatment is the result of twelve years of teaching experience, and the descriptions of most of the exercises have been used in typewritten form for the past six years. The grade of work is that of the course in general physics in colleges and technical schools. It is presumed that the student has had a course in preparatory laboratory physics, and that these exercises will be accompanied, or preceded, by a full course of lectures and recitations in general physics, including instruction in the principles and manipulations of the various experimental operations.

One hundred and twenty-eight exercises are described. Some of these, however, such as reading the barometer, determining the heat equivalent of a calorimeter, etc., are usually performed in connection with other problems. There is a break in the serial numbers of the exercises at the end of each part, to permit the assignment of numbers to additional exercises in their proper groups.

The definite problems given under each heading have been formulated with much care, to cover as wide a range of the subject as is possible. They are based upon the work of the sophomore class in the Case School of Applied Science, and each one represents what may be expected of one or two students in a laboratory exercise of three hours' duration. Shorter exercises, or exercises covered by particular articles, may be conveniently assigned in the manner explained in the Introduction.

The book is not merely a compilation, though of course most of the exercises have been used and described before. The writer has made free use of all sources of information, sufficient acknowledgment for which will be found in the many references given. It is thought that several important exercises appear for the first time in a laboratory manual, and that others have been made more efficient by the method of treatment. Among these may be mentioned those concerning the comparison and calibration of graduated scales, the balance, density by hydrostatic weighing, Reed's method for rating a fork, thermometry and calibration of thermometers, high-temperature measurements, calorimetry by heating, mechanical equivalent of heat, hygrometry, photometry, spectrometry, concave-grating spectroscope, interferometer, silvering glass, refractometer, capillary electrometer, and magnetic variometer.

In making references the endeavor has been not to refer to original sources but rather to those books which contain information in the most useful form for the experiments described, and which are most likely to be available to the student. It is believed that the descriptions are ample for the performance of the exercises; the references indicate where other methods are explained, or where information for advanced work may be found.

The writer is greatly indebted to Professor Edward W. Morley for many valuable suggestions in general, and in particular in connection with the balance, the thermometer, and the interferometer. Messrs. H. W. Springsteen, H. S. Hower, S. R. Cook, and J. W. Easton, present or former Instructors in Physics in Case School of Applied Science, have given efficient aid in reading the manuscript and proof sheets of the book.

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LABORATORY PHYSICS

PART I — INTRODUCTION

CHAPTER I

GENERAL INSTRUCTIONS

1. Objects of Laboratory Practice. — A course in general physics has for its primary object the inculcation of the fundamental principles of the science. This is best accomplished for college classes by the proper coördination of lectures and demonstrations by the instructor, and of recitations and laboratory practice by the student. The lectures, demonstrations, and recitations should consume half or more of the time allotted to physics, and should be founded upon a complete systematic text-book. This part of the course should be in advance of the laboratory work, so that the student shall have studied the theory of an experiment before attempting to perform it.

A course of physical experimentation by the student has for its objects the fixing in mind of the essential principles studied in the class room, the training of the student's thinking and reasoning powers, and the furnishing of direct proof of some of the fundamental laws of the science; and it has the further objects, of almost equal importance, of giving the student an acquaintance with the methods and instruments of physical and technical operations, and of developing skill in the manipulation of delicate apparatus and in the making and reducing of measurements of precision.

This book is designed as a student's handbook for use in the laboratory; it contains such practical explanations of the principles involved in the experiments, and such details of manipulation and reduction, as experience has shown to be necessary and helpful to the student.

With these objects in view, a laboratory course is not considered as consisting of a certain number of exercises to be worked out by each student, and to be complete when these are finished, but rather as consisting of a definite amount of time spent in judicious experimenting. Several days of work put upon some troublesome process which fails at first to give the desired results is as valuable and as much to the student's credit, if honestly done, as the completion of several problems in which no difficulties were encountered.

2. Available Problems. — Certain problems in each branch of the subject are fundamental, and should be required of each student; but many may be given to some students and not to others, as circumstances determine. Only those problems which have a very clear object in view, and which demand some skill in manipulation and the use of apparatus neither too simple nor too delicate, should be used for a general course; and usually an experiment should require not less than three nor more than six hours for its completion. These conditions greatly limit the number of available problems.

Mechanics furnishes a larger number of valuable laboratory exercises than any other branch of physics, and some of these are best introduced at the end of the course after the student has acquired considerable skill. If each problem contains a distinct lesson, either in principle or in manipulation, it matters not whether one student receives this exercise at the same time that it is assigned to another. It is often advantageous to have students at work in several branches of the subject at the same time. With the work properly systematized, this method is not extravagant of teaching force.

3. Equipment. — The duplication of sets of apparatus of cheap and simple construction, in order that all students may be at work upon the same experiment at the same time, is not

advocated for college classes. The laboratory equipment should be of as great a variety as is possible, and the student should be trained in the use of the actual instruments employed in scientific work. If several instruments for one purpose are required, let them be of all varieties as regards size, adjustments, and attachments, and though a student may work with but one form, he cannot fail to obtain information from merely seeing other forms in use.

Each piece of important apparatus should have a definite place in the laboratory; it should be in position and in perfect order at the beginning of an exercise, and the student should leave it in the same condition at the completion of his work. Subsidiary apparatus, such as calipers, weights, thermometers, and tools, may be obtained from the instructor in charge, and these are to be returned by the one who borrowed them.

Much of the apparatus of the physical laboratory is delicate and costly, and is liable to disarrangement by improper handling. If any instrument is out of order, the fact should be at once reported; the student will be held accountable for all damages which result from neglect or carelessness. When a piece of apparatus is to be used with which the student is unfamiliar, the peculiarities of construction and adjustment should be carefully studied before manipulation is begun, both to save time and to prevent injury to the instrument.

With large classes the making of apparatus by the individual student is not practicable. An apparatus made from the odds and ends of the laboratory will not command the beginner's respect and careful attention, even though it would do good work in the hands of an experienced manipulator.

An essential part of the equipment of every laboratory is a reference library, which need not be large, but should contain the principal standard works on practical and theoretical physics. The student should have free access to the reference books, and should use the privilege constantly to obtain information in regard to the various forms of apparatus and the different methods of observation and reduction. A short list of reference books is given in the Appendix.

4. Descriptions of Exercises. — The titles to the various experiments might all be preceded by the phrase, "Determination of," "Measurement of," or "Proof of"; the titles themselves mention what is to be determined, and specify the method or the principal piece of apparatus which is to be employed.

There follows a concise and specific statement of just what is required of the student. In some cases more than one problem is given under one heading; these are then designated as (a), (b), etc. A problem is referred to in the laboratory work, both in assignment and in records, by its number; as 8, or 117 (b).

The articles grouped under each exercise describe the particular methods of work to be adopted, and, together with the articles to which reference is made, they constitute sufficient information to enable the student who has already studied the principles involved to perform the experiment intelligently. These descriptions are largely explanations of methods of work rather than of particular pieces of apparatus. If more than one important form of instrument is used with a given method, brief descriptions of these are given. This makes the manual useful in laboratories of various equipment, and it is also advantageous for the student using one form to know that other forms are often employed.

5. Assignment of Exercises. — For assigning the problems, small cards bearing the numbers of the exercises to be used are prepared; the assignment may be announced in two ways. On each card is written the name of the student to whom the corresponding experiment is assigned and the card is given to him at the beginning of the exercise, to be returned at the end. These names are then crossed off, and for the next day new names are written on the same cards. Or there may be in the laboratory a bulletin board containing a list of the names of the students, and the cards with the numbers may be placed opposite the names selected; then the students have only to refer to the book for the statements of the problems.

The card system has the advantage that any variation desired in the specific statement of a problem may be written on the card, while the description of the method would be the same as

given under the corresponding exercise in the book. Shorter exercises may thus be announced, or entirely different ones, according to the requirements of various laboratories.

6. Notes and Records. — Blank forms arranged for the records of each experiment are not recommended; instead the student is required to arrange for himself the data and results of his work in a systematic way. The efficiency of experimental work will be greatly increased by intelligent and skillful records. Before beginning an experiment the student should carefully study the various operations and measurements involved, and plan a method of procedure and of recording the observations. Tabular and symmetrical forms are to be used whenever possible. Numerical results of several exercises are given in forms suitable for the reports; the original notes should be similarly arranged, though they may of necessity contain more of detail and computation.

Make the first right-hand page of the notebook a general title-page; several following leaves are to be used for a table of contents, which will be filled in as the experiments are performed throughout the course. When the book is filled with notes an index should be prepared and placed at the end. Begin the record of each experiment on a new right-hand page, giving a liberal amount of space to the title. There should be recorded all the original data of the experiment: date, fellow-observer, object of experiment, description, with the identifying numbers or names of the particular pieces of apparatus used, method, and all data of the observations, calculations, and results. All observations, whether used in calculating the final result or not, should be preserved, and any reason for suspecting certain results as being inaccurate should be noted. Never erase the result of an observation once recorded, even if it is deemed worthless.

Important items should be conspicuous enough to be readily found, and they should be so arranged as to make comparisons and computations convenient. The record of every measurement should specify the unit employed. The scales used in graphical representations are to be explained. There should be diagrams

showing the arrangement of apparatus; special formulæ used should be analyzed and proved.

Numerical calculations in multiplication, division, involution, and evolution must not be made by arithmetical processes. For the greater part of the work in this course, four-place logarithms (Appendix) or the slide rule are sufficient, though for a few exercises, five-place, or even six- and seven-place logarithms are required. The figures of calculation, logarithms, etc., must be arranged in an orderly manner. The final result and its exact meaning should be made conspicuous; if it is the mean of several determinations, compute and specify the probable error (Art. 9). When a result has been obtained it should be compared with what is considered the correct result, obtained by some independent method of proving, or from tables of constants, or from published results of research work.

A notebook is valuable only in so far as it is a complete record of all that has been done in making the original experiment, and it should contain just so much of detail as will make it intelligible to the writer or to others at any time after the completion of the work. The necessity for full, unaltered, original notes cannot be too strongly impressed upon the beginner.

Upon the completion of each exercise, before the student leaves the laboratory, he should present his notebook to the instructor for examination, criticism, and grading. If the work has been carelessly or improperly done, a part or all of it may have to be repeated. When the notes and report mentioned in the next article are satisfactory, both will be approved and signed by the instructor.

The notebooks for the entire class should be uniform in style and of a permanent kind, having a cloth or leather cover, and paper of good quality. The leaves are preferably of quadrille paper, while an unruled paper is better than that with lines in one direction only. The quadrille ruling facilitates tabulation and is convenient for graphic solutions. A suitable size for the page is fifteen by twenty-three centimeters.

The original notebook is to remain the permanent property of the student, and if the records are properly made it will have become a valuable possession at the end of the course.

7. Reports. — Upon the completion of each experiment the student must make out a full and systematic report to be given to the instructor, for grading and permanent filing in the records of the laboratory. To secure uniformity, this report should be made upon a blank form which, while containing no form for the experiment proper, bears the title of the laboratory, and has a place for the student's name, title of experiment, and sufficient space for the data mentioned as necessary in the original notes. The report may be somewhat condensed in the data of observations, giving only the essential results, and in the calculation, which may be indicated. The numerical results of several of the exercises which are given exhibit the proper forms for the reports. Credits and grades for the exercises will be given only after both notes and reports have been approved.

8. Observations, Errors, and Corrections. — When the highest precision is attempted it is found that repeated measures of the same quantity differ by appreciable amounts. The measures are affected by errors, some of which are accidental and variable in magnitude, while there are others which are constant. The constant errors may be due to imperfections in the measuring instrument, to a faulty method, or to bias in the observer. Study will often discover the errors of instruments and methods, and they may be eliminated or made inappreciable by determining their values and applying corrections for them.

Errors due to bias or prejudice are only to be reduced by the exercise of special care and skill, or by a change of observers. Whenever practicable the observations of one person should be checked by another. While it is not necessary that the student be in ignorance of the nature or value of a result to be obtained, yet it may require an effort on his part to prevent this knowledge from prejudicing the result, and to check the tendency to make a second measure agree with the first.

Besides the constant errors there are always present accidental errors of observation, which in a large series of independent, equally trustworthy measures, are likely to be half positive and half negative; most of these errors will be of small magnitude, and only a very few will be large. The arithmetical mean is

the best result that can be obtained from such a series, and it is much more trustworthy than is a single measure. Theory indicates that the probability of the mean increases as the square root of the number of observations; hence the mean of nine measures has a probable error one third as large as the probable error of one measure, and the mean of twenty-five measures has a probable error one fifth as large. The probability increases so slowly in comparison with the labor of observation that a practical limit is soon reached. Usually ten measures of the same quantity are sufficient.

The accidental errors are often small in comparison with the constant errors, and time which might be spent in making a longer series of measures is more profitably used in obtaining a second result after making a complete readjustment of the apparatus, or with new apparatus, or with a new method. Since in laboratory practice mere numerical results are not primary objects, the obtaining of a measure which by accident is correct will not excuse the student from making the repeated adjustments required in original work. Usually much more time is consumed in arranging and adjusting a set of apparatus than is required to make the final observations; for the student this is the most instructive part of laboratory work, and it should be performed thoroughly and understandingly.

Another class of errors which may be called mistakes are sometimes present. These generally result from inattention and lack of skill, which are inexcusable even in the beginner.

The methods of procedure given in connection with the laboratory problems are largely concerned with the elimination and correction of errors.

Reference. — *Lupton*, Notes on Observations.

9. Probable Error.—From a number of observations, depending upon their concordance, there may be computed a criterion of the precision with which the measures can be made, which is called the probable error. *The probable error of a single observation* is the quantity which when added to and subtracted from the mean gives limiting values, such that if a

single measure of the same kind is made, it is as likely to lie outside of these limits, on either the positive or negative side, as it is to lie between them. *The probable error of the mean* is a quantity which when added to and subtracted from the mean gives limiting values such that if another mean value is determined as was the first, its value is as likely to lie outside of these limits, in either the positive or negative direction, as it is to lie between them.

The probable error is not a limiting value such that there is no probability of a greater error; it is such a value that in a large series there are certain to be as many errors greater than it as there are smaller ones. In other words, a result is as likely to have an error (compared with the mean) larger than the probable error as it is to have a smaller one. Probability shows that an error twice as large as the probable error is likely to occur in one out of 5 measures, that an error three times the probable error may be expected once in 23 cases, an error four times as large once in 142, while an error five times as large occurs only once in 1314 observations.

To give reasonable security, in the ratio of 142 to 1, that the mean may not have an error exceeding four times the probable error of a single observation, the mean must be the result of 16 measures.

If m is the arithmetical mean of a series of n measures, a_1, a_2, a_3 . . . a_n, the probable error of a single observation is

$$e = \pm\, 0.6745 \sqrt{\frac{(m-a_1)^2 + (m-a_2)^2 + \cdots (m-a_n)^2}{n-1}};$$

and the probable error of the mean is

$$E = \pm\, 0.6745 \sqrt{\frac{(m-a_1)^2 + (m-a_2)^2 + \cdots (m-a_n)^2}{n(n-1)}} = \frac{e}{\sqrt{n}}.$$

The numerical coefficient usually may be taken as $\frac{2}{3}$.

The differences between the mean and the several measures are called the residuals or the deviations. The average of the deviations, without regard to sign, is often used as an indication of precision. The percentage deviation, the value of which is

the deviation divided by the quantity, is useful when only a few measures are made.

These precision measures, the probable error, the percentage deviation, etc., only indicate the agreement of the measures *among themselves* as regards the variable accidental errors; they indicate nothing with respect to constant errors, which may be very large, and hence they do not show the relation of the result to the absolute truth, a relation which usually is wholly indeterminate.

10. Least Squares. — When a number of independent direct measurements of the same quantity have been made, the arithmetical mean is the most probable value. But if the quantity is measured indirectly, being involved with others, the observations furnish equations expressing the relations of these quantities. If the number of the equations equals that of the unknown quantities, only one solution is possible. If there are more equations than unknown quantities, no one set of values for the unknown quantities is likely to satisfy all the equations; that is, an exact solution is impossible. Various approximate solutions may be made, and theory shows that the most probable one is that which makes the sum of the squares of the residual errors the least possible. This method of solution is known as the Method of Least Squares. The calculation is apt to be laborious, and is required only in important reductions. The student may consult the references for descriptions of the operations, and mathematical text-books for the full theory.

References. — Concerning the reduction of observations, the student is referred to *Stewart and Gee*, Practical Physics; *Kohlrausch*, Physical Measurements; *Nichols*, A Laboratory Manual of Physics; *Holman*, Precision of Measurements; *Merriman*, Method of Least Squares.

11. Weighted Observations. — When a final result is to be calculated from a number of different determinations of the quantity, made perhaps by different observers, by different methods, with different apparatus, or resulting from different numbers of separate observations, these determinations will not all be equally trustworthy. In the computation each one must

be allowed to affect the result only in proportion to its trustworthiness, and *weights* must be assigned to them according to their probable accuracy. When the separate values are each the result of a number of observations, the weight is set equal to this number. If the probable error, e, is known (Art. 9), the weight is proportional to $\frac{1}{e^2}$. In other cases the weights must be assigned according to the judgment of the computer. In the calculation each value is multiplied by its weight, and the sum of these products is divided by the sum of the weights. The final mean, r, of several results, r_1, r_2, etc., having weights, p_1, p_2, etc., is

$$r = \frac{p_1r_1 + p_2r_2 + \cdots}{p_1 + p_2 + \cdots}.$$

12. Significant Figures. — In order that numerical operations should have all the accuracy possible and yet not appear to claim an unwarranted precision, and to save time, the data of observation, computation, and results should contain only the proper number of significant figures. A significant figure is any one of the ten digits other than a zero which is used merely to locate the decimal point.

In a series of measurements the significant places should be carried one place farther than that in which differences first occur. In casting off figures, increase by 1 the last figure retained when the following was 5 or more.

When several numbers are to be added or subtracted, whose significant figures by the above rule reach to different decimal places, no significant places are to be retained beyond those of the number whose last significant figure is in the highest place.

When numbers are to be multiplied or divided, there should be the same number of significant figures in all the data, constants, products, quotients, and result, as there are in that factor which has the fewest significant figures.

In logarithmic computation there should be as many significant figures in the mantissæ as there are significant places in the numbers operated upon. The characteristic is not considered in counting the significant places.

All records of measurements should show clearly the full degree of accuracy attained, and to this end final zeros in a decimal fraction should be considered as significant as any other figure in the same place. For example, if in weighing a mass with a balance and by a method capable of determining single milligrams, the result is exactly twenty-one and six-tenths grams, record the result thus: 21.600 g. If the last figure in a number representing a quantity is 9, this figure would of course be written; if the figure is 10, in the decimal system, the 1 of the figure is "carried" and added to the preceding figures, but the 0 is as significant now as was the 9 in the first case. If the 0 in the last place is dropped, it would indicate that no value had been found for this place, which is contrary to the supposition.

13. Graphical Methods. — When a series of observations has been made of the dependence of one quantity upon another, often the most perspicuous presentation of the results is by a curve representing this dependence. Using the values of the independent variable as abscissæ, and the corresponding values of the dependent variable as ordinates, a series of points may be plotted. A curve drawn through these points as nearly as possible, so that very few points are far from the curve, and that with respect to any small portion of the curve there are as many points on one side of it as on the other, will graphically represent the observations (Art. 74). The exact form of the curve will be a matter of judgment, and hence the solution is somewhat indeterminate. An error in an observation may be indicated by the erratic location of a point. The values of unknown quantities may often be measured from such a plot; that is, a graphic solution may be made. Whenever it is possible, even though the solution has not been a graphic one, the results of an exercise should be shown graphically.

14. Units. — All measurements in mechanics are to be recorded in centimeter-gram-second units unless distinctly required otherwise. Do not record some measurements in meters and others in millimeters, but use the centimeter as the universal unit of length, and the gram as the only unit of mass. In final results

all forces are to be expressed in dynes. Attention to these points may save much confusion in calculation.

The centimeter is $\frac{1}{100}$ of the distance, at the temperature of 0°, between two certain marks on a certain bar of platinum-iridium, known as the International Prototype Meter, which is preserved at the International Bureau of Weights and Measures near Paris.

The gram is $\frac{1}{1000}$ of the mass of a certain piece of platinum-iridium, known as the International Prototype Kilogram, which is preserved at the International Bureau of Weights and Measures near Paris.

The second is $\frac{1}{86400}$ of a mean solar day.

Upon these three fundamental units are founded many derived units, such as the dyne, erg, and joule; there are also related units, such as the calorie, and many independent units, such as the degree centigrade, candle power, etc. When deemed necessary these units are described in connection with their first use in the experimental work.

The employment of one system of units for all quantities sometimes results in numbers which are very large or very small. Such a number is best expressed as the product of two factors, one of which is the number with one integral place and as many decimal places as are significant, the other factor being a power of 10. Since logarithmic computation is almost universal, this method is simple; the first factor is the number corresponding to the mantissa alone, while the second is 10 with an exponent equal to the characteristic of the logarithm. An illustration is the answer to the example in Art. 74.

Reference. — For a complete treatment of the units of physics together with tables of constants, reference is made to *Everett*, Illustrations of the C. G. S. System of Units.

15. Reading Divided Scales.—In quantitative work the reading of some form of divided scale is almost universal, and when the highest precision is desired fractions of the smallest divisions must be determined. Often this is accomplished by means of either the vernier or the micrometer screw, described in Arts. 17 and 19. When this is not the case it is a general rule that the

fraction of a division is to be estimated in tenths. Never estimate in fourths or eighths, but always in tenths for recording decimally. This is to apply to straight and circular measuring scales; to the divided heads of measuring screws; to balance, galvanometer, and thermometer scales; to the time intervals represented by pendulum beats; and to all other readings where such an estimation is possible.

Part II—Mechanics

CHAPTER II

GENERAL MEASUREMENTS, LENGTH, AND MASS

I. LENGTH WITH THE CALIPERS

Measure the length of a steel rod with the vernier caliper, in both inches and centimeters, and find the ratio of the inch to the centimeter. Determine the dimensions of a steel cylinder with English and metric micrometer calipers, and find the ratio of the inch to the centimeter. Calculate the probable error of a single observation and of the mean for each set of measurements.

16. Length. — Measurements of length are always referred to real or virtual divided scales, and when precision is desired fractions of divisions must be determined. One of three methods is employed for this purpose. The simplest is that of estimation, mentioned in Art. 15, which depends upon the judgment of the observer. A second method, more precise and largely independent of judgment, is by means of the vernier. The greatest precision is attained through the application of the micrometer screw. These latter methods are directly applied in the calipers described below, and their use in more elaborate instruments for length measures and comparisons is described in connection with other experiments.

17. The Vernier. — To facilitate the reading of fractions of a division of a divided scale, the vernier is often employed. It consists of a short subsidiary scale, movable along the main

scale, the zero line of which is the index to the main scale. If the smallest indication desired is $\frac{1}{n}$ of a division, the vernier is usually $n-1$ scale divisions in length, and is divided into n parts.

If a vernier is applied to determine tenths of a scale division, it may be constructed so that its total length is nine scale divisions, while it is divided into ten equal parts; each vernier part will therefore be equal to nine tenths of a scale division. Suppose, as is represented in Fig. 1, that the index or 0 vernier line lies between the main scale graduations corresponding to 1.4 and 1.5, the fraction of a division from 1.4 to the index line can be determined as follows. The vernier line No. 1 is nearer to its corresponding scale line by one tenth of a division (a vernier

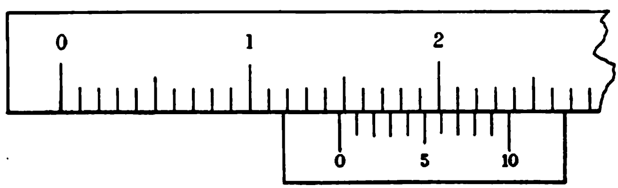


FIG. 1. SCALE WITH VERNIER

division being nine tenths of a scale division), and the vernier line No. 2 is two tenths of a division nearer the next scale line, and so on. Each succeeding vernier line being one tenth of a division nearer its corresponding scale line, the number of the vernier line which exactly coincides with a scale line will represent the number of tenths of a division by which the first, or 0, line is separated from the preceding scale line. If, as shown, line No. 7 of the vernier thus coincides with a scale line, since it is seven tenths of a division nearer to this line than the 0 vernier line is to its corresponding scale line, the reading to the nearest tenth of a division is 1.47.

The following rule for reading vernier scales is usually applicable. From an examination of the main scale determine the value of its smallest division. This smallest division will be further subdivided by the vernier into as many parts as there are divisions in the entire vernier scale: determine what this

value is; it may be called the value of a vernier division. A reading then consists of the sum of two parts. The first part is the exact reading of the main scale up to the line which immediately precedes the index or 0 line of the vernier. The second part is the vernier reading for the fraction of a division. Look along the vernier until a line is discovered which exactly coincides with some line (no matter which) on the main scale. The vernier reading is then the product of the number, *on the vernier*, of this coinciding line, and the value of one vernier division.

Fig. 2 represents a circular vernier in which the smallest division of the circle is $\frac{1}{6}°$, or $10'$; the vernier subdivides this into sixty parts, therefore indicating $10''$. The first part of the reading (at a) is $158° 40'$, the vernier

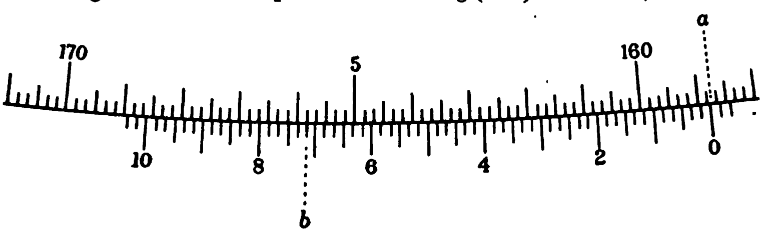


FIG. 2. VERNIER FOR DIVIDED CIRCLE READING TO 10″

reading (at b) is $7' 10''$, and the whole reading is $158° 47' 10''$. Other forms of verniers are sometimes used, but it is not necessary to describe them, as their indications may be analyzed in a manner similar to that given above.

REFERENCE. — *Staley*, Gillespie's Surveying, Vol. I, pp. 201–213.

18. The Vernier Caliper. — For quickly measuring various small lengths with moderate precision, the vernier caliper is useful. As compared with the screw caliper described in the next article, it is not so precise, but it has the advantage of greater range and of facility in setting for measures of different lengths. The vernier caliper (Fig. 3) consists of a straight scale having two jaws, whose plane and parallel surfaces are at right angles to the length of the scale. One jaw is fixed, and the other, movable along the length of the scale, carries an index and vernier, which indicates the distance between the jaws. The object whose length is to be measured is placed between the jaws, which are adjusted by the slow-motion screw until it

is very lightly held between them. The movable jaw is then clamped in position, and the reading of the vernier taken. The scale is sometimes graduated on one side in inches and on the other in centimeters. In this problem read both sides at each setting of the caliper. Each reading must be corrected for the *index error* by adding, with the sign changed, the reading of the vernier when the jaws are closed.

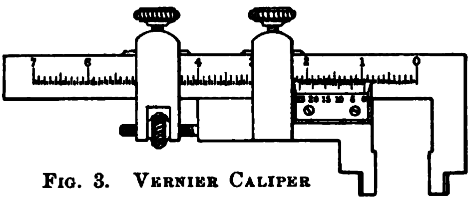


Fig. 3. Vernier Caliper

19. The Micrometer Caliper. — One of the most useful devices for the measurement of ordinary small lengths is the micrometer screw, which depends upon the principle that if an accurate screw is turned in a fixed nut, the end of the screw will move through spaces proportional to the rotation. The distance in terms of the pitch of the screw is thus obtained, which may be expressed in any desired unit if the pitch is known.

The micrometer caliper consists of a screw moving in a fixed nut toward or from a stop carried by the frame that holds the nut. The distance between the end of the screw and the stop can be determined in terms of the pitch of the screw, there being a longitudinal scale (Fig. 4) to indicate whole turns of the screw, and divisions upon the head to indicate fractions of a turn. The screws of such calipers often have forty threads per inch, or twenty per centimeter. The heads of the former being divided into twenty-five parts, and those of the latter into fifty parts, the instruments read, by estimating tenths of the divisions, to a ten-thousandth of an inch, or a ten-thousandth of a centimeter.

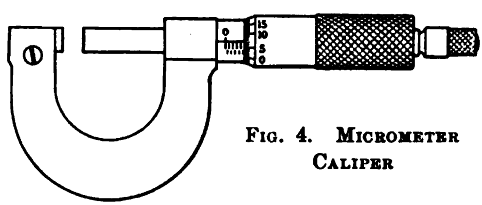


Fig. 4. Micrometer Caliper

Place the object to be measured between the screw and the stop, and turn the screw upon it to hold it lightly. The reading is then taken in a manner which will be obvious from an

examination of the caliper. The error of the zero, with its sign changed, must be applied as a correction to all readings, and it should be determined both before and after making a series of measurements by taking the reading when the caliper is closed. Often the zero error can be eliminated by learning, from several trials, just how much pressure must be put upon the screw head to bring the index exactly to zero. If the screw is provided with a ratchet head for securing uniform pressure, it should be turned gently until the ratchet slips *one* notch.

II. LENGTH AND RADIUS OF CURVATURE WITH THE SPHEROMETER

Measure the thickness of a metal disk, and the depth of a hole in it. Determine the radius of curvature of one surface of a lens. Calculate the probable error of a single observation and of the mean for each set of measurements.

20. The Spherometer. — For certain kinds of measurements the micrometer screw is conveniently made in the form of a spherometer, which consists of a tripod (Fig. 5) carrying an accurate screw, the axis of which passes through the center of the circle in which lie the three feet. The head of the screw is a large divided disk, and attached to the tripod is an upright scale which serves as an index to indicate the number of whole turns given to the screw.

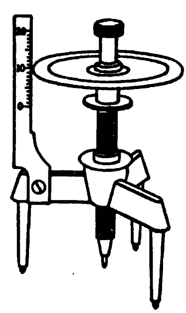


Fig. 5
Spherometer

The screw may have a pitch of one twentieth of a centimeter, and the disk be divided into five hundred parts. Then one whole turn of the screw represents a longitudinal motion of the point of five hundredths of a centimeter, and a turn through one division of the large disk represents a motion of the point through one ten-thousandth of a centimeter. By estimating tenths of a division the reading can be taken to a hundred-thousandth of a centimeter.

An essential part of the instrument is a plane surface for it to stand upon; usually a piece of plate glass may be employed. A series of readings should first be taken to determine the *zero point*, by placing the spherometer upon the plane and so adjusting the screw that its point shall be in the plane of the three fixed points. This can be done with great accuracy by attempting to rotate the instrument about the central point; if it rests more heavily upon this point than upon the others, it will easily rotate about it, and the screw should be so turned that this tendency just disappears.

The object whose thickness is required is placed under the central point, and the screw again adjusted as described. A series of readings equal in number to those made for the zero point should be taken, and the difference between the means of the two series will be the thickness.

The spherometer is especially useful in determining the radius of curvature of spherical surfaces. Make a series of settings with the instrument standing upon the spherical surface. Let d be the difference between this reading and the zero, determined as before, l the mean of the lengths of the sides of the triangle formed by the points of the three legs, and R the radius of curvature of the surface; then

$$R = \frac{l^2}{6\,d} + \frac{d}{2}.$$

The length l may be determined by holding the edge of a steel scale to the different points, or by measuring between the impressions of the points upon a piece of smooth paper.

III. CONSTANT OF A MICROMETER MICROSCOPE

Calibrate a microscope in both inches and centimeters, and find the ratio of the inch to the centimeter. Calculate the probable error of a single observation and of the mean for each set of measurements.

21. The Micrometer Microscope. — A most useful instrument for measuring very small lengths with precision is the micrometer

microscope, which is a combination of the micrometer screw and the ordinary microscope. The objective of the microscope produces an enlarged image of the object, which is adjusted, by focusing, to lie in the plane MN (Fig. 6) of a system of fixed and movable spider lines. These spider lines and the image are observed with a magnifying eyepiece. The position of the movable cross wires is controlled by a fine screw, M. The measurement of a length consists in determining exactly the number of turns of the screw required to move the cross wires over the magnified image. This value will vary with the focal length of the objective and also with the length of the microscope body.

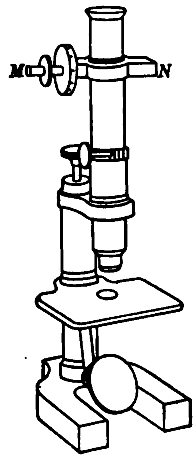


FIG. 6. MICROMETER MICROSCOPE

The head of the screw is usually a disk divided into one hundred parts to facilitate the reading of fractions of a turn. Estimate tenths of these divisions. To facilitate the counting of whole turns there is in the field of view a saw-toothed index (Fig. 7), the distance from one tooth to another corresponding to the motion of the wire in one complete turn of the screw. Every fifth notch is cut deeper than the others to aid in counting.

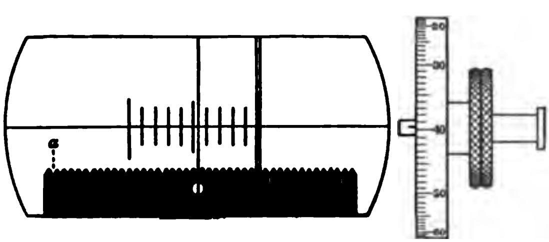


FIG. 7. FIELD OF VIEW OF MICROMETER MICROSCOPE

The deep notch farthest from the screw head (a, Fig. 7) is taken as the zero position for the cross wires. When the index has been set upon any mark in the field of view, the position is to be referred to this zero. The reading consists of the number of notches between the initial deep notch and the position of the index (which equals the number of whole turns of the screw)

plus the reading on the disk for the fractional turn. If the appearance of the field of view and of the head of the screw is as shown in the figure, the reading is 28.397.

In measuring a length, set the movable wire first on one end and then on the other, reading each position as described; the difference between the two readings is the required length. Do not try to set one end of the space at the zero position of the cross wires; the space to be measured should as nearly as possible be central in the field of view. The fixed cross wires will assist in adjusting the object to be measured so that it is parallel to the line of motion.

The disk on the screw head is often held in position by a friction washer, to permit setting it to position. When the index wire is exactly over the central fixed wire the disk should read zero.

When a micrometer screw is used for measurement, the final motions of the screw in making settings should always be in the same direction, to eliminate errors due to lost motion; usually that direction is best which gives increasing numbers on the divided head of the screw.

Where the readings to be made are the distances between lines of a divided scale instead of a single wire or cross wires, it is desirable to have two parallel wires for the movable index, the distance between them being a little more than the width of the line. To set upon a line, adjust the parallel wires so that the line bisects the space between them.

Whenever using a microscope or telescope with cross wires in the field of view, always *first* focus the eyepiece, by a sliding motion, upon the cross wires; *then* focus upon the object by the rack and pinion, or by a second draw tube. The perfection of focus is indicated by the absence of parallax; that is, when the eye is moved sideways there should be no apparent motion between the cross wires and the image. If there is such motion the adjustments must be repeated. To obtain a measurement in any given system of units the microscope must be calibrated by determining how many turns of the screw would be necessary to move the cross wires over the enlarged image of a standard

unit of this system. This number is a constant for a given microscope under fixed conditions. For metric measures a standard length of one tenth of a centimeter will be convenient, since this is about the length easily seen in the field of view. Any measurement made in terms of the pitch of the screw will be reduced to centimeters by dividing the number of turns by the calibration constant.

Thus, if it is found that to move the cross wires over one tenth of a centimeter requires 16.43 turns of the screw, the calibration constant is 164.3 turns = 1 cm; and if to move the wires over the bore of a small tube seen endwise requires 12.46 turns, the diameter of the bore is 12.46 ÷ 164.3 = 0.0758 cm.

IV. LENGTH WITH THE COMPARATOR

Compare a steel meter with a standard meter at each decimeter of its length, and plot the results. Determine the correction to the steel meter at 0°.

22. The Comparator. — An apparatus for facilitating the comparison of two nearly equal lengths, and for determining the difference between them, is a comparator. It is essentially a rigid bar (Fig. 8) to which two micrometer microscopes can be

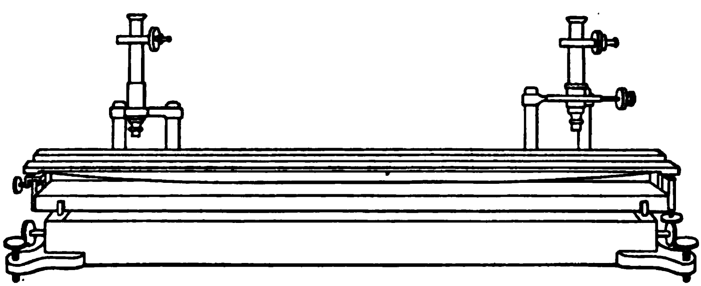


Fig. 8. Comparator

firmly attached at any desired distance apart. This bar is fastened to a base on which rolls a carriage. The carriage is provided with two adjustable tables on which the scales to be compared are placed, and its movement permits either scale to be quickly brought into view under the microscopes.

The microscopes are set at a distance apart approximately equal to the lengths to be compared, and are adjusted so that both shall focus upon a line parallel to the supported bar. The scales are placed upon their supports, and are so adjusted that when brought under the microscopes the end graduations will be visible near the center of the field of view, and in perfect focus.

The microscopes are first set upon the standard scale, then upon the other, and the difference between their lengths read in turns of the micrometer screws; make a second determination of this difference by resetting upon the standard scale. Often one microscope serves as an index to which one end of each scale can be set by means of suitable screws, and the difference is measured with the other microscope. The value of the micrometer screw may be determined from a known distance graduated on the standard bar.

If the two bars are of different materials, their coefficients of expansion will be different, and it will be necessary to reduce the observations to 0° in finding the correction to the unknown bar. If t is the temperature of the two bars when compared, d the difference in their lengths, positive when the standard is the longer, δ_1 the correction to the standard at 0°, e_1 its coefficient of expansion, and δ_2 and e_2 the corresponding constants for the other bar, then the correction for the unknown bar at 0° is

$$\delta_2 = d + \delta_1 + t(e_2 - e_1).$$

The constants of the standard bar are supposed to be known, and the coefficient of expansion of the material of the unknown bar may be taken from tables, or it may be experimentally determined.

The relation between these various quantities is apparent from the following diagram (Fig. 9), in which M represents the true length of the meter at 0°, S the actual length of the standard at $t°$, and L the length of the bar to be compared. The signs in the diagram indicate the algebraic signs of the quantities for the case supposed.

In work of precision great care is necessary to ascertain exactly the temperatures of the bars. The comparator should be mounted in a constant-temperature room, and the bars adjusted the day

before the comparisons are made. Several thermometers whose bulbs lie upon the bars at various points may be used to show their temperatures.

In the comparison of long bars flexure may introduce serious error. For some work it is sufficient to assume that the surfaces of the supports are plane. If the bars are of the H form, the

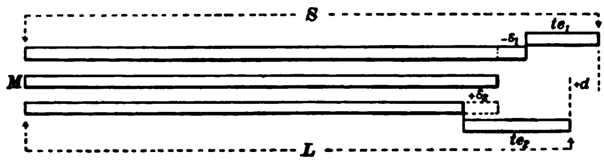


FIG. 9. COMPARISON OF LENGTHS

flexure effects are negligible. In other cases the bars should be supported upon two rollers placed at the neutral points, which for meter bars are 54 cm apart.

V. LENGTH WITH THE DIVIDING ENGINE

(a) Divide 10 cm in half-millimeter spaces, and an inch in twentieths and thirty-seconds.
(b) Make a 30-cm brass scale divided to single millimeters.
(c) Determine the pitch of the screw, and measure an unknown length.

23. The Dividing Engine. — A machine for ruling lines which shall divide a given length into any required number of subdivisions is a dividing engine. It usually consists of a screw of known pitch, *S* (Fig. 10), which moves a table, *T*, designed to carry the object to be graduated. A steel or diamond graver is supported by an adjustable mechanism, *G*, which permits the cutting of lines at right angles to the axis of the screw. An arrangement of disks and screws regulates the length of the stroke so that long and short marks may be made in such groups as are desired.

Suitable stops and ratchets on the divided head of the screw permit the latter to be turned repeatedly through any determined angle.

Having determined the style of graduation, the stops on the divided ratchet head are set to allow the screw to move the table forward the length of the smallest division by each motion of the crank. The disks and screws in the tool holder must

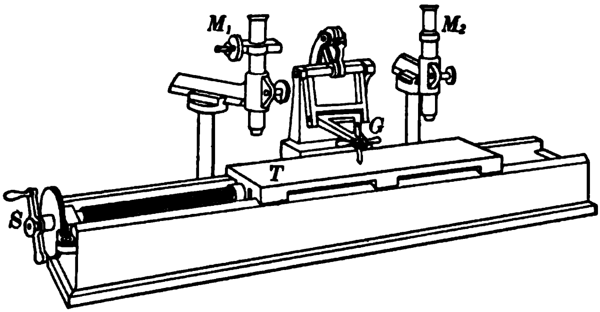


Fig. 10. Dividing Engine for Straight Scales

be properly set, and the blank scale secured to the bed with its edge parallel to the screw. A mark is made with the graver, and the scale is moved forward one division by a motion of the crank; this operation is repeated to the end of the scale.

Care must be taken to avoid all lost motion in the ratchet, and to turn it against the stops gently. An index is provided by which any error in turning the screw may be detected. The steel graver should be ground and set as shown in Fig. 11, the front surface of the cutting point being perpendicular to the scale. Often the piece to be graduated is coated with wax, in which the graduations are made, and afterwards etched with acid.

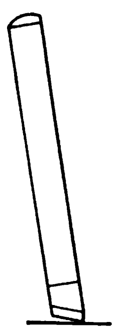


Fig. 11. Graver for Dividing Engine

24. Dividing-Engine Comparator. — The dividing engine is often provided with microscopes, M_1 and M_2 (Fig. 10), so that it may be used as a comparator, or for measuring lengths if the exact pitch of the screw is known. The pitch is determined by placing a standard length upon the table and observing the number

of turns of the screw necessary to move the table this length. Such measurements should be made for different parts of the screw, applying corrections for the temperature of the scale and the screw. The dividing engine is adapted to the calibration of thermometer and other tubes, and to the measurement of all small lengths which are too large for the micrometer microscope.

VI. CIRCULAR ARC WITH THE DIVIDING ENGINE

Graduate an arc of a circle in half degrees, and make a vernier reading to single minutes.

25. Dividing Circles. — In experimental work divided circles, arcs, or verniers are often required. The circle dividing engine is a machine for facilitating their production. It consists of a master circle, *C* (Fig. 12), arranged to be rotated by a worm gear, *G*. The worm is provided with a ratchet head having adjustable stops, to permit the turning of the circle any required amount. An adjustable tool carriage, *T*, holds the graver, and a microscope, *M*, attached to the bedplate serves as an index to the master circle.

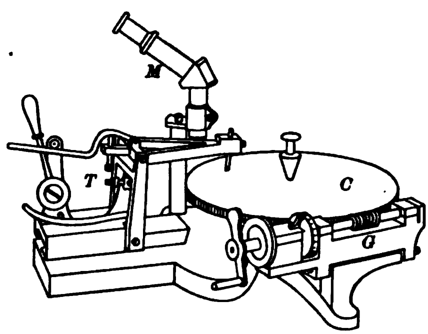


FIG. 12. DIVIDING ENGINE FOR CIRCLES

The blank circle is centered and clamped upon the master circle, the ratchet is set to give the required divisions, and the disks and screws regulating the motion of the graver are adjusted. The graver is set as explained in Art. 23. Make a mark with the graver, and then turn the worm; repeat the operation until the circle is graduated, watching the master circle to avoid mistakes.

For the vernier the ratchet stops are adjusted to make the divisions of the proper size, according to the principles of Art. 17.

VII. ERRORS OF A GRADUATED SCALE BY GAY-LUSSAC'S METHOD

(a) Determine the corrections to the ten-degree divisions of a thermometer scale from 0° to 100°.
(b) Determine the corrections to the decimeter divisions of a meter scale.

26. Calibration of Graduated Scales. — Before any graduated scale, such as a scale of lengths, a thermometer scale, or a divided circle, can be used as an instrument of precision, the errors of its subdivisions must be investigated and corresponding corrections must be applied to the observations made with the scale. The fundamental error in the total length of the scale, or of its principal length, must be determined by comparison with known standards; as by the method of Art. 22 for scales of length. The following method, due to Gay-Lussac, is the simplest which can be employed to determine the errors of the subdivisions of a scale, considered as equal parts of a known length. (See Art. 30.) Though the method is described as applied to the calibration of a scale in ten subdivisions, it may be readily adapted to any number of parts; however, when this number is large, more than ten for instance, the cumulative errors in the results may be serious. The probable errors in the positions of the lines increase from the ends toward the center of the scale.

27. Calibration with a Dividing Engine and One Microscope. — Let it be required to find the corrections to the *scale* of a thermometer reading from 0° to 100°, at each of the ten-degree points. (For the calibration of a thermometer, see Art. 129.) A dividing engine used as a comparator is a convenient form of measuring apparatus (Art. 24). The thermometer may be conveniently supported, parallel to the line of motion, in grooves cut in two large, flat corks placed upon the dividing-engine table. By means of the engine screw bring the zero line of the scale under the stationary index microscope. The stops being removed, the divided screw head, to which the crank is attached, may be turned backward to the zero of its index without turning the screw. Now turn the screw forward till the 10° mark is under the microscope, noting the number of

whole turns given to the screw. The direct reading of the screw head will give the fraction of a turn in excess of this number as it started from zero. With a screw of the usual pitch, the threads being 1 mm apart, this reading is the length of the space in millimeters. Without disturbing the thermometer, turn the screw head backward again to zero, and then forward till the 20° mark is under the microscope, noting the full turns and fractions of a turn, as before. Repeat this process for each of the ten spaces on the scale, from 0° to 100°. If the spaces are all of equal length within the limits of the errors of observation, the graduation errors are negligible. Otherwise the correction to each space is the quantity which must be algebraically added to it to make it equal to the average length of all the spaces. The corrections usually required are those to the spaces included between each of the several lines and the zero line of the scale. These are equal to the sums, taken successively, of the corrections to each of the ten-degree spaces. Thus the correction to the space between 0° and 30° is equal to the sum of the corrections to the spaces between 0° and 10°, between 10° and 20°, and between 20° and 30°.

The following is an example of this method.

Calibration of the Scale of Thermometer, Gerhardt 4248, at the Ten-Degree Points from 100° to 200°

May 7, 1898

Spaces	Observed Lengths	Corrections	Sums of Corrections
100°–110°	25.600 mm	− 0.116 mm	− 0.116 mm
110 –120	.615	− 0.131	− 0.247
120 –130	.420	+ 0.064	− 0.183
130 –140	.475	+ 0.009	− 0.174
140 –150	.405	+ 0.079	− 0.095
150 –160	.395	+ 0.089	− 0.006
160 –170	.470	+ 0.014	+ 0.008
170 –180	.480	+ 0.004	+ 0.012
180 –190	.505	− 0.021	− 0.009
190 –200	.475	+ 0.009	0.000
Average 25.484			

28. Calibration with a Dividing Engine and Two Microscopes. — The method just described is useful only for short scales and those in which great precision in the measurement of length is not required. The following method, when it can be applied, is more expeditious, and is more accurate, as it does not involve the cumulative errors of many turns of the dividing engine screw.

Two fixed index microscopes are placed a distance apart slightly less than the length to be calibrated. Designate the marks defining these various spaces by (0), (1), (2), etc. Adjust the scale on the dividing-engine table, parallel to the line of motion, so that mark (1) is exactly under the microscope farthest from the screw head, and mark (0) is nearly under the other microscope. Turn the screw head backward to the zero of its index, and then forward till the mark (0) is under the corresponding microscope. The fraction of a turn given to the screw measures the excess of the length of the space on the scale over the distance between the microscopes. Now move the scale on the table about the length of one subdivision till line (3) is exactly under one microscope and line (2) is nearly under the other. Turn the screw head backward to zero reading, and then forward till line (2) is in position; the amount the screw is turned measures the excess of the second space over the fixed length between the microscopes. Measure each subdivision of the scale in this manner. Each space is thus found to consist of an unknown constant length plus a known length which differs for the several spaces. The corrections to the scale parts are such quantities as, when algebraically added to the observed excess lengths, will make them, and therefore the separate spaces on the scale, all equal. The solution will be the same as that given above in the numerical example, except that instead of the total lengths of the subdivisions, the second column will contain the small excess lengths. Average these excess lengths, and the difference between the average and each observed length is the correction to the space. The corrections are to be summed successively as before.

29. Calibration with a Micrometer-Microscope Comparator. — When the highest precision of calibration is required a micrometer microscope is substituted for one of the index microscopes mentioned in the previous article, and the small excess lengths are measured with the micrometer screw instead of with the dividing-engine screw. The observations are thus given in terms of turns of the micrometer screw, which must be calibrated (Art. 21) and the readings, or the results obtained from them, reduced to terms of microns or centimeters. Instead of a dividing engine any other convenient form of comparator (Art. 22) for conveniently carrying the scale and microscopes may be used.

The following is an example of this method.

CALIBRATION OF SOCIÉTÉ GENEVOISE NICKEL METER No. 11

FOR THE DECIMETER SPACES

WITH COMPARATOR AND MICROMETER MICROSCOPE No. 3

May 13, 1898

SPACES cm	OBSERVATIONS Turns of Screw	CORRECTIONS Turns of Screw	CORRECTIONS μ 20.791 turns = 1 mm	Σ CORRECTIONS μ
0–10	23.256	+ 0.091	+ 4.4	+ 4.4
10–20	.391	− 0.044	− 2.1	+ 2.3
20–30	.347	0.000	0.0	+ 2.3
30–40	.348	+ 0.001	0.0	+ 2.3
40–50	.291	+ 0.056	+ 2.6	+ 4.9
50–60	.412	− 0.065	− 3.2	+ 1.7
60–70	.465	− 0.118	− 5.7	− 4.0
70–80	.145	+ 0.202	+ 9.7	+ 5.7
80–90	.416	− 0.069	− 3.3	+ 2.4
90–100	.396	− 0.049	− 2.4	0.0
Average 23.347				

VIII. ERRORS OF A GRADUATED SCALE BY PRECISION METHODS

Determine the corrections to the decimeter divisions of a meter scale.

30. Calibration of Scales by Precision Method. — A complete method for the calibration of a divided scale which determines the positions of all the lines of the scale with equal precision, while requiring a much larger number of observations than Gay-Lussac's method (Art. 27), is, however, usually to be preferred. The method, including a least-squares solution, is due to Hansen; while a much simpler solution of the same observations, which can in every case be substituted for the least-squares solution without lessening the precision of the result, has been devised by Neumann and Thiesen. This latter method is so useful and simple that, while no attempt will be made to explain the theory, a numerical example is given of the calibration of the Société Genevoise Nickel Meter No. 11, by means of which one can adapt the method to any problem in calibration that is likely to occur.

To determine the corrections to the decimeter subdivisions of a meter scale, the following observations are required. With the microscopes of the comparator set approximately 1 dm apart, each of the ten decimeter spaces is measured as described in Art. 29, the example there given being a part of the present example. With the microscopes approximately 2 dm apart, the spaces from (0) to (2), from (1) to (3), and so on to (8) to (10) are measured. With the microscopes approximately 3 dm apart, the spaces (0) to (3) . . . (7) to (10) are measured. The process is continued until, with the microscopes 9 dm apart, the observations are completed by measuring the spaces (0) to (9) and (1) to (10).

Table I contains these observations, which are fifty-four in number for a ten-space calibration. The first column in each group designates the space measured; the second column contains the readings expressed in thousandths of a turn of the micrometer-microscope screw, which represent the excesses in

the lengths of the several spaces over the fixed distance between the microscopes; the third column in each group contains the numbers that must be added to each number of column 2 to give the number next below it in the same column.

To obtain the corrections to the graduations from these observations, arrange a square form of ten rows and ten columns, Table II; place a row of ciphers in the diagonal from the upper left to the lower right corners; write the numbers in the third column of group 1 of Table I, in the diagonal next below the cipher diagonal, and write the same numbers, *with their signs changed*, in the diagonal just above it; write the numbers of the third column of group 2 of Table I in the next lower diagonal, and the same numbers, *with their signs changed*, in the

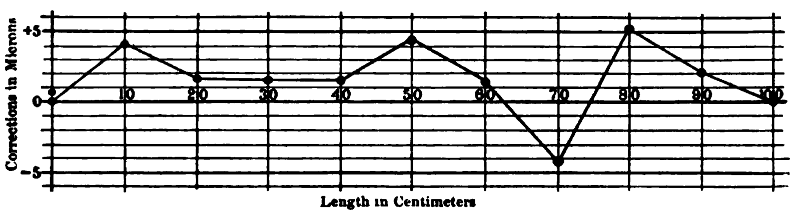


FIG. 13. CORRECTIONS TO GENEVA NICKEL METER No. 11

next upper diagonal; and so continue until the square form is filled. The algebraic sum of all the numbers in each column of the square, divided by ten, is the correction to the corresponding decimeter space, expressed in thousandths of a turn of the micrometer screw. These corrections are reduced to microns by dividing by 20.791, the number of turns of the micrometer screw per millimeter. The successive sums of these numbers are the corrections to the spaces 1, 2, 3, . . . decimeters as measured from the zero line; these are given in the bottom row of Table II. Fig. 13 is a graphic representation of these results.

The actual temperature of the scale during these observations is of no importance, but it is necessary that, during any one set of measurements, the temperature should not change. This is particularly important when the microscopes are far apart. The comparator should therefore be in a constant-temperature room.

Calibration of Société Genevoise Nickel Meter No. 11

COMPLETE METHOD FOR THE DECIMETER SPACES

With Comparator and Micrometer Microscope No. 3

May 13, 1898

Table I. Observations

Spaces in decimeters. Readings in thousandths of a turn of the screw. One turn = 20.791 mm

0–1	256	+135	0–2	304	+ 88	0–3	251	+ 89	0–4	160	+ 31	0–5	203	+147	0–6	107	+209	0–7	314	−113	0–8	147	+154	0–9	243	+131
1–2	391	− 44	1–3	392	− 44	1–4	340	−108	1–5	191	+ 9	1–6	350	+ 74	1–7	316	−251	1–8	201	+ 7	1–9	301	− 13	1–10	374	
2–3	347	+ 1	2–4	348	− 65	2–5	232	+ 54	2–6	200	+115	2–7	424	−198	2–8	65	+ 60	2–9	208	+ 50	2–10	288				
3–4	348	− 57	3–5	283	+ 65	3–6	286	+114	3–7	315	−198	3–8	226	+ 70	3–9	125	+ 45	3–10	258							
4–5	291	+121	4–6	348	+181	4–7	400	−130	4–8	117	+136	4–9	296	+110	4–10	170										
5–6	412	+ 53	5–7	529	−264	5–8	270	+ 4	5–9	253	− 27	5–10	406													
6–7	465	−320	6–8	265	− 52	6–9	274	− 89	6–10	226																
7–8	145	+271	7–9	213	+248	7–10	185																			
8–9	416	− 20	8–10	461																						
9–10	396																									

Table II. Calculation

Spaces Centimeters	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Observations Unit: thousandth of a turn of the screw. 1 turn = 20.791 mm	0	−135	− 88	− 89	− 31	−147	−209	+113	−154	−131
	+135	0	+ 44	+ 44	+108	− 9	− 74	+251	− 7	+ 13
	+ 88	− 44	0	− 1	+ 65	− 54	−115	+198	− 60	− 50
	+ 89	− 44	+ 1	0	+ 57	− 65	−114	+198	− 70	− 45
	+ 31	−108	− 65	− 57	0	−121	−181	+130	−136	−110
	+147	+ 9	+ 54	+ 65	+121	0	− 53	+264	− 4	+ 27
	+209	+ 74	+115	+114	+181	+ 53	0	+320	+ 52	+ 89
	−113	−251	−198	−198	−130	−264	−320	0	−271	−248
	+154	+ 7	+ 60	+ 70	+136	+ 4	− 52	+271	0	+ 20
	+131	− 13	+ 50	+ 45	+110	− 27	− 89	+248	− 20	0
Corrections	+ 87.1	− 50.5	− 2.7	− 0.7	+ 61.7	− 63.0	−120.7	+199.3	− 67.0	− 43.5
Corrections μ	+ 4.19	− 2.43	− 0.13	− 0.03	+ 2.97	− 3.03	− 5.81	+ 9.58	− 3.22	− 2.09
Σ Corrections μ	+ 4.19	+ 1.76	+ 1.63	+ 1.60	+ 4.57	+ 1.54	− 4.27	+ 5.81	+ 2.09	0.00

To eliminate, as well as may be, the unavoidable small progressive changes produced while observations are being made, the measurements of the several parts of the scale with one spacing of the microscopes should be made successively from one end to the other of the scale, and repeated in reverse order to the starting point. The average of these two sets will be free from progressive errors.

The correction to the total length of the scale must be determined by an independent comparison with a standard, as explained in Art. 22. See also Art. 129 for thermometer calibration. An error in the total length of a scale affects subdivisions in proportion to their lengths.

REFERENCE. — For a full explanation of this method of calibration, also for the calculation of the probable errors, and for references to the original memoirs, see *Guillaume*, Thermométrie, pp. 46–73.

IX. LENGTH WITH THE OPTICAL MICROMETER

Measure the thickness of two pieces of thin glass.

31. The Optical Micrometer. — A ray of light may be made to serve as a very long pointer without mass, for indicating small angular displacements. A mirror is attached to the body whose motion is to be measured, by which light is reflected from a fixed source to a suitable scale. The optical micrometer employs this device for measuring small lengths. It consists of a rod supported by four short legs, one at each end, and two, close together, so placed that the line joining them bisects at right angles the line joining the end legs. The instrument must stand upon a plane surface, such as a piece of thick plate glass. One of the end legs is adjustable in length, so that by the sense of touch the four supporting points may be brought into the same plane. This adjustment should be made with great care.

A reading telescope with a vertical scale is used for determining the deflection of the ray of light. Place the telescope at a distance from the micrometer of several meters, more or less, according to the degree of accuracy required, with its line of

sight in the direction of the length of the micrometer. Adjust the telescope, scale, and mirror, any one or all, so that the image of the scale, reflected by the mirror, is seen in the telescope (Art. 33). The scale should be made perpendicular to the line joining the mirror and that part of the scale seen in the telescope. Place the object to be measured under the two central legs of the micrometer; the instrument not being balanced about the line of the central legs, one end will be raised above the plane and the micrometer will stand upon three legs only. In this position read in the telescope the division of the scale which coincides with the cross wires. Next place a small weight upon the raised end of the micrometer, causing it to rest upon the plane, and again read the scale.

Let d (Fig. 14) represent the thickness of the object under the central legs, l the half length of the base, L the distance of the scale from the mirror, and n the difference between the scale readings. MC and MD represent the normal to the mirror in

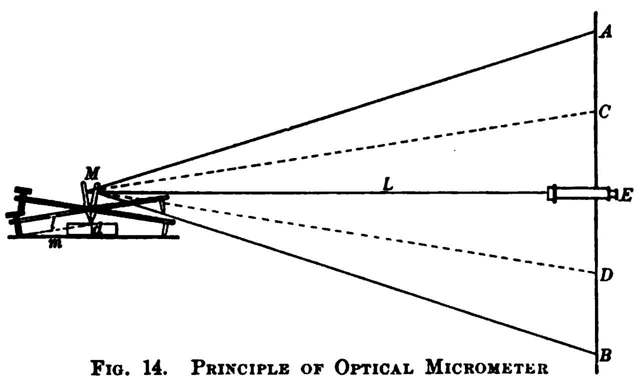


FIG. 14. PRINCIPLE OF OPTICAL MICROMETER

its two positions, and A and B the two divisions of the scale seen in the telescope. In the similar triangles, $d : m = CE : L$. But in any case in which this method is useful, m may be taken as equal to l without sensible error, and CE as equal to $\frac{AB}{4} = \frac{n}{4}$; therefore

$$d = \frac{nl}{4L}.$$

The length of the micrometer may be determined by measuring between impressions of its four points on a firm piece of paper.

REFERENCE. — When the thickness of the object is considerable as compared with the half length of the base, the complete trigonometric formula should be applied. See *Stewart and Gee*, Practical Physics, Vol. I, p. 54.

32. Three-Legged Optical Micrometer. — A modification of this method may be used in case the object to be measured is too small to permit both central legs to rest upon it. The instrument is allowed to stand upon its three fixed legs only, and the telescope reading taken while it rests upon the plane. The object is then placed under the end leg, and the reading again taken. Using the same notation as before,

$$d = \frac{nl}{2L}.$$

33. The Reading Telescope. — In experimental work the reading telescope is extensively used to give definite direction to a line of sight, or for magnifying a divided scale or a small displacement. A small mirror may be attached to the body to be observed, by which the image of a stationary scale is reflected to the telescope. Any rotation of the mirror results in an apparent displacement of the scale.

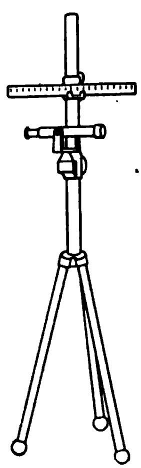


FIG. 15. READING TELESCOPE

Usually the telescope and scale are separately attached to the same support so that they may be set at various heights and adjusted as to direction. A convenient support is a tall, upright rod on a tripod base, as shown in Fig. 15. The telescope is often mounted on a short stand, which is then supported on a small tripod table of variable height. If possible, the mirror should be so turned that the scale when in position shall be well illuminated. To find quickly the positions for the telescope and scale, the observer, holding a piece of white paper in the hand, moves about in front of the mirror till the reflection is seen. Then, following the reflection with the eye, the motion is

continued till the paper and eye are in positions convenient for the scale and telescope respectively.

After locating the telescope and scale, first focus the sliding eyepiece so that the cross wires are distinct and then alter the focus of the telescope by the rack and pinion or the large draw tube till the scale is clearly seen. In searching for the scale, remember that it is optically twice as far away as the mirror, and the telescope must be focused accordingly.

Telescopes usually will focus only upon objects distant a meter or more. A subsidiary convex lens may be placed over the object glass to facilitate the focusing on nearer objects and to increase the magnifying power; the telescope then becomes in effect a low-power microscope.

X. CONSTANTS OF A LEVEL WITH THE LEVEL TRIER

(a) Determine the value of one division of a level, in seconds of arc, for various parts of the scale. Find the radius of curvature of the level.
(b) Calculate the value of one turn of the screw of the level trier.

34. The Spirit Level. — A very sensitive instrument for measuring minute angles, particularly the deviation of lines from the true horizontal, is the spirit level. Its essential part is a closed glass tube nearly filled with a mixture of ether and alcohol. This tube has its inner surface, on the upper side, accurately ground so that a longitudinal section (Fig. 16) would show an arc of a circle of large radius. The longer this radius the more sensitive the level. If the tube is inclined through a small angle, the bubble will rise to the highest part of the curve in the vial, and by the change in position will measure the angle. The tube is held in a suitable frame, which carries a graduated scale for indicating the position of the bubble. Changes in temperature will vary the length of the bubble, and a fine level may have a chamber at one end which permits the length of the bubble to be altered.

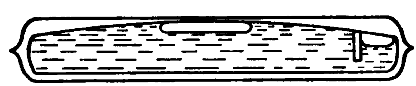


FIG. 16. CHAMBERED LEVEL VIAL

To eliminate errors due to improper adjustments, a level should be read in direct and reversed positions. Half the difference between the readings is the inclination of the line. If the surface is altered till the readings in the two positions are the same, the surface is level, and if the level is then made to read zero, it is in adjustment. With sensitive levels this is difficult to accomplish, and the two readings should always be taken. The level should be at a uniform temperature, as the bubble tends to move toward the warmest portion of the vial.

For any particular measurement, a level of suitable sensitiveness should be chosen. It is therefore necessary to determine the inclination corresponding to a displacement of the bubble by one division, and to determine that this relation is constant throughout the scale. This measurement is made by means of the level trier, which consists of a horizontal plate, arranged to carry all forms of levels, pivoted at one end, and movable through very small known arcs by means of a screw placed at the other end. The level having been adjusted in position (Fig. 17), the greater part of the weight is taken from the point of the screw by the counterpoise, and a series of observations is made to determine how much the screw must be turned to move the bubble through a certain number of level divisions. Do not attempt to set the bubble on a particular mark, but read the positions of both its ends, and consider the mean the level reading. Turn the screw to move the bubble approximately ten scale divisions, and record the positions of both screw and bubble; move the bubble another ten divisions, and continue until the limit of the scale is reached. Reverse the level in the Y's, and repeat the observations. The final motion of the screw in making settings should always be in a direction to lift the arm carrying the level, as this will diminish the errors due to lost motion.

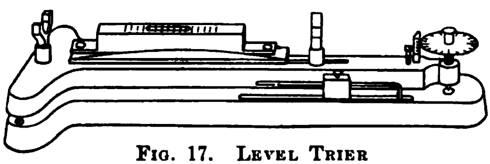


Fig. 17. Level Trier

Compute the value of one division of the level, in seconds of arc, from each observation. If there is no indication of

irregularity, the mean of all may be taken as the value of one division. Usually the head of the screw is so graduated that it indicates seconds of arc directly; if this is not so, the value of the screw may be determined as described below.

If the scale is l centimeters long and has n divisions, and if a is the value in seconds of one division, then the length of one second of arc of the level vial is $\frac{l}{an}$, and the radius of the level is

$$R = \frac{206265\, l}{an},$$

206265 being the number of seconds in a radian.

If s is the distance between the threads of the screw, and L the length of the arm, the value of one turn of the screw, in seconds of arc, is

$$t = \frac{206265\, s}{L}.$$

The length of the arm may be obtained from direct measurements of the triangle formed by the point of the screw, and the two pivots, or by measuring the impressions of these points on paper.

The nominal value of the pitch of the screw, known from its construction, is usually sufficiently accurate for use in the above formula. If not, it may be determined in the following manner. Having noted the reading of the level, a small, thick piece of glass, the thickness of which has been measured with the micrometer caliper, is placed under the screw, and the level reading is restored by turning the screw. The thickness divided by the number of turns is the required distance between the threads. The pitch may be determined by direct measurement, as described in Art. 24, the screw and nut being taken from the level trier and arranged to move either the microscope or the standard scale of the dividing-engine comparator.

The angular motion produced by the screw may also be measured by a ray of light reflected by a mirror attached to the

level trier. Calculate the angle from the measured deflection caused, at a known distance, by a certain number of turns of the screw. If the incident and reflected rays and the arm of the level trier are in the same plane, the ray will be deflected through twice the required angle.

XI. AREAS WITH THE PLANIMETER

Determine the area of the datum circle in square centimeters, and measure two large and two small areas. Measure the small areas in square inches. Measure the area, in square feet, of a figure drawn to scale.

35. The Planimeter. — A mechanical integrating machine termed the planimeter may be used to measure directly the area of a plane surface. It is employed in engineering work for measuring the areas of cross sections, the contour lines of rivers and ponds, drainage areas, displacement areas of vessels, indicator diagrams, etc. The polar planimeter (Fig. 18) consists of two arms hinged together, the outer end of one being pivoted at P in the plane of the figure, the end of the other arm carrying

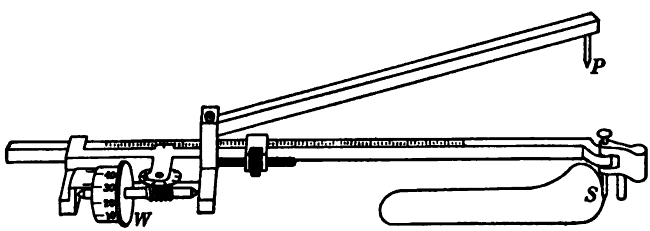


FIG. 18. PLANIMETER

a style, S, is caused to trace the periphery. A graduated wheel and disk, W, partly rolls and partly slides over the surface, and by means of a counting mechanism, indicates the area, the readings usually being given by means of the vernier to one tenth of a square centimeter. The instrument may be set to read in various systems of units and to different scales by varying the length of the arm WS.

When the angle between the two arms is such that the line from the pole to the wheel contact, PW (Fig. 19), is perpendicular to the axis of the wheel, motion of the whole instrument about the pole will produce no rotation of the wheel and hence no record. The style will trace a circle of radius PS, called the *datum circle*, the area of which may be found as explained later, and recorded as a constant of the instrument. If the distance from the pole P to the joint J is a, and from the joint to the style S is b, and from the joint to the wheel contact is c, then the radius PS is

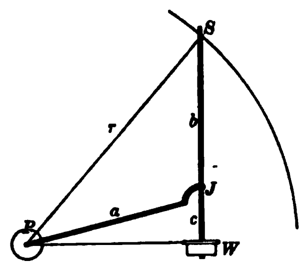


FIG. 19. PLANIMETER TRACING DATUM CIRCLE

$$r_0 = \sqrt{a^2 + b^2 + 2\,bc}.$$

When the style is outside the datum circle and is moved around the pole in a clockwise direction, the wheel gives positive readings, while motion in the same direction inside the circle gives reversed rotation to the wheel.

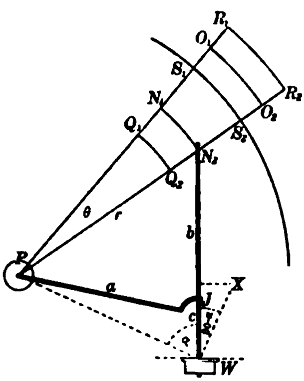


FIG. 20. MEASUREMENT OF AREAS

To explain the theory of the planimeter an element of area will be assumed which is bounded by an arc of the datum circle, $S_1S_2 = \theta$ (Fig. 20), by the arc of a concentric circle, N_2N_1, of radius r, and by two radial lines, N_1S_1 and N_2S_2.

As already noted, tracing S_1S_2 will produce no indication. Tracing S_2N_2 will cause some positive rotation, while tracing N_1S_1 will cause an equal contrary rotation. Therefore when the style traces the entire periphery, $S_1S_2N_2N_1S_1$, the final reading will be due to rotation produced while tracing the arc N_2N_1 only. It is to be shown that this is proportional to the inclosed area.

When the arc $N_2N_1 = \theta$ is traced, W revolves around P in the direction WX through an equal arc of length $PW\theta$. If a is the

angle between PW and the axle, the axle makes the angle $90° - a$ with the direction of motion. This motion may be resolved into the component $PW\theta \cos (90° - a)$ in the direction of the axle, producing only sliding, and

$$PW\theta \sin (90° - a) = PW\theta \cos a,$$

perpendicular to the axle, which turns the wheel.

In the triangles PWN_2 and PWJ,

$$r^2 = (b + c)^2 + PW^2 - 2(b + c) PW \cos a,$$

and $$a^2 = c^2 + PW^2 - 2cPW \cos a.$$

$$\therefore PW \cos a = \frac{1}{2b}(a^2 + b^2 + 2bc - r^2) = \frac{1}{2b}(r_0^2 - r^2),$$

and the turning of the wheel equals $\frac{1}{2b}(r_0^2 - r^2)\theta$.

$$\begin{aligned} \text{Area } S_1S_2N_2N_1 &= \text{sector } PS_1S_2 - \text{sector } PN_1N_2 \\ &= \tfrac{1}{2}(r_0^2 - r^2)\theta. \end{aligned}$$

This is b times the distance traveled by the roller; b is constant, therefore the area is proportional to the travel of the roller.

If the element of area lies outside of the datum circle, as $R_1R_2S_2S_1$, similar reasoning will show that when the style has traced the lines R_2S_2, S_2S_1, and S_1R_1, the wheel will indicate no change, but that when the line R_1R_2 is traced, the wheel will travel a distance proportional to the inclosed area.

Let the element of area be $R_2N_2N_1R_1$, partly within and partly without the datum circle. Tracings on the radial lines will neutralize each other; the motion (counter-clockwise around the pole) while tracing N_2N_1 will indicate positively the area within the circle, and the tracing of R_1R_2 will add to this the area of the part without the circle; the final reading will therefore indicate the total area.

If the element of area is $R_1R_2O_2O_1$, entirely without the datum circle, tracing R_1R_2 will give a positive reading for the area between this line and the datum circle, while the tracing of O_2O_1 will give a negative reading for the area $O_1O_2S_2S_1$, and

the final result is the area of $R_1R_2O_2O_1$. Similarly tracing the element $N_1N_2Q_2Q_1$ will give the difference between $Q_2Q_1S_1S_2$ and $N_2N_1S_1S_2$, or the area $Q_2Q_1N_1N_2$.

Since any area which does not inclose the pole may be represented by the sum of elements such as have been considered, it follows that the tracing of the periphery by the style moving around it in a clockwise direction will cause the wheel to indicate directly the area inclosed.

If the area boundary incloses the entire datum circle and the pole, it will be made up of elementary lines such as R_1R_2; the tracing of this line entirely around the circumference will cause the wheel to indicate, in the manner explained, only the area between the boundary and the datum circle. In this case, therefore, the wheel indication must be added to the known area of the datum circle.

When the boundary incloses the pole and lies within the datum circle, the wheel gives a negative indication (for clockwise motion of the style) which is the area between the boundary and the circle; hence the desired area, being the difference between that of the circle and the ring, is equal to the algebraic sum of the circle area and the wheel indication. The same rule naturally applies if the boundary inclosing the pole is partly inside and partly outside the circle.

To find the Area of the Datum Circle.— In a strip of cardboard pierce two needle holes at a measured distance apart. Fasten the pole through one hole, and by resting the style in the other a circle of known area may be readily traced. The wheel reading is the area to be subtracted algebraically from this known circle to give the area of the datum circle. It is best to trace two known circles, one a little smaller than the datum circle, and one larger.

General Rule for measuring Areas. — If possible place the pole outside the area; trace the periphery with a clockwise motion of the style; the wheel reading is the area. When the pole is inside the area trace the boundary with a clockwise motion of the style; the wheel indication, attention being given to the sign, algebraically added to the area of the datum circle is the required area.

If in the last case a trial shows a negative turning of the wheel, it may be convenient to trace the boundary in a counter-clockwise direction when the direct reading of the wheel is the area to be subtracted from the datum circle.

The setting of the wheel is so easily accomplished that it is usually best to start with the index at 0. Use a straightedge to facilitate the tracing of straight lines. Large areas may be subdivided and measured in parts.

The details of setting the instrument to various scales will be obvious from the marks and figures engraved on the arm, or from the maker's instructions. The setting for a particular scale may be found or tested by trial measurements of a known area, such as a rectangle, or a circular groove in a metal plate.

36. Mensuration of Areas.—For figures of regular shape the following formulæ may be used, the symbols requiring no explanation.

Triangle $= \text{base} \times \frac{1}{2}$ altitude.
Circle $= \pi r^2$.
Parabola $= \text{base} \times \frac{2}{3}$ height.
Ellipse $= \pi ab$.
Cylinder $= 2\ \pi rl + 2\ \pi r^2$.
Cone $= \pi r^2 + 2\ \pi r \times \frac{1}{2}$ slant height.
Sphere $= 4\ \pi r^2$.

For irregular plane figures the curved boundary may be drawn upon cross-section paper, and the number of squares inclosed counted, estimating fractions of squares; or the figure may be cut out of a sheet of paper or metal of uniform thickness, and its weight compared with the weight of a piece of the same material whose area is known.

XII. ANGULAR DISTANCE WITH THE SEXTANT

Determine the angle between two distant points from two slightly different stations. Determine the angular elevation of a distant object.

37. The Sextant.—The angle in any plane between two points may be very conveniently measured, with considerable precision,

by means of the sextant. It is a small, light-weight instrument, with which the angle between the two objects is measured not by pointing first at one and then at the other but by observing both at the same time, the objects being brought into apparent coincidence by mirrors. The adjustment for coincidence can be made while the instrument is held in the hand, and even when the observer is in motion, as on board ship. This portability makes the instrument very useful in the laboratory and field as well as at sea.

The sextant consists of an arc of a graduated circle of about 60°, at the center of which is pivoted an index arm with a vernier, *V* (Fig. 21), carrying the index mirror *M*. Attached to the frame of the sextant is a small telescope, *T*, and the horizon glass, *H*, both fixed in position. The horizon glass is half silvered and half transparent, so that upon looking through the telescope one sees objects in the direction *HB*, and also, by reflection from the mirrors *M* and *H*, objects in the direction *MA*.

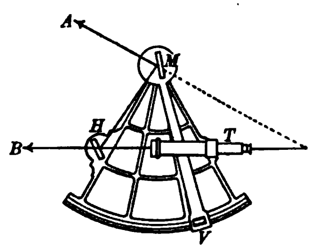


FIG. 21. SEXTANT

If the two mirrors are parallel the two lines of sight are parallel, while if one mirror is turned through any angle the lines of sight are inclined at twice this angle; hence every half-degree division on the arc is numbered as a whole degree, and the readings give directly the required angle.

Colored shade glasses are provided to reduce the intensity of one or the other of the beams of light entering the telescope, in order that both objects may be seen equally well.

The parts of a sextant, properly constructed, should fulfill the following conditions: the index glass and horizon glass should be perpendicular to the plane of the circle, the axis of the telescope should be parallel to this plane, and when the mirrors are parallel the vernier should read 0°. Sextants are provided with means for making these adjustments, and the following methods may be used to verify them; but having been once properly made they should not need altering.

Set the index at about 100°, and place the eye close to the index glass so that part of the divided circle is seen reflected by the glass and part by direct vision. These two portions of the arc should appear to be in the same plane. If they do not so appear, alter the inclination of the mirror till the condition is satisfied; then the index mirror is perpendicular to the plane of the arc.

Point the telescope at a small, distant object, and move the index arm till the reflected image of the same object comes into the field of view. If the horizon glass is perpendicular to the plane of the arc, the two images can be made to coincide by moving the index arm; if not, alter the inclination of the horizon glass till this is possible.

That the axis of the telescope is parallel to the plane of the sextant is sufficiently tested as follows. Support the sextant in a horizontal position, and place two sights on the ends of the divided arc. The sights should be of the same height and such that their tops are in a plane with the axis of the telescope, parallel to the plane of the arc. Observe a distant point in line with the sights, which should also be visible in the center of the field of the telescope. The method of correcting, if necessary, will be obvious from the construction of the instrument.

To test the fourth condition, point the telescope to a distant object and turn the index arm till the reflected image coincides with the direct one, as for the second correction. The two mirrors are now parallel and the vernier should read 0°. If it does not, the construction may permit the correction of the error. Usually it is better to determine the amount of this index error, and to apply to all angle readings the index correction, which is the vernier reading for the above setting with its sign changed.

When altitudes above the horizon are to be measured, the observer, if at sea, points the telescope to the visible horizon and brings the image of the object tangent to this line by moving the index arm; but if the observer is on land the true horizon can rarely be seen, and an artificial horizon consisting of a shallow basin of mercury is used. Such a position is taken that the image of the sun, for instance, may be seen reflected from the mercury,

and the angle between the apparent direction of the image and of the sun is measured, which is twice the altitude of the sun.

Measure the angle between two selected points distant half a mile or more, as seen from a given point. Make three settings to determine the index correction; the sum of the means of the two determinations is the required angle.

Move ten feet towards or from the objects (a half mile distant) and repeat the measurements.

Using an artificial horizon, measure the altitude of an assigned point.

REFERENCES. — *Chauvenet*, Practical Astronomy, Vol. II, pp. 92–118; *Johnson*, Surveying, pp. 108–112.

XIII. LAWS OF THE EQUILIBRIUM OF FORCES BY THE TRIANGLE OF FORCES

Prove the laws for the equilibrium of three forces about a point, using various combinations of forces.

38. Equilibrium of Forces. — The relation between three forces which, acting simultaneously at a point, result in equilibrium is of great importance in engineering work. By the polygon law, such forces are in equilibrium if they are proportional to the sides of a triangle, the sides taken in order having directions parallel to the forces. Or the forces are in equilibrium if each one is equal and opposite to the resultant of the other two.

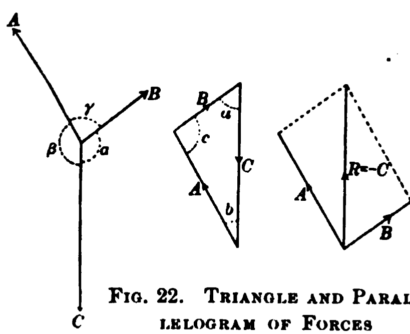


FIG. 22. TRIANGLE AND PARALLELOGRAM OF FORCES

These relations are illustrated by Fig. 22, in which A, B, and C represent the three forces, and R the resultant of A and B, which is equal to $-C$. In a triangle the sides are proportional to the sines of the opposite angles. It is evident that

the angles a, b, and c are the supplements respectively of α, β, and γ, the angles between the forces as originally applied to the point. Hence if these forces applied to a point are in equilibrium, they are proportional to the sines of the opposite angles; that is

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}.$$

This relation may be proved as follows.

Arrange apparatus consisting of ball-bearing pulleys, cords, and weights in the manner shown in Fig. 23, preferably in front of a blackboard. The friction of the pulleys and the stiffness of the cords may cause the position of equilibrium to be a little uncertain, but by moving the system out of equilibrium and allowing it to return in various directions, a mean position can be found which will be sufficiently exact. On the blackboard draw lines parallel to the three cords, and with a large protractor measure the three angles, α, β, γ.

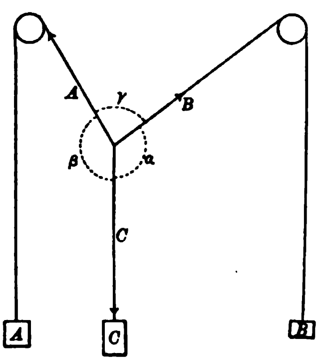


Fig. 23. Equilibrium of Forces

Use different combinations of weights, varying their relations as much as possible, and record the results in the following tabular form.

Equilibrium of Forces

December 20, 1901

Series	Forces, grams			Angles, degrees			Ratios		
No.	A	B	C	α	β	γ	$\frac{A}{\sin \alpha}$	$\frac{B}{\sin \beta}$	$\frac{C}{\sin \gamma}$
1	59.8	69.8	102.8	146	139	75	106.5	106.9	106.4
2									
3									
4									
5									

XIV. CONSERVATION OF MOMENTUM WITH THE BALLISTIC PENDULUM

Prove the conservation of momentum by the impact of elastic bodies, a larger body striking a smaller one, and also the smaller striking the larger; find the coefficient of restitution in each case. Prove the principle for the impact of inelastic bodies. Compare the kinetic energies before and after impact.

39. Momentum and Impact. — The changes in motion impressed upon bodies by their impact is in general a complicated problem, but the study of several simple cases gives valuable illustrations of Newton's Second and Third Laws of Motion. The cases of impact to be considered are those of bodies whose centers of mass may move only in the same line, and for which the point of contact is also in this line.

By the Second Law of Motion any change in momentum is the result of the action of a force; if the only forces acting are those due to impact, then, since by the Third Law action and reaction are equal and opposite, regard being had to algebraic sign, the quantity of motion is unchanged by impact. In other words, the sum of the momenta before impact is equal to the sum of the momenta after impact; this is the principle of the Conservation of Momentum.

Perfectly elastic bodies after collision would separate with the same velocity as that with which they approach; inelastic bodies would not separate at all. Actual bodies lie between these two extremes, and the ratio of the relative velocity after collision to the relative velocity before is the *coefficient of restitution.*

The fact of imperfect restitution is not contrary to the Conservation of Momentum nor to the Third Law of Motion; it may diminish the action, but the reaction is diminished by the same amount, always remaining equal to the action; thus the change in momentum of the system is zero.

The usual result of imperfect restitution is a change in the kinetic energy of the system at the time of impact, this energy causing vibration, change of shape, etc., and being largely dissipated in the form of heat.

The conditions of collision above mentioned are very approximately secured by using bodies of spherical shape supported from parallel axes by bifilar suspensions, so that when they are at rest the spheres are in contact without pressure, and their centers are in the same horizontal line in the plane of vibration.

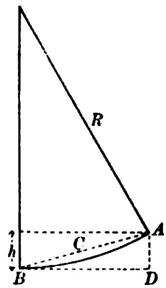


FIG. 24. RELATION OF FALL AND AMPLITUDE OF PENDULUM

One body may be cylindrical, when four cords from the same axis are required for its proper support. These cords may be about a meter and a half long, and a graduated arc of 40° is suitable for measuring the swings of the pendulums. The separation of the two axes will slightly displace them from the center of the arc, but the errors resulting are inappreciable. Pointers on the under sides of the balls may move paper riders placed on the edge of the arc to indicate the extreme points of motion.

If a sphere under the influence of gravity swings through an arc AB (Fig. 24), it will have acquired at B a velocity the same as if it had fallen vertically from A to D, that is, through the height h, and its velocity will be $\sqrt{2gh}$, or it is proportional to $\sqrt{h}$. By geometry, $h = \frac{C^2}{2R}$, C being the chord of the arc and R its radius; hence, since $\sqrt{h}$ is proportional to C, the velocity is proportional to the chord of the arc through which the ball swings. But further, for arcs not exceeding 20°, since the length of the chord of 20° is 0.348 and of the arc 0.349, the velocities may in this exercise be taken as proportional to the arcs for the purposes of comparing momenta and energies.

Elastic Bodies.—Suspend two steel balls of about 3.5 cm and 5 cm diameter, so that the line joining the centers, the balls being at rest and in contact without pressure, is horizontal and in the plane of vibration. Withdraw the smaller ball to the end of the arc and allow it to fall through a measured arc, a_1 (Fig. 25), against the larger ball at rest. After collision the

large ball will move forward through an arc, a_2, while the small ball rebounds through the arc a_3. The momenta are proportional to the products of the masses M and m into the respective arcs; and it is to be shown, by allowing m to fall from different heights, that in each case

$$ma_1 = Ma_2 - ma_3.$$

The coefficient of restitution is

$$\frac{a_2 + a_3}{a_1}.$$

Calculate the relations of the kinetic energies before and after collision. In this particular case the energy before collision is proportional to ma_1^2, and after collision to $Ma_2^2 + ma_3^2$.

Make similar observations for the momenta, kinetic energies, and the restitution when the large ball is allowed to strike the small one.

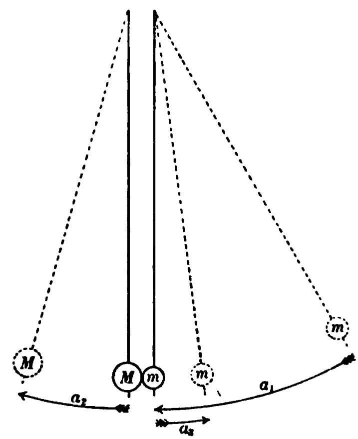


FIG. 25. COLLISION PENDULUMS

Inelastic Bodies. — A brass sphere and a lead cylinder serve for these experiments, being made inelastic in effect by placing a piece of soft wax at the point of contact. They are to be supported as before, so that when the two hang at rest they are in contact without pressure, and the line joining the centers of mass is horizontal and in the plane of vibration. Allow the sphere to strike the cylinder, falling from various heights, and calculate the relations between the momenta before and after impact, and also between the kinetic energies.

XV. MASS BY THE EQUILIBRIUM OF MOMENTS

Determine an unknown mass with a balanced bar; determine the weight of the bar, using known masses; determine an unknown mass, the bar not balanced.

40. Equilibrium of Moments. — A rigid body free to rotate about a fixed point will be in equilibrium only when the sum of the moments of the forces acting upon it is zero; that is, when the sum of the moments tending to produce rotation in one direction is equal to the sum of the moments tending to produce rotation in the opposite direction.

A sliding knife-edge is provided to support a meter stick from a stirrup attached to a suitable stand, as shown in Fig. 26. Slide the bar in its holder till it rests in a horizontal position.

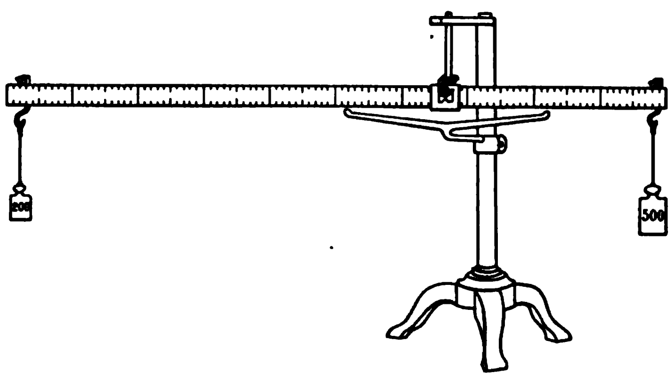


FIG. 26. EQUILIBRIUM OF MOMENTS

By means of knife-edge hooks suspend a known mass near one end and an unknown mass near the other. Alter the position of the hooks to restore equilibrium, keeping them as near the ends as possible. Calculate the value of the unknown mass. The knife-edge support being balanced, its mass need not be considered; but the hangers must be considered parts of the suspended masses.

Suspend masses of 200 g and 500 g 98 cm apart (Fig. 26), and shift the bar in its support until the moments are again in

equilibrium. The three moments, due to the suspended masses and the mass of the bar, are now in equilibrium. With the aid of the principle that the weight of a body may be considered as concentrated at its center of gravity, the mass of the bar is to be calculated.

Substitute an unknown mass for one of the known masses, and shift the point of support to secure equilibrium; determine the unknown mass, taking into account the weight of the unbalanced bar.

Verify the results by weighing the unknown masses on a balance.

XVI. MASS WITH THE BALANCE BY THE METHOD OF VIBRATIONS AND DOUBLE WEIGHING

Determine an unknown mass by vibrations. Determine a mass by double weighing, and find the ratio of the arms of the balance.

41. The Balance. — Perhaps the most generally useful and the most precise of scientific measuring instruments is the beam balance, with which masses are compared by the principle of the equilibrium of moments.

The balance consists of a double lever or *beam*, B (Fig. 27), supported in a horizontal position upon a knife-edge through the center; near the ends of the beam are knife-edges from which are suspended pans, P, in which can be placed the masses to be compared. The distances from the center knife-edge to the end knife-edges are the *arms* of the balance. A pointer attached to the beam and moving over a scale, S, indicates rotational displacements of the beam. An *arrest* is provided which removes the beam and pan supports from the knife-edges, and in addition it often firmly supports the pans from beneath. It is usually operated by a milled head or lever at A. A system of lever rods, R, is provided for placing small weights called *riders* upon the beam at any point of its length, and for lifting them when they are not needed. A case to protect the balance from air disturbances is necessary. There

are numerous adjusting devices and conveniences, which need not be described.

The theoretical conditions which a balance should fulfill to secure sensitiveness and precision are the following: the three knife-edges should be in the same plane and parallel to each other; the beam should be inflexible; the arms should be of equal length; the center of gravity should be below the central knife-edge, and as near to it as possible. That the balance may be operated rapidly the beam should be short, and its center of gravity far from its support. Some of these requirements are practically inconsistent with each other, and one or another may be sacrificed according to the use for which the balance is desired.

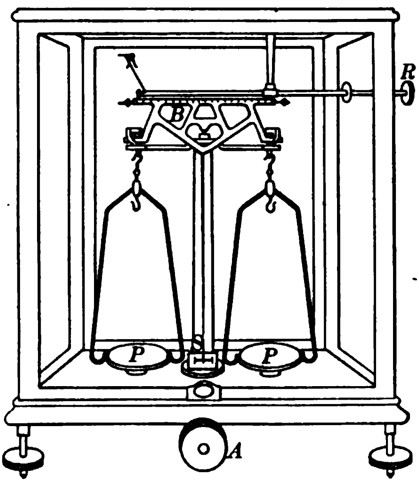


Fig. 27. The Balance

The highest sensibility and precision are attained in balances with long and heavy beams. Such balances indicate variations of 1 part in 500 000000 in the maximum load of 500 g or 1000 g. The period of vibration is very long, being as much as 60 seconds in some instances. A balance which will indicate a variation of 1 part in 1 000000 for the maximum load of 200 g is usually amply sensitive. Such balances may be made with short and light beams and with a period of vibration of 10 seconds.

If a perfect balance were used, a theoretical weighing would consist simply in placing the unknown mass in one pan and known masses or weights in the other pan, and in adjusting the latter to bring the balance to a position of rest exactly the same as its position of rest when no loads are in the pans; but in practice it would be very difficult or impossible to

secure these conditions. In general it is easier to make a precise determination of the difference between two quantities than to secure their equality. The practical method of weighing is, then, to select two groups of weights differing by a very small quantity, as for instance by 1 mg, such that one weight is smaller than the unknown mass and the other larger; and after comparing each of these with the unknown mass, to find the true value of the latter by interpolation. The practice of this method will be developed in the following articles.

42. Weighing by Vibrations. — A concrete example will aid in explaining the method of weighing by vibrations. Let it be required to determine the apparent weight of a brass cylinder, to the nearest tenth of a milligram, the smallest weight used being one milligram.

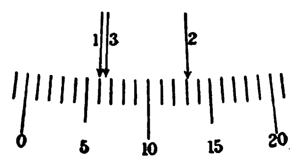


FIG. 28. SCALE OF BALANCE

The scale of the balance is usually graduated as shown in Fig. 28. The figures are often omitted, which fact, however, will not interfere with making the readings described. Allow the pointer to swing two or three divisions each side of the center. Determine the *zero point* by reading one swing to the left, one to the right, and a second one to the left, estimating tenths of a division; or the readings may begin and end with a swing to the right. Suppose the reading, indicated in the figure, as the pointer swings to the left, is 6.0; then that it swings to the right to 13.1, and back to the left to 6.6. The average of the two left readings is 6.3, and the sum of the average left swing and the right swing, that is the *sum of the excursions*, is 19.4. The resting point is half the sum of the excursions; but it is shorter and somewhat more precise to use the sums of the excursions instead of the equilibrium points in calculating the weights. Record the observations on one line as follows:

Zero point	6.0	13.1	6.6	19.4

When beginning work with the balance it will be well to determine the zero point three times, arresting the beam after

each determination. The proper agreement of the results will indicate that the balance is in good condition.

Now place the brass cylinder in the left pan, and weights to balance it as nearly as can be estimated, 93.217 g, in the right pan. Allow the balance to swing, and read three excursions as before, — to the right 10.4, to the left 8.6, and again to the right 10.2. Record thus:

Cylinder left	93.217 g right	10.4	8.6	10.2	18.8

The weight in the right pan is evidently too great, as indicated by the sum of the excursions being smaller than for the zero point. Remove one milligram and observe another set of vibrations:

	93.216 g right	13.0	7.9	12.7	20.7

Now the weight in the pan is too small, and the apparent weight of the cylinder lies between the two amounts tried. From these observations the exact weight can be calculated. The observations show that the removal of 1 mg caused a change in the sum of the excursions of (20.7 − 18.8) 1.9 divisions, while the change desired was (19.4 − 18.8) 0.6 divisions. Therefore, if the weight removed had been $\frac{6}{19}$ mg, the sum of the excursions would have been the same as for the zero point. This shows that a weight of (93.217 g − $\frac{6}{19}$ mg) 93.2166 g would exactly balance the cylinder.

Collecting the observations, the whole record for a complete single weighing by vibrations is as follows:

Mass of Cylinder No. 5

METHOD OF SINGLE WEIGHING BY VIBRATIONS

Balance No. 7, Weights No. 1

May 21, 1895

Zero point		6.0	13.1	6.6	19.4
Cylinder left	93.217 g right	10.4	8.6	10.2	18.8
	93.216 g right	13.0	7.9	12.7	20.7

Cylinder in left pan = 93.217 g − $\frac{6}{19}$ mg = 93.2166 g.

The assumptions made in this article, that the arms of the balance are of equal length, that the density of the weights and

of the body weighed are equal, and that the weights are perfectly adjusted, are never realized. The method must be extended to suit these circumstances, as explained in the next article and in Arts. 47, 48, and 49.

43. Double Weighing and Ratio of Balance Arms. — The method of double weighing described below theoretically gives the true apparent mass of a body by the elimination of the effects of inequality of the balance arms. It assumes that the position of equilibrium of the loaded balance is the same as that of the unloaded balance. Since this assumption is not justified, the method is not trustworthy for the determination of the true mass. The method of weighing by reversal, described in Art. 47, is always to be preferred for this purpose, as it is precise in practice and requires fewer observations.

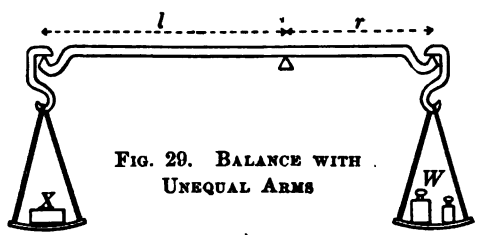


FIG. 29. BALANCE WITH UNEQUAL ARMS

The method of this article is the only one possible for finding the ratio of the balance arms, and it must therefore be used in investigating the constants of a balance.

Let a body be weighed by vibrations, as described in the preceding article, with a balance whose arms are unequal; let the length of the left arm be represented by l, and of the right by r (Fig. 29); let X be the unknown mass in the left pan, and W_1 the weight that balances it. The body is transferred to the right pan and a second complete weighing is made by vibrations; let W_2 be the weight balancing it in this position. Then from the principle of the equilibrium of moments the following equations are true.

$$Xl = W_1 r, \quad Xr = W_2 l.$$

The product of these equations is $X^2 lr = W_1 W_2 lr$; therefore

$$X = \sqrt{W_1 W_2},$$

which is the weight of the body independent of the lengths of the arms. Instead of the geometrical mean, the arithmetical mean,

$$X = \frac{W_1 + W_2}{2},$$

is sufficiently precise for a balance in which the arms are made as nearly equal as possible.

Dividing one of the equations for the moments by the other,

$$\frac{l}{r} = \sqrt{\frac{W_1}{W_2}},$$

which gives the ratio of the arms. When the arms are of nearly equal length their ratio is more easily computed, and with sufficient accuracy, by the approximate formula,

$$\frac{l}{r} = 1 + \frac{W_1 - W_2}{2\,W_2}.$$

A numerical illustration follows.

Mass of Cylinder No. 5 and Ratio of Balance Arms

Method of Double Weighing by Vibrations

Balance No. 7, Weights No. 1

May 21, 1895

Zero point		6.0	13.1	6.6	19.4
Cylinder left	93.217 g right	10.4	8.6	10.2	18.8
	93.216 g right	13.0	7.9	12.7	20.7

Cylinder in left pan = 93.217 g − $\frac{6}{15}$ mg = 93.2166 g.

Zero point		6.8	12.3	7.1	19.3
Cylinder right	93.210 g left	11.4	7.7	11.2	19.0
	93.211 g left	15.0	6.1	14.4	20.8

Cylinder in right pan = 93.210 g + $\frac{3}{15}$ mg = 93.2102 g.

Apparent mass = $\sqrt{93.2102 \times 93.2166}$ g = 93.2134 g.

Ratio of balance arms, $\frac{l}{r} = \sqrt{\frac{93.2166}{93.2102}} = 1.000034.$

44. Weighing by the Rider Method. — A more rapid method of weighing than that described in Art. 42 is often required. For the determination of specific gravity, and for chemical analysis, relative mass only is necessary. If bodies are always weighed on the same side of the balance, their masses are affected in the same proportion by inequality in the arms, and weighing by reversal is not necessary. The method of single weighing by vibrations may be simplified for these purposes.

Small weights are troublesome to handle, and those of less than 10 mg value may be replaced by a *rider*, which is a piece of wire usually weighing exactly 10 mg, of such a shape that it may ride astride the beam of the balance at different distances from the central knife-edge. It may be manipulated by the handle shown at R in Fig. 27. If it rides over the end knife-edge it produces the same effect as though it were in the pan. But if it rides at a tenth of the distance from the center to the end its effect is one tenth as great; that is, it may be used in this position as a substitute for 1 mg in the pan. The arm may be divided into ten parts with subdivisions, and then when the rider is moved to a position for equilibrium, its effect in milligrams and fractions is read from the figures engraved on the beam.

Often two riders are used, one on each arm; or one rider may move over both arms. The effect of the rider, whether to be added to or subtracted from the weights in the pan, will be evident from inspection.

The complete method of vibrations with interpolation may be employed as described, or it may be shortened with only a slight loss of precision, as follows. The sensibility of the balance for various loads (Art. 45) is carefully determined once for all, and a table is constructed showing the sensibility for all loads within the capacity of the instrument. When weighing, the zero point of the unloaded balance is determined as before. The body is placed in the pan and balanced as nearly as may be with weights and the rider. The difference between the equilibrium point and zero point, taken in connection with the table of sensibilities, will determine the amount by which the weight in the pan differs from the true weight.

Instead of finding the zero point in this manner, the unloaded balance may be made to swing equally on each side of the center of the scale by means of one rider; the center of the scale is then the zero point, which simplifies the readings. A second rider is used in balancing the body. Instead of reading three turning points on the scale, the equilibrium position may be taken as the mean of two readings.

Each simplification slightly diminishes the precision, and the nature of the work must determine in each case the method to be adopted.

45. Sensibility of Balance. — The scale displacement of the equilibrium position of the pointer of a balance, caused by a change of one milligram in the load in either pan, is the sensibility. The sensibility varies greatly with the loads in the pans; it may be determined for various loads, and a table or curve prepared for reference. It is somewhat more precise to determine the sensibility at each weighing, as explained in Arts. 42 and 47.

The sensibility of a balance may be adjusted by altering the position of the center of gravity of the beam. A small screw is often attached above the central knife-edge for this purpose. The center of gravity of the beam must be below the point of support, otherwise the beam will be in unstable equilibrium. The nearer the center of gravity is brought to the point of support, the more sensitive the balance; but increase of sensitiveness results in a longer period of vibration and a less rapid use. The limits of precision of a balance vary greatly according to the design and quality of construction, and it is useless to increase the sensitiveness beyond the point of trustworthy indications. Often it is advantageous to make the sensibility less than its maximum value, altering it so that it is not greater than is required for the particular work in hand.

46. Practical Hints on the Use of the Balance. — In addition to the general methods already described, the student should carefully observe the following hints.

Remove dust from the pans and weights with a camel's-hair brush.

Level the base.

The unloaded balance should be in equilibrium when the pointer is near the center of the scale; this condition may be secured by adjusting the small screw attached to the beam. The student should not alter this, or any other adjustment of the balance, except upon definite orders.

The beam and pans should be arrested when the balance is not in use, and also whenever weights are added or removed.

Arrest the beam only when it is passing through its position of equilibrium.

Avoid parallax in reading by having the line of sight perpendicular to the scale.

The release of the arrest will usually cause the beam to vibrate as desired. If not, a breath of air under one of the pans will be sufficient. A small stream of air from a rubber bulb held in the hand may be directed against the pans to regulate the swing. The pans should have no pendular motion.

Weighings should be made with the balance case closed, and precautions are required to avoid the effects of air currents in the closed case caused by uneven temperature.

The larger weights should be placed near the center of the pan, and the others so distributed that the pan will not swing to one side when released.

The weights should be handled only with the pincers and forks provided. Small weights may be removed from the pan with a camel's-hair pencil.

To avoid mistakes, the weights should be counted several times. Count them while they are in the pan; remove them from the pan, arranging them in groups in the order of magnitude, and count again; and finally they may be counted by the unoccupied places in the box, or as they are returned to their places in the box.

When using gilded weights care should be taken that they do not come into contact with small particles of mercury.

Volatile liquids and fuming acids should be weighed only in closed vessels. It is sometimes necessary, as when weighing hygroscopic bodies, to keep drying material in the balance case.

Arrest the balance, remove weights from the pan, arranging them in the proper order in the box, and close the balance case, upon the completion of the weighing.

XVII. ABSOLUTE MASS WITH THE BALANCE BY THE METHOD OF VIBRATIONS AND REVERSAL

Determine the true mass of a body.

47. Weighing by Reversal. — Any weighing which depends upon the determination of the zero point must be untrustworthy, as the position of equilibrium of the unloaded balance may not coincide with its position of equilibrium when loaded. The method of reversal here described does not require the position of the zero point to be known; it eliminates the effect of the inequality of the arms, and is both short and accurate.

For example, let it be required to compare the kilogram weight $(1000)_Q$ with the normal kilogram K_N. With K_N in the right pan and $(1000)_Q$ in the left, determine the sum of the excursions as described in Art. 42. Interchange the loads in the pans and again find the sum of the excursions. If the apparent weights of the two masses are equal, the sums of the excursions will be identical, regardless of the lengths of the beam arms; while if the weights are unequal, the difference between the sums of the excursions will correspond to *twice* the difference between these weights.

This difference will be expressed in milligrams by making the observed change in the sum of the excursions, caused by reversal, the numerator of a fraction whose denominator is the change in the sum of the excursions produced by adding a two-milligram weight to *either* side. The two-milligram weight should be dropped into the pan without arresting the balance.

The complete record of such a comparison is given in the next article.

48. Weight in Vacuo. — When a weighing is performed in the air, as is usually necessary, all bodies in the pans are buoyed up by forces proportional to their volumes. If the volumes in

the two pans are unequal, the apparent weight is in error because of this buoyancy. A correction for this is not always applied, though it is of sufficient importance to be regarded even in ordinary weighing. If m represents the weight which balances a body in air, l the density of the air, s the density of the body, and d the density of the weights, the true weight *in vacuo* is (Table 1, Appendix),

$$M = m\left(1 + \frac{l}{s} - \frac{l}{d}\right).$$

Frequently this correction can be made with greater ease and precision from the known volumes of the bodies and the weights. Its amount is equal to the difference of volumes in the two pans multiplied by the density of the air at the temperature and pressure of the experiment, the apparent weight being too small by this amount, if the body has the larger volume.

When the masses to be compared are of different materials, it is also necessary to consider their volumetric expansions. The volumes are to be reduced to 0°, using the coefficients of cubical expansion, which are three times the coefficients of linear expansion.

A numerical illustration follows.

ABSOLUTE MASS OF KILOGRAM $(1000)_Q$

BY COMPARISON WITH NORMAL KILOGRAM K_N, METHOD OF REVERSAL

Staudinger Balance, Rueprecht Weights

June 15, 1895

The true mass of the standard kilogram, K_N, is known to be 999.9998 g. Barometer 74.6 cm; attached thermometer 22°.4; thermometer, air, 20°.7.

$(1000)_Q$ left	K_N right	7.7	13.3	8.0	21.1
K_N left	$(1000)_Q$ right	8.1	11.6	8.3	19.8
Added 2 mg left		8.7	14.0	8.9	22.8

$$(1000)_Q = K_N + \tfrac{13}{30}\ \text{mg}.$$

Volume of K_N at 0° referred to water at 4° = 118.876 ccm.
Volume of $(1000)_Q$ at 18°.6 referred to water at 4° = 120.75 ccm.
Both masses made of brass.

To compare the volumes of the two bodies, that of $(1000)_Q$ observed at 18°.6 must be reduced to 0°. The cubical expansion of brass is 0.000057 for 1°, from which the volume at 0° is found to be 120.62 ccm. The difference in volume of the bodies, 1.74 ccm, gives for the reduction to vacuum, the density of air being 0.00118, + 0.00208 g. Hence

$$\begin{aligned}(1000)_Q &= K_N + 0.00043\,\text{g} + 0.00208\,\text{g}\\ K_N &= 999.9998\,\text{g}\\ (1000)_Q &= 1000.0023\,\text{g}\end{aligned}$$

This represents the absolute mass of the weight, since it refers to the body when it is in a vacuum.

XVIII. ERRORS OF A SET OF WEIGHTS BY COMPARISON

Determine the correction to each weight of a set by comparisons among themselves and by comparison with a normal weight.

49. Calibration of Weights. — The errors of adjustment in a set of weights are frequently of considerable magnitude, and it cannot be assumed that they are so small as to be negligible, even in weights presumed to be of the finest quality. Weights for use in work of precision must always have their apparent relative masses determined, and often their absolute masses. The latter requires that the volume of each weight be known; this may be ascertained by hydrostatic weighing (Art. 96), though it may be sufficient to assume a density for the material of which the weights are constructed and to compute the volumes from this and their nominal values.

All the weights are to be compared among themselves according to a systematic scheme, in which there are as many weighings as separate weights, each weight being compared with another single weight or combination of weights. These weighings furnish equations from which may be calculated the errors of each weight. When the absolute mass is required, one weight must be compared with a standard weight. A numerical example will illustrate the method.

Calibration of Rueprecht Platinized Weights

Spoerhase Balance No. 1

December 6, 1895

Designate each weight by its nominal value in parenthesis; where there are several weights of the same nominal value distinguishing marks must be placed upon them. Let the weights in the set be

$$(100) = a,\ (50) = b,\ (20) = c,\ (10) = d,\ (10') = e,$$

and

$$(10'') = (5) + (2) + (1) + (1') + (1'') = f,$$

the letters representing the true values.

The weight a is compared with the standard 100-gram weight, and the weights are compared among themselves, as indicated in the following equations, by the method of double weighing described in Art. 47.

$$\begin{aligned} 100\text{ g} &= a & -\ 0.07\text{ mg} \\ a &= b+c+d+e+f & -\ 0.91 \\ b &= c+d+e+f & -\ 0.95 \\ c &= d+e & -\ 0.76 \\ d &= e & -\ 0.03 \\ e &= f & -\ 0.04 \end{aligned}$$

Solving these equations,

$$\begin{aligned} f &= f \\ e &= f - 0.04\text{ mg} \\ d &= f - 0.07 \\ c &= 2f - 0.87 \\ b &= 5f - 1.93 \\ a &= 10f - 3.82 \\ a &= 100.00007\text{ g} \end{aligned}$$

From the last two equations,

$$\begin{aligned} 10f &= 100.00389\text{ g} \\ f &= 10.00039\text{ g} \end{aligned}$$

and therefore

$a =$	100.00007 g,	correction + 0.00007 g
$b =$	50.00002	+ 0.00002
$c =$	19.99991	− 0.00009
$d =$	10.00032	+ 0.00032
$e =$	10.00035	+ 0.00035
$f =$	10.00039	+ 0.00039

A repetition of the process with the smaller weights contained in the set f will determine their values; and the values of the subdivisions of the gram are to be found in the same manner.

50. Adjustment of Weights. — Usually it will be more convenient to make a table of corrections for a set of weights than to adjust them. When adjustment is necessary the following method may be used; if other methods are required the circumstances will determine the procedure.

Analytical weights are usually constructed with knobs screwed into the bodies; the knobs may be removed, exposing cavities in which small bodies are placed in adjusting. Make each weight a trifle lighter than its normal value, removing small particles from the interior if necessary. The entire set of weights is then calibrated by comparison with a normal weight and among themselves, as described in the preceding article. The error of each weight is thus determined. Weigh a measured length of fine wire which may have mass of about one milligram per centimeter of length; by measurement it will then be possible to cut pieces from this wire which will very nearly correct the several errors. Place these pieces of wire inside the respective weights.

After adjustment the whole set of weights should again be calibrated. The errors will be very small if the work has been carefully done.

XIX. VOLUME BY WEIGHING

(a) Determine the volume of a glass bulb, and the volume per centimeter of length of its tubular stem.
(b) Determine the radius of bore of a capillary tube.

51. Volume by Weighing. — This method is useful for verifying or correcting the volumes of measuring vessels and for determining the volumes of bulbs and tubes.

If the fluid used to fill the vessel weighs m grams in air and has the density s, its volume is

$$v = \frac{m}{s}\left(1 + \frac{l}{s} - \frac{l}{d}\right),$$

l being the density of the air and d that of the weights.

For water and brass weights in air it will be sufficient to take the volume as

$$v = m(2.00106 - s).$$

For mercury weighed with brass weights in air the volume is

$$v = 0.999945\,\frac{m}{s},$$

the density of the mercury at $t°$ being (Appendix, Table 3)

$$s = 13.595\,(1 - 0.000182\,t).$$

The mass of water which balances 1 g of brass in air has at 20° the volume of 1.0029 ccm.

The mass of mercury which balances 1 g of brass in air has at 20° the volume 0.07382 ccm.

Tables 3 and 4 in the Appendix facilitate the calculation of volumes for the conditions which occur most frequently.

52. Volume and Radius of a Tube. — A mass of mercury of m grams at the temperature $t°$ has the volume

$$v = \frac{m\,(1 + 0.000182\,t)}{13.595}.$$

To find the volume, c, per centimeter of length of a cylindrical tube, determine the weight, m grams, of the column of mercury at the temperature $t°$ which occupies l centimeters of length of the tube; then

$$c = \frac{m\,(1 + 0.00018\,t)}{13.595\,l}.$$

If m is the weight of the mercury which fills l centimeters of the cylindrical tube at 20°, the radius is

$$r = 0.1534\sqrt{\frac{m}{l}}.$$

XX. ATMOSPHERIC PRESSURE AND ALTITUDES WITH THE BAROMETER

(a) Read the barometer and reduce the reading to standard conditions.
(b) Determine altitudes from the barometric reading.

53. The Barometer. — In its usual form the barometer consists essentially of a glass tube closed at one end, filled with mercury, and then inverted in a cistern of mercury. The tube

is of such a length that in its upper part there is a vacuum. Instead of the cistern, the open lower end of the tube is sometimes extended and bent upward, forming the siphon barometer. The mercury column being maintained by atmospheric pressure, variations in its height will measure changes in this pressure.

By the height of the barometer is meant the height of the mercury column at 0°, which at the specified point would exert the same pressure as is exerted by the atmosphere at that point. If the barometric pressures at different localities are to be compared, it is not sufficient to compare the heights as defined; but the actual pressures in dynes per square centimeter must be compared, since the weight of a given column of mercury varies with the intensity of gravity.

FIG. 30. CISTERN OF BAROMETER

The Fortin cistern barometer is the form usually employed for laboratory and meteorological observations. In reading it the following operations should be performed in the order given. Read the *attached thermometer.* Gently tap the tube near the top of the column to avoid adhesion of the mercury. Adjust the surface of the mercury in the cistern (Fig. 30) by the screw S at the bottom of the instrument until it just touches the tip of the ivory pointer P. Again gently tap the tube and set the vernier index so that its lower edge is *tangent* to the meniscus. Read the vernier.

In the siphon barometer the scale is often etched on the glass tube, with its zero near the middle; readings are then to be made upwards and downwards to the mercury surfaces, and the apparent height is the sum of the two readings.

54. Reduction of Barometer Readings. — A reading, taken as described, requires several corrections; viz. for the temperature of the mercury, the temperature of the scale which is supposed to be correct at 0°, the capillary depression of the mercury column, and for the depression due to the presence of air or

water vapor in the tube above the mercury. Sometimes it may also be necessary to include a correction for the index error of the scale. The temperature correction is usually large and should never be omitted, while for many laboratory purposes the other corrections are so small as to be negligible.

Temperature Corrections. — If l is the height of the column as read at $t°$, and e the coefficient of expansion of the material of the scale, the height corrected for the two temperature errors is

$$h = l - (0.000182 - e)\, lt;$$

from which,

$$\text{for a brass scale, } h = l - 0.000163\, lt,$$
$$\text{for a glass scale, } h = l - 0.000174\, lt.$$

The values of the temperature correction for a barometer with a brass scale are given in the Appendix, Table 14.

Index and Other Errors. — The tubes of barometers are often so small that the capillary depression is considerable; and often the barometer contains air or water vapor above the mercury, causing an appreciable depression. The makers usually eliminate these errors from the readings by setting the scale so that it reads correctly, as compared with a true standard barometer, though it does not give the actual height of the mercury above the index. If such an apparent index error is discovered, it does not necessarily introduce an error into the readings; it probably corrects several errors. Whether it does so correct the errors can be conveniently determined only by recomparison with a standard barometer. In the siphon barometer it is not difficult to determine such an error, by altering the quantity of mercury so that the space in the tube above the mercury is changed. If this space be diminished to one half, the pressure due to inclosed air will be doubled, and it may be detected by comparison of the readings.

If a true index error is present it may be corrected in a manner which will be evident from the construction of the instrument.

Correction for Capillary Depression. — If this has been corrected in setting the scale as described above, it will need no further consideration. But mercury columns are used for many measurements where independent correction is desired.

A table is given in the Appendix (Table 13) from which the correction for capillary depression may be found. Its amount is somewhat uncertain, and for tubes of more than one centimeter internal diameter it may usually be neglected.

55. Barometric Pressure.—The observed height of a mercury column, corrected as described in the previous article, will be reduced to the true pressure in dynes per square centimeter when it is multiplied by the density of mercury and by the intensity of gravity at the place of observation.

The density of mercury at 0° is 13.5950, while the values of g for various localities may be found in tables (Table 10, Appendix).

56. The Barye and Standard Pressure. — The unit of pressure is a pressure of one dyne per square centimeter; it is called the *barye*. For measurements of the elasticity of gases, and for all other cases where pressures have been heretofore expressed in atmospheres, it has been decided (International Congress of Physics, Paris, 1900) to adopt the *megabarye*, equal to a pressure of 10^6 dynes per square centimeter, as the standard pressure. For Cleveland, where g has the value 980.240, this corresponds to a mercury column having at 0° the height of 75.04 cm.

57. Altitudes by the Barometer.—If the height of the barometer be observed at the same time at two different stations, the difference in altitude may be calculated by the following formula. Let b_1 and b_2 denote the two corrected barometer readings, t the mean of the temperatures t_1 and t_2 of the air at the two stations, e_1 and e_2 the tensions of the vapor of water at the two stations, h the mean height in centimeters above the sea level of the two places, ϕ the latitude, and H the required difference in altitude in centimeters; then,

$$H = 1\,843000\,(\log b_1 - \log b_2)\,(1 + 0.00367\,t)$$
$$(1 + 0.0026 \cos 2\phi + 0.00002\,h + \tfrac{3}{8}\,k),$$

k being $\frac{1}{2}\left(\frac{e_1}{b_1} + \frac{e_2}{b_2}\right)$.

The following approximate formula, which assumes the value of gravity as that of latitude 45°, that gravity does not

vary with altitudes, and that the air is half saturated with moisture, is sufficient for differences of altitude not exceeding 100000 cm.

$$H = 1\,600000\,\frac{b_1 - b_2}{b_1 + b_2}\,(1 + 0.004\,t).$$

REFERENCES. — *Kohlrausch*, Physical Measurements, pp. 76–80; *Staley*, Gillespie's Surveying, Vol. II, pp. 273–296; *Everett*, C. G. S. System of Units, pp. 43–47.

CHAPTER III

TIME, ACCELERATION, AND GRAVITY

XXI. LAWS OF ACCELERATED MOTION WITH A FALLING TUNING FORK

Verify the laws of accelerated motion and find the acceleration of gravity.

58. Time. — Of the three fundamental units of mechanics, determinations of length and mass are considered as belonging to the physicist, while the determination of time is usually delegated to the astronomer. In the physical laboratory it is required either to find the ratio of relatively short time intervals or to compare a periodic interval with the indications of a standard timepiece. The methods for such comparisons are described in connection with the several exercises of this chapter.

59. Accelerated Motion. — A tuning fork is attached to a frame arranged to fall vertically about a meter, between guides which offer the least possible resistance. The fork may be held at the top of the guides by a catch having a trigger, which will release the fork and at the same time set it in vibration. One prong of the fork carries a style for recording the vibrations in a sinuous line upon a long plate of smoked glass. The glass may be lightly smoked with burning camphor gum. Make the guides vertical, so that the slide may fall freely, tracing a sinuous line. If necessary adjust the style and repeat the tracing after having moved the glass plate sideways about a centimeter. In this way obtain three good tracings to be measured as directed below.

Remove the plate from the frame; select a point, *a* (Fig. 31), near the beginning of a tracing where the waves are perfectly formed. From this point mark off spaces each containing exactly

ten waves, as b, c, d, etc. When the fork was at a it had already acquired some velocity in falling, which is represented by v_0. In the interval of time t, corresponding to ten vibrations, the fork, because of this initial velocity alone, would pass over a space, v_0t; but gravity also acts, and adds to this the space $\frac{1}{2}gt^2$. Hence in the first interval, from a to b, the fork passes over a space,

$$s_1 = v_0t + \tfrac{1}{2}gt^2.$$

In two intervals of time it would pass over a space, from a to c, represented by the same formula if for t we substitute $2t$:

$$s_2 = 2v_0t + \tfrac{4}{2}gt^2.$$

Similarly for the successive intervals, ad, ae, etc., we must substitute $3t$, $4t$, etc., obtaining

$$s_3 = 3v_0t + \tfrac{9}{2}gt^2,$$
$$s_4 = 4v_0t + \tfrac{16}{2}gt^2,$$
$$\cdots\cdots\cdots$$

The differences between these distances are the lengths of the spaces passed over in the successive ten-vibration intervals. These are

$$s_1 - s_0 = ab = v_0t + \tfrac{1}{2}gt^2,$$
$$s_2 - s_1 = bc = v_0t + \tfrac{3}{2}gt^2,$$
$$s_3 - s_2 = cd = v_0t + \tfrac{5}{2}gt^2,$$
$$\cdots\cdots\cdots\cdots$$

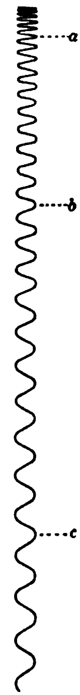


FIG. 31. TRACE MADE BY A TUNING FORK

These equations show that

$$bc - ab = cd - bc = \cdots\cdots = gt^2.$$

Measure as many of the ten-vibration spaces of the trace as is possible, and find the differences between successive distances. The differences should all be equal, proving that the force acting on the fork, gravity, is constant. Compute the mean of these measured accelerations, which is equal to $f = gt^2$. If the frequency of the fork, n, is known, the time interval is

$$t = \frac{10}{n}.$$

Compute g, the acceleration due to gravity; the result will be a little too small, because of the friction of the falling slide.

Having found t and g, calculate the initial velocity, v_0, from the value of any of the measured intervals; as

$$ab = v_0 t + \tfrac{1}{2} g t^2.$$

60. Acceleration with Variable Time Unit.—Using the second and third wave traces, made as above described, mark off on one spaces of twenty and on the other spaces of thirty wave lengths. Find the average increase in lengths of the successive equal time intervals. Compare the accelerations thus obtained for the double and triple time intervals with that for the single interval obtained before. The relations should be

$$f_1 : f_2 : f_3 = t_1^2 : t_2^2 : t_3^2 = 1 : 4 : 9.$$

In other words, the acceleration is proportional to the squares of the lengths of the time unit. A further analysis of the concrete case of the experiment will make the meaning clear. The acceleration is obtained for two different *units* of time which have the ratio of 1 : 3. In the first instance the acceleration is measured by the increase of velocity per single interval. If, now, the force acts during the triple interval, it will produce three times as much increase in velocity as before, the velocity being measured, *as before, per single interval.* But if the velocity is now measured by the space passed over in the triple interval, its value will be again multiplied by three, or in all by nine; that is, the acceleration increases as the square of the unit of time taken for measuring. But having a fixed unit of time, as the second, the effect of a constant acceleration is proportional to the first power of the time during which it acts. Gravity in three seconds will increase the velocity of a falling body only three times as much, expressed in centimeters per second, as it would do in one second.

XXII. LAWS OF ACCELERATED MOTION WITH ATWOOD'S MACHINE

Verify the laws of uniformly accelerated motion. Determine gravity and the moment of inertia of the pulley.

61. Atwood's Machine. — The essential features of Atwood's machine are two equal weights, of mass m, suspended by a fine cord passing over a light pulley mounted on antifriction bearings. A third mass, r, is allowed to ride upon one of the masses, m, causing it to descend. After falling for a known time interval, the rider is removed by a ring through which m passes. A stop is placed to receive m a known time after the removal of the rider. The distance through which the rider falls is measured, and also that through which the weight falls after the lifting of the rider.

The time intervals may be determined with a metronome, with a sounder in connection with a break-circuit clock, or with a chronograph. Many machines have pendulums or clock attachments for giving the time signals.

Neglecting the inertia of the pulley, the moving mass may be represented by $2m + r$; and, not considering friction, the force moving this mass is the weight, $f = rg$, of the rider. If a is the acceleration imparted to the moving mass by the rider, measured by the distance moved over in one succeeding second after the rider is lifted, divided by the time during which the rider acted,

$$f = rg = \text{mass} \times \text{acceleration} = (2m + r)\,a.$$

Prove from measurements of the spaces passed over in one, two, and three seconds, that after the rider is lifted the masses move with constant velocity.

Allow the rider to act for one, two, and three seconds, and show that the spaces through which it falls are as the squares of the times.

Increase the size of the masses and that of the rider in the same proportion, and show that the fall in three seconds is the same as with the small masses, proving that heavy and light bodies fall equally fast.

Show that the acquired velocities are proportional directly to the times of the fall, by allowing the rider to act for one, two, and three seconds and measuring the velocities produced in each instance.

Prove that a force is proportional to the product of the mass and the acceleration, as expressed by the above equation, by using one rider on two different masses.

Prove the preceding proposition by measuring the accelerations produced by two different riders on the same mass. In this experiment the sum $2m + r$ should be kept constant by means of small auxiliary weights.

62. Gravity and Inertia of Pulley with Atwood's Machine. — The friction of the wheelwork and of the air is very difficult to determine, and it is therefore reduced to a minimum by suitable construction. The remaining friction may be compensated for by adding to the descending mass small pieces of tin foil of such a weight that after the motion is started by hand the mass will descend uniformly. The amount of foil required should be determined for each different load carried by the pulley. This adjustment is best made when the two movable masses are at the same level, so that the weights of the cord on the two sides of the pulley are equal.

The rider in falling not only accelerates the masses m but it also must turn the wheel with increasing speed. The pulley is equivalent to a small additional balanced mass, P, for which allowance should be made.

The equation of motion may now be written

$$rg = (2m + r + P)a_1.$$

Alter the masses m (for instance, double them) and represent the new masses by M. With the same rider as before, the equation of motion is

$$rg = (2M + r + P)a_2.$$

By eliminating P from these two equations the following result is obtained for the acceleration due to gravity.

$$g = \frac{a_1 a_2}{a_1 - a_2} \cdot \frac{2M - 2m}{r}.$$

Determine a_1 and a_2 as explained in the preceding article and calculate g. When the rider under the influence of gravity falls through a space s, it does work represented by rgs. This work imparts velocity to the masses and the wheel, and it is measured by their kinetic energies. The masses, $2m + r$, acquire a velocity v, and the wheel, whose radius is R and moment of inertia is I, acquires an angular velocity ω. Therefore

$$rgs = \tfrac{1}{2}(2m + r)v^2 + \tfrac{1}{2}I\omega^2.$$

But $$v^2 = 2as = R^2\omega^2;$$

hence $$rg = \left(2m + r + \frac{I}{R^2}\right)a,$$

and $$P = \frac{I}{R^2}.$$

That is, the equivalent mass of the pulley is its moment of inertia divided by the square of its radius.

By eliminating g from the first two equations,

$$P = \frac{(a_2 - a_1)r + a_2 2M - a_1 2m}{a_1 - a_2}.$$

Measure the radius of the wheel and compute I.

$$I = PR^2.$$

The moment of inertia might be experimentally determined with the torsion pendulum, as described in Art. 84.

XXIII. PROPERTIES OF THE PATH OF A PROJECTILE

Prove that the acceleration of gravity is constant and show that the path of a projectile is a parabola.

63. The Trajectory.—Arrange apparatus for projecting a ball horizontally in front of a blackboard or a sheet of paper, by allowing it to roll down a semicycloidal curve, as indicated in Fig. 32. The height of this curve may be 30 cm. A mechanical trigger should be arranged to release the ball uniformly.

Determine the path of the ball by holding a straightedge in different parts of the path, so that the ball just grazes it in falling.

Fifteen or more points should be determined, and a smooth curve drawn through them. A parallel curve separated from this by the radius of the ball will be the path of the center. From the level of the center of the ball at the lowest point of its cycloidal path draw a horizontal line, and measure a series of distances, t, increasing by two centimeters each. Since the ball will move with a uniform velocity in a horizontal direction, these distances represent both the times of flight and the horizontal motion in these times. Measure the vertical distances, l, from each division to the plotted path. These represent the spaces fallen in the several intervals of time, due to gravity. A level and thumb line will aid in plotting the coördinates.

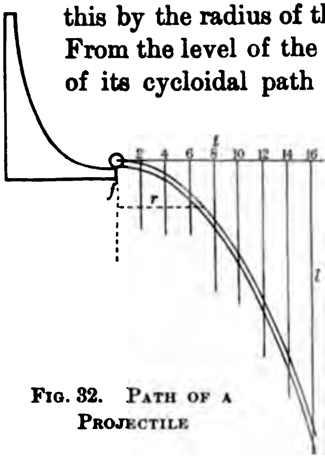


FIG. 32. PATH OF A PROJECTILE

From the laws of falling bodies each pair of values of l and t should satisfy the equation $l = \frac{1}{2} at^2$; prove this by showing that the values of

$$a = \frac{2\,l}{t^2},$$

computed for each point measured, are all equal. This value of a represents the acceleration produced by gravity in the time taken for the ball to move over 1 cm in a horizontal direction; this would be equal to g if the unit of horizontal measure, instead of being 1 cm, was of such a length that it represented the horizontal distance traveled in one second.

Since the acceleration produced is proportional to the square of the time, the unit of time represented by t is

$$t = \sqrt{\frac{a}{g}},$$

and the velocity of the horizontal projection of the ball is

$$v = \frac{1}{t}.$$

Calculate both t and v.

The constancy of a, as determined above, proves the path to be a parabola, whose equation in the usual form is $t^2 = \frac{2}{a} l$, from which it appears that the distance of the focus from the origin is

$$f = \frac{1}{2a}.$$

The length of the semi-latus rectum is

$$r = 2f.$$

Plot the focus and measure the latus rectum.

XXIV. GRAVITY BY THE SIMPLE PENDULUM

Determine the intensity of gravity.

64. Simple Pendulum. — An excellent pendulum consists of a metal sphere of from 3 to 5 cm diameter, suspended from a rigid support by a fine steel wire about 0.01 cm in diameter. A simple and sufficient support is made by fastening the wire to a light steel ring, such as a screw eye, and hanging this on a knife-edge, which may be a three-cornered file, ground smooth, as indicated in Fig. 33.

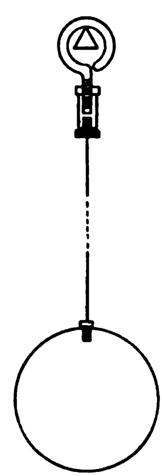


FIG. 33. SIMPLE PENDULUM

The length of the pendulum for the purposes of this experiment may be about 99 cm or 396 cm. This length must be determined as accurately as possible: if it is about a meter, a meter bar and a beam compass may be used for measuring; if longer, a steel tape is convenient. The distance, d, from the point of support to the center of the sphere is required. The distance to the bottom of the ball may be measured, from which is to be subtracted the radius of the ball, r, determined with the

calipers. The length of the equivalent simple pendulum is, neglecting the effect of the suspending wire,

$$l = d + \frac{2\,r^2}{5\,d}.$$

The pendulum is made to vibrate through a small arc, a, not exceeding 4°, and its period, p, is determined, preferably by the method of coincidences as described in the next article. The time of vibration in an infinitely small arc is approximately

$$t = \frac{p}{1 + \frac{1}{4}\sin^2\frac{a}{4}}.$$

From the theory of the pendulum the acceleration of gravity is

$$g = \frac{\pi^2 l}{t^2} = \frac{\pi^2}{p^2}\left(d + \frac{2\,r^2}{5\,d}\right)\left(1 + \tfrac{1}{4}\sin^2\frac{a}{4}\right)^2.$$

Since a dyne is the force which imparts unit acceleration to unit mass, and gravity imparts g units of acceleration, its intensity in dynes is represented by the numerical value of g.

This result might be further corrected for the buoyancy of the air and the mass of the suspension wire; but these are both very small, and moreover are of opposite sign, and they may be neglected. The error due to motion of the support is difficult of determination.

If the formula without corrections,

$$g = \frac{\pi^2 d}{p^2},$$

is used, the errors may be one part in two thousand.

A pendulum about four meters long will swing in about two seconds, and will be convenient when the clock signals are given once in two seconds, as is frequently the case.

65. Periodic Time by the Method of Coincidences. — Arrange a telescope with its line of sight at right angles to the line of vibration, so that the vertical wire in the field of view coincides with the pendulum wire when it is at rest. The vibrations are

then compared with the audible beats of the seconds pendulum of a standard clock, or, better, with the ticking of a sounder in connection with a break-circuit clock or chronometer. For many purposes this latter arrangement is very useful, as sounders may be placed in all parts of a laboratory to distribute accurate time signals. In the absence of these devices an ordinary watch having a second-hand can be made to serve satisfactorily. An assistant looking at the second-hand may count the seconds to himself, and in any convenient manner beat them audibly for the observer.

The comparisons between the pendulum and the clock can best be made by the valuable method of coincidences, which is useful for comparing two periodic phenomena whose periods are nearly equal. Let the pendulum be of such length that its period is very nearly one second. If the signals occur every second, a time will come when the pendulum will pass the center of its swing, as seen in the telescope, exactly as one of the signals is given; the transit of the pendulum and the signal coincide. But at the next swing they will not coincide, as the period of the pendulum differs from the interval between the two signals. This lack of coincidence will increase until the transit occurs midway between the two signals; then it will apparently decrease until coincidence again occurs. The interval between these two coincidences should be determined as accurately as possible; it will be an integral number of seconds. In this interval the number of vibrations of the pendulum is one more or less than the number of time signals. Hence the period is obtained by dividing the number of seconds by this number plus or minus one, according as the pendulum is shorter or longer than the seconds pendulum. Whether the pendulum gains or loses on the time signals may be determined by a simple auxiliary observation. If there appears to be coincidence at several transits, note the first and last, and consider the mean as the true time of coincidence. Observe three or more coincidence intervals, and compute the period from each independently. If the rate of the clock is known, and it exceeds one second per day, a further correction should be made because of

it; if the clock is gaining, the computed period is too large. A numerical example of a gravity determination follows.

Gravity with the Simple Pendulum

Riefler Mean-Time Clock

March 5, 1902

Time of Coincidence			Interval		Period
11^h	58^m	15^s			
			10^m	10^s	$\frac{610}{611} = 0.99836$ s
12	8	25			
			9	55	$\frac{595}{596} = 0.99832$
12	18	20			
			9	59	$\frac{599}{600} = 0.99833$
12	28	19			
					Mean $= 0.99834$

Rate of clock known to be less than 0.5 s per day.

Average arc $= 3°$. Corrected period,	$t =$ 0.99830 s
From the point of support to bottom of sphere,	101.41 cm
Radius of sphere,	2.45
Length of equivalent simple pendulum,	$l =$ 98.985

$$g = \frac{\pi^2 l}{t^2} = 980.30 \ \frac{\text{cm}}{\text{s}^2}.$$

If the pendulum has a period of nearly two seconds, and the coincidences of its transits with a time signal occurring once in two seconds are observed, the number of vibrations made in the coincidence interval is *half* the number of seconds, plus or minus one. Otherwise the method is as above described.

XXV. GRAVITY WITH THE REVERSIBLE PENDULUM

Determine the intensity of gravity.

66. Kater's Pendulum. — The reversible pendulum has the advantage over other forms, that it is easier to obtain an accurate value of the length of the equivalent simple pendulum. It consists of a rigid bar (Fig. 34) with a fixed knife-edge near each end. A heavy mass is fastened at one end, and lighter masses are adjustable along its length. The pendulum is to be swung first upon one knife-edge and then upon the other, and the adjustable masses moved nearer that knife-edge about which

the time of swing is greater, until the times of vibration about the two knife-edges become equal. When this condition is secured the distance between the knife-edges is equal to the length of an ideal pendulum of equal period.

To determine this distance the pendulum may be placed upon a comparator (Art. 22) and the microscopes set upon the knife-edges; a standard scale is substituted for the pendulum and the length read directly. If proper apparatus is available, the length may be determined with a cathetometer while the pendulum is suspended. But probably the best method is to provide a steel rod with a screw end, whose length can be adjusted to reach from one knife-edge to the other while the pendulum is suspended. The length of this rod is then measured with an end comparator, or in any other convenient manner.

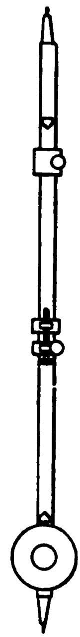


FIG. 34. REVERSIBLE PENDULUM

The period may be determined by the method of coincidences described in Art. 65. The comparisons with the standard clock will be facilitated by the use of the optical method of Art. 69, or by making the comparison with the chronograph (Art. 68). For the latter method a platinum wire is attached to each end of the pendulum, so that when it is stationary the wire at the lower end just touches a small globule of mercury held in a small tube, completing an electric circuit through the chronograph. The swinging of the pendulum may thus be recorded. If the contact does not occur exactly at the middle of the swing, the resulting error will be eliminated by considering only intervals containing an even number of vibrations. It is advantageous to have an adjustment by which the mercury globule can be withdrawn from the contact wire after one coincidence, to be replaced when another approaches. The pendulum may be allowed to swing during several coincidence intervals, and

by taking the mean, the uncertainty due to the difficulty of determining the exact coincidence will be somewhat reduced. The period thus determined should be corrected for the amplitude of swing by the formula given in Art. 64.

If the period of the pendulum about one knife-edge is exactly equal to that about the other, the time of a single oscillation, reduced to an infinitely small arc, being t, and the distance between the knife-edges being l, the acceleration of gravity is

$$g = \frac{\pi^2 l}{t^2}.$$

67. Formula for the Compound Pendulum. — Instead of continuing the adjustments until exact reversal is secured, when the times are approximately equal the following formula may be applied. Let h_1 and h_2 be the distances from the center of gravity to the knife-edges, about which the times of vibration are respectively t_1 and t_2. Then

$$\frac{\pi^2}{g} = \frac{t_1^2 + t_2^2}{2\,l} + \frac{t_1^2 - t_2^2}{2\,(h_1 - h_2)}.$$

The center of gravity can be determined with sufficient exactness by balancing the pendulum upon a metal rod and measuring the distances h_1 and h_2 with a meter stick.

Further corrections for the amplitude of swing, motion of the support, the drag of the air, etc., may be made when extreme precision is required, by methods described in the references.

References. — *Stewart and Gee*, Practical Physics, Vol. I, pp. 247–256; *Routh*, Rigid Dynamics, pp. 72–85; *Price*, Integral Calculus, Vol. III, p. 582; Vol. IV, p. 290.

68. The Chronograph. — When it is desired to make a permanent record of the time of the occurrence of a phenomenon, or of its duration, some form of chronograph may be used. This may print the time in plain figures, or the interval of time may be determined by comparison with the vibrations of a tuning fork (Art. 71), or by measuring a length. The latter form is the more common. It consists of a cylinder (Fig. 35) covered

with paper, which is made to revolve upon its axis at a uniform rate, usually one turn per minute. A pen carried on the armature of an electromagnet is moved slowly along the length of the cylinder as it revolves, tracing a spiral mark upon the

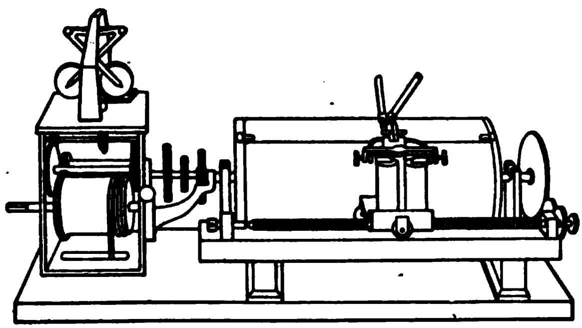


FIG. 35. CYLINDER CHRONOGRAPH

paper. Making or breaking an electric circuit through this magnet causes a little indentation in the trace, as represented in Fig. 36, which shows part of a chronograph record. The reading at a is $2^h\ 26^m\ 4^s.7$.

The chronograph is used in connection with a standard timepiece, having an attachment for either momentarily making or breaking the circuit at intervals of one or two seconds. To

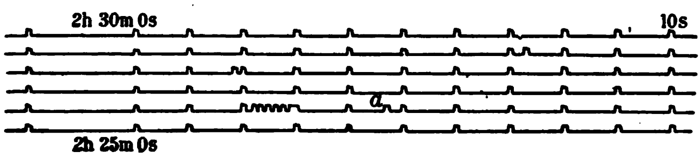


FIG. 36. RECORD OF CYLINDER CHRONOGRAPH

indicate the beginning of each minute there may be some peculiarity in the circuit breaking. If the signals occur at intervals of one second, the fifty-ninth in each minute may be omitted; while for an interval of two seconds, the odd second's signals, except the fifty-ninth in each minute, may be omitted.

A signal key, either to be held in the hand or attached to any apparatus as a pendulum, is included in the pen circuit, and permits the recording of the time of any phenomenon along with the clock signals. From this record the actual time of the occurrence, or the interval of time between two phenomena, can be determined by comparing the positions of the special signal marks with the nearest second's mark. A scale which divides the interval between two successive clock signals into parts corresponding to tenths of a second is often convenient for measuring a chronograph record.

Two pens may be used, one for the clock signals only, and the other for the observation signals. Sometimes provision is made for rotating the drum at higher rates, as one turn in

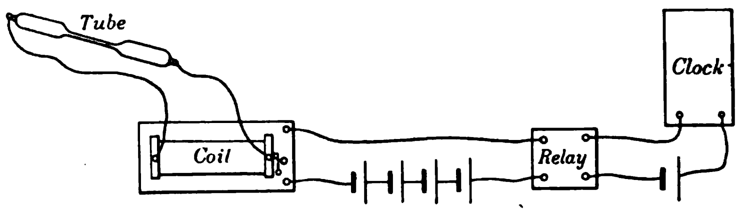


FIG. 37. FLASHING APPARATUS

ten seconds, or even one turn in one second. It is difficult to maintain uniform rotation at these high speeds. Observations which would require a rapid rotation, if the ordinary chronograph is employed, are usually better made with the tuning-fork chronograph.

69. Mendenhall's Optical Method for comparing Two Pendulums. — This method employs the principle of coincidences (Art. 65), the coincidences being easily and accurately observed by optical means.

The standard pendulum should have a break-circuit attachment, which produces a flashing line signal of the pendulum beats in the following manner. The clock breaks the primary circuit of an induction coil, the secondary circuit of which is connected to a Plücker tube (Fig. 37). As the amount of current required in the primary of the coil would be likely

to ruin the break-circuit mechanism, or to interrupt the clock, a relay is introduced. In the clock circuit a single cell of gravity battery is sufficient, while the coil circuit may contain one or more storage cells, or a few dry cells, for this experiment. When properly adjusted each beat of the pendulum will cause the tube to flash.

If the pendulum breaks the circuit by swinging through a globule of mercury, a large current will do no injury, and the relay may be dispensed with. Instead of the coil and tube, an electrically controlled shutter may illuminate a slit with flashes of light.

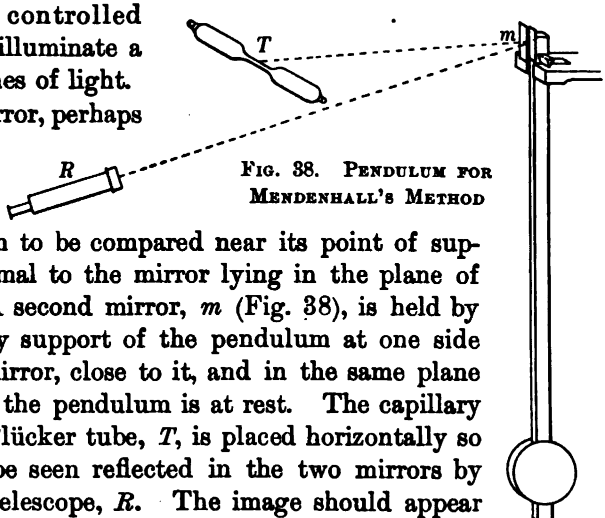


Fig. 38. Pendulum for Mendenhall's Method

A small mirror, perhaps 2 cm wide and 5 cm long, is attached to the pendulum to be compared near its point of support, the normal to the mirror lying in the plane of vibration. A second mirror, *m* (Fig. 38), is held by the stationary support of the pendulum at one side of the first mirror, close to it, and in the same plane with it when the pendulum is at rest. The capillary part of the Plücker tube, *T*, is placed horizontally so that it may be seen reflected in the two mirrors by means of a telescope, *R*. The image should appear straight and unbroken, one mirror being adjusted to secure this if necessary. If, now, the tube flashing with the clock beats, the pendulum is set in vibration, the two parts of the image of the flash made by the mirrors will appear discontinuous except when the two pendulums coincide; this coincidence is thus determined by noting that flash which gives a perfectly continuous image.

When this method is used with the reversible pendulum in addition to the mirror on the support, two are required on the pendulum, one near each knife-edge.

A numerical example is given of the determination of the period of a pendulum to be used in Exercise LIV.

PERIOD OF A PENDULUM; MENDENHALL'S METHOD

Time from Standard Self-winding Clock

July 26, 1902

An auxiliary pendulum was compared with the standard clock. Pendulum gaining: as nearly as can be judged, it makes 14 vibrations in 13 seconds.

Coincidences			Number of Intervals between Observations
5^h	26^m	10^s	
	27	15	5
	29	14	9
	30	30	6
	32	40	10
	36	09	16
	38	33	11
	40	28	9
Interval	14^m	$18^s = 858^s$	Total 66

PERIOD

$$t = \frac{858}{858 + 66} = 0.9286 \text{ s}$$

XXVI. GRAVITY BY FREE FALL

Determine the acceleration of gravity by observing a freely falling body.

70. Freely Falling Body. — An iron ball is supported by an electromagnet and released by a key which also records a signal upon an electric high-speed chronograph (Art. 68) or upon a tuning-fork chronograph (Art. 71). A contact piece is placed below by which a second signal is recorded when the ball strikes, thus permitting the time of fall to be determined. The contact may be placed at several known distances below the starting point of the ball, as 100, 200, and 300 cm; and with the results the equations of accelerated motion given in Art. 59 may be verified. The method is more useful for a ball dropped from the top of a tall shaft, the acceleration of gravity being directly obtained. The distance fallen is measured with a steel tape, the temperature correction being applied.

If l is the distance fallen in the time t, the acceleration of gravity is

$$g = \frac{2\,l}{t^2}.$$

71. The Tuning-Fork Chronograph. — For accurately determining an interval of time not exceeding a few seconds in duration, the tuning-fork chronograph is suitable. It is a machine by means of which a piece of smoked paper is moved under a style attached to one prong of a tuning fork of known vibration number, the vibrations of the fork causing the style to trace a sinuous line upon the paper. The paper may be attached to the surface of a rotating cylinder or may be in the form of a ribbon upon reels. There are two methods by which an observation may be recorded. The cylinder and the fork may be connected to the terminals of the secondary circuit of an induction coil, so that making or breaking the primary circuit causes a spark to pass from the style through the paper to the cylinder, producing a record. Or a second style attached to the armature of an electromagnet may trace a line beside the sinuous mark of the fork, and making or breaking the circuit through this magnet will produce an indentation in the line. Fig. 39 shows such a record made by an instrument having two recording styles, one on either side of the fork. Lines are drawn from the notches on one side of the wave to those on the other side; by counting the number of waves between two cross lines, the time interval is determined with great precision.

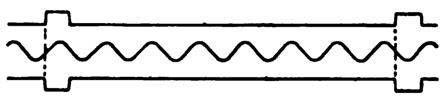


FIG. 39. RECORD OF TUNING-FORK CHRONOGRAPH

A fork commonly used gives one hundred complete vibrations per second, and is maintained in motion automatically by an electromagnetic attachment. The paper may be moved by clockwork, by hand, or in any other convenient manner. Uniform motion is desirable but not essential.

In the absence of a tuning-fork chronograph, a pen chronograph, arranged for a high-speed rotation of the cylinder, may be used (Art. 68).

72. Smoked Paper and Glass. — Glass or paper may be smoked, as is required in many experiments, by passing it through the smoking flame of a candle or, better, by holding it in the smoke

produced by burning camphor gum. Usually a very thin coating is preferable to a thick one.

A paper ribbon may be conveniently smoked by passing it over the surface of a brass cylinder filled with cold water, while a smoking kerosene flame plays against it. The water absorbs the heat, preventing burning and condensing the smoke.

CHAPTER IV

ELASTICITY, AND PROPERTIES OF MATTER

XXVII. YOUNG'S MODULUS BY STRETCHING

(a) Determine the modulus of elasticity and the elastic limit of a short iron wire.
(b) Determine the modulus of elasticity of a long steel wire.

73. Coefficients of Elasticity. — The elasticity of an isotropic body under any form of stress may be described with the aid of two coefficients or moduli of elasticity, the *modulus of bulk elasticity*, the reciprocal of which is the compressibility, and the modulus of elasticity of shape or the *modulus of rigidity*. Only solids possess the latter form of elasticity, hence rigidity is chiefly used in describing them; the modulus of bulk elasticity is used for fluids, though it applies also to solids.

Besides the above a somewhat simpler modulus is frequently employed, called *Young's modulus;* it is the modulus for simple longitudinal strain, in which change of lateral dimensions is not considered.

Each modulus is measured by the ratio of the applied stress to the resulting strain. The bulk modulus is denoted by K, the modulus of rigidity by R, and Young's modulus by M.

When a varying stress not exceeding a certain limit is applied to a solid, the resulting strain is proportional to the stress. That limiting value of the stress beyond which the strain is no longer proportional to the stress is called the *elastic limit* of the substance. A body subjected to a stress greater than the elastic limit is permanently deformed.

74. Young's Modulus and Elastic Limit. — Attach the wire to be tested to a comparator, in the manner shown in Fig 40. The

wire is held at one end by a firm screw, *S*, which permits of longitudinal adjustment; while the other end is attached to a right-angled arm in order that the weight of a load, *F*, placed in the pan shall be transmitted as a horizontal stress on the wire. Sufficient load should be placed in the pan to take the kinks out of the wire. An index microscope, *X*, is adjusted upon a fine mark placed near the end of the wire, and a micrometer microscope, *M*, is set upon a mark at the other end, about a meter distant. Note the reading of the micrometer microscope. Place a weight, for instance a kilogram, in the pan; readjust the mark under the index microscope by the screw *S*, set the micrometer microscope again on the mark, and note reading. The

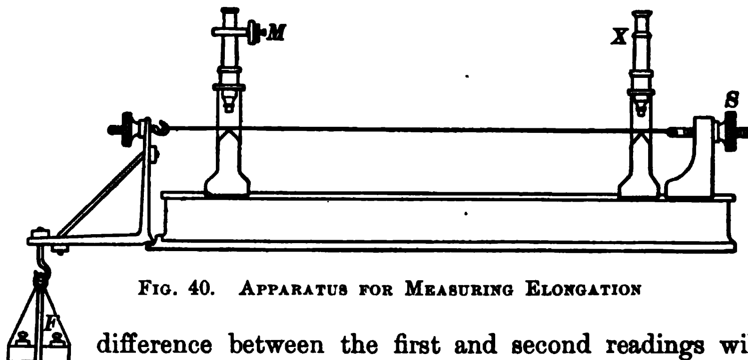


Fig. 40. Apparatus for Measuring Elongation

difference between the first and second readings will give the elongation produced in the length of wire between the two marks by a weight of one kilogram. Remove the kilogram, readjust the screw, and repeat the zero reading. Make the load two kilograms, then one kilogram; three kilograms, and then two; adjusting the screw *S*, and observing the elongation produced in each case. In this manner increase the load a kilogram at a time, removing the last added kilogram to determine whether the wire has been permanently stretched. Continue until the elastic limit is reached.

Plot the results, the abscissæ representing the stresses, and the ordinates the strains.

From the observations within the elastic limit find the elongation produced by each kilogram of change in the load,

expressing the mean value in centimeters (Art. 21). This is the quantity s of the formula, and F is the load producing the stretch, in this case 1000 g. Measure a number (ten) of diameters of the wire in different directions and in different parts, and let r be the mean radius. With a meter bar placed upon the table of the comparator determine the original length, l, of the wire between the marks.

Then the stress in dynes per square centimeter is $\frac{Fg}{\pi r^2}$, and the strain per unit length is $\frac{s}{l}$. Young's modulus is

$$M = \frac{\text{stress}}{\text{strain}} = \frac{Fgl}{\pi r^2 s}.$$

The report may be in the following form.

Young's Modulus of Annealed Iron Wire

WITH A MICROMETER-MICROSCOPE COMPARATOR

August 15, 1902

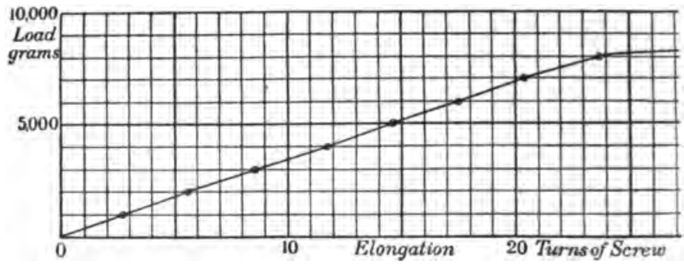
Loads, grams 2100 +	Readings 1st	Readings 2d	Averages Turns	Elongations per 1000 g	Diameters cm
0	0.00	0.08	0.04		
1000	2.67	2.69	2.68	2.64	0.0568
2000	5.63	5.50	5.56	2.88	9
3000	8.62	8.50	8.56	3.00	5
4000	11.72	11.84	11.78	3.22	9
5000	14.66	14.60	14.63	2.85	5
6000	17.46	17.44	17.45	2.82	8
7000	20.36	20.41	20.39	2.94	6
8000	23.69	61.4	elastic limit		7
9000	61.4				9
10000	wire broke				6
Means				2.907	0.0567

Micrometer microscope, No. 2, 169.7 turns per cm.

Elongation, $s = \dfrac{2.907}{169.7} = 0.01712$ cm. Load, $F = 1000$ g.

Radius, $r = 0.0284$ cm. Length, $l = 89.3$ cm.

$$M = \frac{Fgl}{\pi r^2 s} = 2.02 \times 10^{12}.$$



Plot of the Observations

75. Young's Modulus with a Suspended Wire and Scale. — Suspend from the same support two pieces of wire, each from 3 m to 10 m long, and to one piece attach at its lower end a millimeter scale and a small weight to keep it stretched. To the other wire fasten a vernier arranged to move over the scale, and below all a pan for weights. Measure with a steel tape the length of the wire to be stretched when there is sufficient weight in the pan to keep it straight. Then proceed to determine the elongation produced by the successive addition of weights in the manner described in the preceding article. The modulus of elasticity is calculated by the same formula.

Reference. — *Glazebrook and Shaw*, Practical Physics, p. 141.

76. Young's Modulus with the Cathetometer. — Suspend vertically from a firm support a wire about a meter long, to the lower end of which is attached a pan for weights. The elongation produced by the addition of weights is then measured with the cathetometer (Art. 77), having one or two reading telescopes. Needles may be fastened to the wire with wax, to serve as indices. The method of procedure and calculation is as described in Art. 74.

Reference. — *Nichols*, A Laboratory Manual of Physics, Vol. I, p. 74.

77. Adjustment of the Cathetometer. — A cathetometer (Fig. 41) is essentially a vertical scale carrying a horizontal telescope capable of motion up and down, the amount of this motion being determined by a vernier attached to the telescope. The following are the adjustments usually required. The second and third, having once been carefully made, should not thereafter be disturbed, so that ordinarily it is necessary only, to adjust for parallax, to set the scale vertical and the telescope horizontal.

To adjust for Parallax. — Adjust the sliding eyepiece until the cross wires are seen with perfect distinctness; then focus by the rack and pinion, or long draw tube, upon the object or mark. If, now, a motion of the eye produces a relative motion of the cross wires and the image of the object, the preceding adjustments must be altered and the process repeated until no such motion is produced by moving the eye, and both the cross wires and object are seen distinctly.

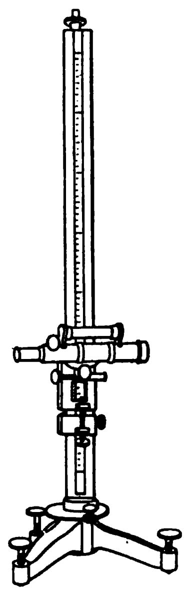


FIG. 41
CATHETOMETER

To adjust the Line of Collimation. — Set the cross wires exactly upon some well-defined point, and rotate the telescope upon its own axis. If the cross wires move away from the point, they must be brought back partly by adjusting the reticule held by small screws in the eyepiece and partly by altering the direction of the telescope. Repeat this process until the cross wires remain exactly upon the point during a complete rotation of the telescope.

To set the Level Parallel to the Optical Axis of the Telescope. — Move the telescope until the level bubble is central. Reverse the telescope with the attached level in its Y's. If the bubble is not central it must be made so, partly by means of the screws that attach it to the telescope and partly by moving the whole telescope. Repeat this operation until the bubble remains central, when the telescope is reversed.

To set the Scale Vertical and the Telescope Horizontal. — Turn the column till the level is parallel to the line joining two of the leveling screws in the base, and make the bubble central. Turn the column through 180°, and if the telescope is not level, adjust it, partly by means of the leveling screws and partly by altering the angle between telescope and scale. Turn the column through 90° and level by the third leveling screw. Repeat the whole process until the bubble remains central during an entire revolution of the column.

REFERENCE. — *Stewart and Gee*, Practical Physics, Vol. I, pp. 27–35.

XXVIII. MODULUS OF ELASTICITY BY FLEXURE

Determine the modulus of elasticity of a rectangular steel bar when it is resting on its broader side; also when resting on its edge. Make the same measurements for a wood bar. Find the modulus of elasticity of a round brass rod.

78. Flexure of a Rectangular Bar supported at Both Ends. — The ends of the bar are supported upon knife-edges resting on

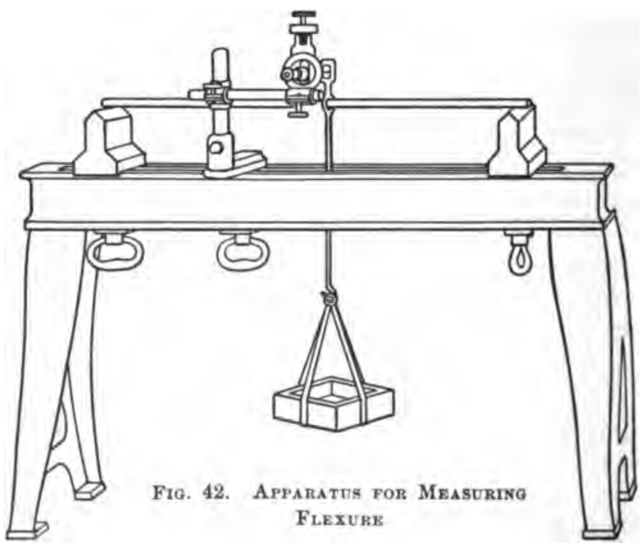


FIG. 42. APPARATUS FOR MEASURING FLEXURE

a rigid bedplate, as shown in Fig. 42. Midway between the knife-edges there hangs upon the bar a hook or pan for carrying weights. Above the bar, supported by this hook, is a divided scale or a fine wire, which serves as an index. Attached to the bedplate is an adjustable microscope with which the flexure may be measured.

Adjust the microscope and observe the reading. Add a weight (500 g, more or less, according to the strength of the beam) and determine the flexure produced; add weights, one at a time until five have been used, observing the increase of flexure produced by each one.

Let P be the value of each of the weights expressed in grams, F the average deflection produced by this load in its successive additions expressed in centimeters, L the distance between the supports of the bar, a the vertical dimension of the cross section of the bar, and b the horizontal dimension; then the modulus of elasticity of the bar is

$$M = \frac{PgL^3}{4\,Fa^3b}.$$

79. Bar supported at One End; Cylindrical Bar. — If the bar is fixed at one end and the weight is applied at the other end, the modulus is

$$M = \frac{4\,PgL^3}{Fa^3b}.$$

If the section of the bar in either case is a circle of radius r, instead of a^3b substitute $3\,\pi r^4$.

REFERENCES. — *Stewart and Gee*, Practical Physics, Vol. I, pp. 179–186; *Kohlrausch*, Physical Measurements, pp. 128–130.

XXIX. COEFFICIENT OF RIGIDITY WITH THE TORSION LATHE

Determine the coefficients of rigidity of one steel rod and of two brass rods of different diameters.

80. The Modulus of Rigidity. — When a substance is in the form of a solid rod its modulus of rigidity may be determined from measures of its torsion. If a rod of length l and radius r

is twisted through an angle θ (radians) by a couple C (gram-centimeters), the modulus of rigidity is

$$R = \frac{2\,gCl}{\pi r^4 \theta}.$$

Clamp the ends of a metal rod, from 3 mm to 6 mm in diameter, in the chucks of the torsion lathe, so that the length of the rod to be twisted shall be about a meter. One chuck is rigidly attached to the base, while the other is capable of rotation about the axis of the rod by means of a large pulley (Fig. 43), the amount of rotation being determined by a divided circle. This circle may be divided into two hundred parts, when, since the circumference equals $2\,\pi$ radians, each division is $0.01\,\pi$ radians.

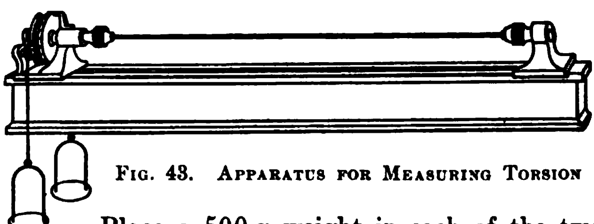


FIG. 43. APPARATUS FOR MEASURING TORSION

Place a 500 g weight in each of the two pans, which are attached by cords to the pulley so that they act as a couple to twist the rod. Observe the amount of twist produced. To eliminate the effects of friction disturb the circle from its position of rest several (five) times, reading its indications each time, and take the average position as a single measure. Add a second 500 g weight to each pan, then a third, and then remove the weights one at a time; determine the twist resulting from each change of couple. Find the average effect. The couple is measured by the mass, 1000 g as described, multiplied by the radius of the pulley.

Rods of the same material, of equal lengths but of different radii, of the same radius but of different lengths, and rods of various materials, should be experimented with. In this manner all the laws of torsion and rigidity may be demonstrated.

81. Simple Torsion Apparatus. — The laws of torsion may be illustrated by simple apparatus, the rod or wire being suspended

vertically in a firm clamp, which is attached to the wall or supported on a tripod. To the bottom is fastened a heavy weight carrying a pointer moving over a divided circle of paper (Fig. 44). Two spring balances or weights are attached to cords passing around the weight, acting as a couple, of any desired moment, tending to twist the wire. Pointers attached to the wire by wax and moving over divided circles, placed at various positions along its length, may be used to show that the angle of torsion varies as the length of the wire. To prove the relation of torsion to the radius of the wire, fasten together two pieces of unequal diameter and of equal length, and suspend as before with a pointer attached to the bottom of each piece. The respective angles of torsion should satisfy the formula,

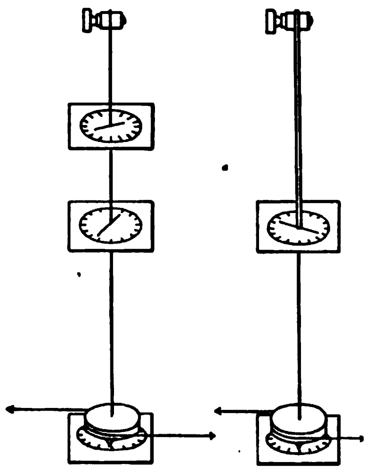


FIG. 44. SIMPLE TORSION APPARATUS

$$\theta = \frac{2gCl}{\pi R r^4},$$

θ being the angle of torsion of a wire whose length is l, radius r, and rigidity R, which is twisted by a couple C.

If θ is expressed in radians, the modulus of rigidity, R, may be computed as described in the preceding article.

XXX. MODULUS OF TORSION BY THE TORSION PENDULUM

Determine the modulus of torsion of several wires with the torsion pendulum.

82. Modulus of Torsion. — Suspend a heavy weight of some simple, regular, geometrical shape by a fine wire (Fig. 45), the upper end of the wire being rigidly clamped. Turn the weight

through an angle of 30°, twisting the wire, and release it; it will oscillate with a rotary motion, and constitutes a torsion pendulum. The formula expressing the condition of vibration is

$$T = 2\pi\sqrt{\frac{I}{\tau}},$$

in which T is the time of a complete vibration, I the moment of inertia of the weight, and τ the modulus of torsion of the wire. The time of vibration is to be determined by the method of the following article, and the moment of inertia may be calculated by the formula given in Art. 85.

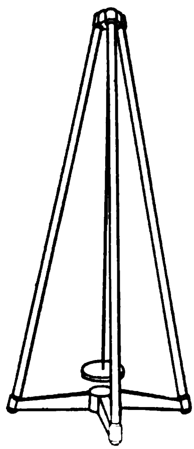


FIG. 45
TORSION PENDULUM

83. Periodic Time by the Method of Transits. — This method of determining the time of a slow periodic movement is so valuable that it will be described in detail. Adjust a telescope so that the vertical cross wire is seen in the prolongation of the torsion wire and covers an index mark placed on the edge of the disk when it is stationary. Cause the disk to vibrate, stopping any pendular vibration with the hand, and the index will move back and forth in the field of view, crossing the wire twice in each complete vibration. Let one observer, watching the vibration, give the signal "tick" each time the index crosses the wire, for ten successive transits; while another observer records the time of each signal from a clock, chronometer, or watch, noting the day, hour, minute, second, and *tenths of a second*. With a clock beating seconds or a chronometer beating half seconds, a single observer can note the time by the eye-and-ear method, observing the index at the second just before and just after transit to assist in estimating the tenths of seconds. Repeat the series of ten readings twice, at intervals of fifteen or twenty minutes, being careful that the disk swings in the *same* direction at the *beginning* of each series.

Add together the fifth and sixth times of transit of each set; also the fourth and seventh, the third and eighth, the second and ninth, and the first and tenth. Divide each sum by two, and average the five quotients of each series. These results will give the times of the *middle elongations*, or the times when the index was at its greatest distance from the cross wires between the fifth and sixth transits of each series. Dividing the interval between the two middle elongations by the number of vibrations will give the periodic time with great accuracy. The number of vibrations may be determined from the observations without the trouble of counting. Between the first and ninth, or second and tenth transits of each set, there were four complete vibrations, and this interval divided by four is approximately the periodic time. If all the observations were exact, the period thus found would be exact. To reduce the unavoidable errors of observation it is desirable to find the period from a longer interval. The interval between any two elongations would contain the exact period an integral number of times, provided the disk moved in the same direction at the beginning of each set. The quotient will be a whole number plus a half, if the disk moved in opposite directions. Divide the interval between two elongations by the approximate period; in general the result will not be an integer, but will be so near a whole number that there will be no doubt as to the true number of vibrations. Having thus found the number of vibrations in the long interval, the interval divided by this number will give a more precise value of the period.

The length of time to be allowed between the sets of observations will depend upon the accuracy of the work and the periodic time; other things being equal, the longer the interval the more exact the final result. Two sets of observations are sufficient, but it is better to make three, as directed, to guard against errors. If the interval between the first and third sets is too long to give a sure result, one of the shorter intervals can be used. The middle set may also aid in determining the number of vibrations in the long interval. A numerical example of such a determination will illustrate the method.

PERIOD OF TORSION PENDULUM; METHOD OF TRANSITS

Time from Chronometer

May 24, 1895

Transit	Time	Elongation	Transit	Time	Elongation
	h m s			h m s	
1	2 : 10 : 8.2		1	2 : 30 : 51.3	
2	2 : 10 : 14.6		2	2 : 30 : 58.0	
3	2 : 10 : 20.8		3	2 : 31 : 4.1	
4	2 : 10 : 27.0		4	2 : 31 : 10.5	
5	2 : 10 : 33.7	h m s	5	2 : 31 : 16.9	h m s
6	2 : 10 : 40.0	5–6 2 : 10 : 36.85	6	2 : 31 : 23.5	5–6 2 : 31 : 20.20
7	2 : 10 : 46.9	4–7 2 : 10 : 36.95	7	2 : 31 : 30.0	4–7 2 : 31 : 20.25
8	2 : 10 : 53.0	3–8 2 : 10 : 36.90	8	2 : 31 : 36.3	3–8 2 : 31 : 20.20
9	2 : 10 : 59.3	2–9 2 : 10 : 36.95	9	2 : 31 : 42.4	2–9 2 : 31 : 20.20
10	2 : 11 : 5.6	1–10 2 : 10 : 36.90	10	2 : 31 : 48.9	1–10 2 : 31 : 20.10
		Average 2 : 10 : 36.91			Average 2 : 31 : 20.19

The interval between the two sets is 20 m 43.28 s, equal to 1243.28 s. The approximate time of vibration, determined from the first and ninth observations of the first set (51.1 ÷ 4), is 12.8 s.

$$1243.28 \div 12.8 = 97.1+.$$

From this result it is certain that there were 97 vibrations in the interval, which gives the final value of the period,

$$P = 1243.28 \div 97 = 12.817 \text{ s}.$$

XXXI. MOMENT OF INERTIA BY THE TORSION PENDULUM

Determine the moment of inertia of an irregularly shaped body.

84. Moment of Inertia by Torsion Pendulum. — If a body is suspended by a wire and caused to vibrate as a torsion pendulum, its moment of inertia is

$$I = \frac{T^2\tau}{4\pi^2},$$

T being the period, and τ the modulus of torsion of the wire. To determine the moment of inertia of a body, it may be suspended by a wire whose modulus of torsion is known (Art. 82), in such a manner that the axis about which the moment of inertia is required shall be in the prolongation of the wire. The system is then made to vibrate as a torsion pendulum, and its period is determined by the method of Art. 83.

If the modulus of torsion of the wire is unknown, it will be necessary to make two observations. The period of the pendulum is determined when the body, the moment of inertia of which is required, is suspended as described above; the second observation is for the period of the pendulum when the suspended body is one whose moment of inertia is known, by calculation from its dimensions (Art. 85) or otherwise. If I_1 is the unknown moment of inertia, and I_2 the known, and T_1 and T_2 the respective periods, by elimination between two equations of the previous form,

$$I_1 = I_2 \frac{T_1^2}{T_2^2}.$$

Instead of substituting the body with known moment of inertia for the one whose moment of inertia is to be determined, the former may be added to the latter, and the period of the pendulum, consisting of the combined bodies, is determined. Let I_1 be the unknown moment of inertia, and I_2 the known moment of inertia, and T_1 the period when the first body only is suspended, and T_{1+2} the period with the combined bodies; then

$$I_1 = I_2 \frac{T_1^2}{T^2_{1+2} - T_1^2}.$$

85. Formulæ for Moments of Inertia. — The moment of inertia of a *solid cylinder* about its axis of figure is

$$I = m\frac{r^2}{2},$$

r being the radius and m the mass.

For a *hollow cylinder* about its axis of figure, the moment of inertia is

$$I = m\,\frac{(r_1^2 + r_2^2)}{2},$$

r_1 and r_2 being the inner and outer radii of the ring, and m its mass.

For a *sphere* of radius r and mass m, the moment of inertia about a diameter is

$$I = \frac{2\,mr^2}{5}.$$

The moment of inertia of a *parallelopipedon* of mass m, about an axis passing through its center of mass and perpendicular to the face whose edges are a and b, is

$$I = m\,\frac{a^2 + b^2}{12}.$$

REFERENCE.—*Stewart and Gee*, Practical Physics, Vol. I, p. 243.

XXXII. COMPRESSIBILITY OF A LIQUID WITH THE PIEZOMETER

Determine the compressibility of water.

86. The Piezometer.—The compressibility of a liquid may be determined with a piezometer, which consists of a strong glass cylinder (Fig. 46) the ends of which are closed by metal caps, so that when the cylinder is filled with water, a great internal pressure may be produced by means of a screw plug. A glass bulb, B, with a narrow, open stem, is provided for containing the liquid whose compressibility is to be determined. This bulb, a thermometer, T, and a manometer, M, are attached to a suitable support, and during the experiment are placed in the water in the cylinder.

It is necessary to know the volume, V, of the liquid experimented upon, and the change in volume, v, which is caused by a change in pressure, P. Carefully weigh the bulb when it is

empty and dry. Then draw into the stem a thread of mercury which is nearly as long as the stem, and measure the length of the thread. Allow the mercury to drop into the bulb, and again weigh to determine the weight of the mercury. Complete the calibration by the method of Art. 52, finding the volume, c, of one centimeter of length of the tube. Now drive the mercury out of the bulb, and fill it and about half of the stem with water. Immerse the bulb in a water bath; notice the temperature and the exact position of the end of the water column in the stem. Wipe the outside of the bulb and again weigh; by the formulæ of Art. 51 find the volume, V, of the bulb.

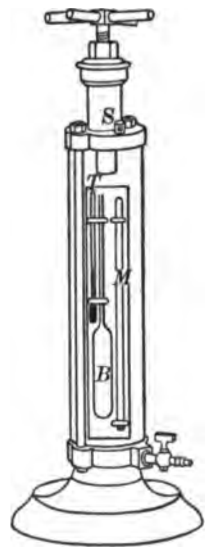


Fig. 46. Piezometer

A drop of mercury is placed in the stem for an index, to show the position of the end of the liquid column. The bulb is attached to its support and is placed in the cylinder, which is filled with water. The caps are firmly fastened, the vent screw, S, being open.

The manometer may be a glass tube, M, closed at the top and open at the bottom, which is filled with air while it is placed in the water. The pressure, H, of the air in M is equal to the barometric pressure plus the pressure due to the depth of the water. The latter is expressed in terms of a mercury column by dividing the distance from S to the bottom of M in centimeters by 13.6. Observe the position of the mercury index, and the length, l, of the air column in M, and the temperature, t. Close the vent and increase the pressure as much as is convenient, to 5 or 10 atmospheres for instance. Again observe the length, l_2, of the air column, and the position of the index. The volumes of the air may be taken as proportional to the lengths of the column, a proper allowance being made for the change in shape of the tube where it is sealed. It would be more precise if the tube were graduated and calibrated as to volume.

Make ten determinations, and let n be the mean change of the index. Then the apparent change in volume is $v = n \times c$. The change of pressure in dynes is

$$P = \frac{l_1 - l_2}{l_2} \cdot H \times 13.6 \times 980.$$

The apparent compressibility is then

$$k = \frac{v}{PV}.$$

The pressure in the piezometer will decrease the volume of the bulb, notwithstanding its being open, by the same amount as though it were of solid glass. Hence the observed compression is the difference between that of the liquid and the glass. The real compressibility, at the temperature $t°$, is

$$K_t = k + k_1,$$

k_1 being the compressibility of glass, which is 2.6×10^{-12} per dyne per square centimeter.

XXXIII. BOYLE'S LAW WITH THE U-TUBE

Verify Boyle's Law for pressures both greater and less than one atmosphere.

87. Boyle's Law. *Pressures greater than One Atmosphere.* — An apparatus for verifying Boyle's Law consists of a U-tube about a meter in length, one branch ending in a funnel, the other capable of being closed by a stopcock (Fig. 47). At the bottom is an outlet closed by a stopcock, and between the tubes is a scale. The upper stopcock being open, pour mercury into the tube till it stands at the zero of the scale in both branches; if too much mercury is introduced, some may be drawn off at the bottom. Close the stopcock at K tightly. Pour mercury in at the funnel till it stands at the height of 10 cm in the open arm. Denote this height by h, and read the height, h', of the column in the left branch. Add mercury to make h 20 cm, and read h'. Continue increasing h by 10 cm at a time till it becomes 100 cm, finding the corresponding values of h'. Then make a set of

measures in reversed order, by drawing off mercury at the bottom so that h is 90 cm, 80 cm, etc. The top of the mercury column is represented in each case by the plane tangent to the meniscus.

The products of the volumes and the corresponding pressures of the inclosed gas should all be equal provided the temperature has remained constant. The volumes may be taken proportional to the lengths of the tube occupied by the air. Let K be the division of the scale at the top of the inclosed part of the tube (so chosen that it corrects for the distortion of the tube where the stopcock is sealed on); then the volumes are represented by $K - h'$. A further correction may be made by adding a small quantity, c, which represents the volume of air contained between the tangent plane to the mercury meniscus and its actual surface. The pressures are equal to the barometer reading plus the differences between heights of the two columns of mercury in the U-tube, $P = B + (h - h')$. Corrections for the temperature of the mercury are not required if the barometer and manometer columns are at nearly the same temperature. Record the observations in tabular form (see table on following page), and plot them, using the pressures as abscissæ and the volumes as ordinates.

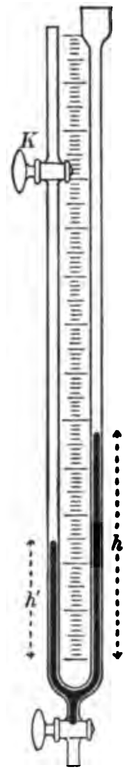


FIG. 47
BOYLE'S LAW

Pressures less than One Atmosphere.—With the stopcock, K, open, fill the U-tube with mercury till the top of the column is about 10 cm below K. Close K, and read the heights, h and h', of the right and left columns. Draw off mercury at the bottom to lower the column h by 10 cm, and record the height h'. Continue lowering h 10 cm at a time till it becomes zero, finding the corresponding values of h'. Repeat the measures by increasing h 10cm at a time till it is again 100 cm. Record and plot as described in the preceding article, the pressures in this case being $P = B - (h' - h)$.

Other forms of apparatus are frequently used; for instance, the mercury may be forced from a cistern into the tubes by compressed air, or the two tubes may be joined by a flexible tube, or a tube closed at one end may be dipped into a jar of mercury. But with any form the measurements outlined are to be made.

h	h'	$P = B + (h - h')$	$V = K - h' + c$	PV
10				
20				
30				
—				
100				

XXXIV. ERRORS OF AN ANEROID WITH AN AIR PUMP

Find the corrections to an aneroid barometer for each centimeter of pressure from 65 to 76 cm.

88. Testing the Aneroid Barometer. — The aneroid barometer has the advantage of portability, and if frequently compared with a mercurial barometer and adjusted, it will serve excellently for approximate determinations of altitude.

The aneroid may be set, by turning the adjusting screw, so that its reading agrees with that of the mercurial barometer, though this will not be necessary if the error is small. Place it in a receiver (Fig. 48) having a plate-glass cover with a ground joint. The receiver connects through a stopcock with an air pump and with a U-tube mercury manometer.

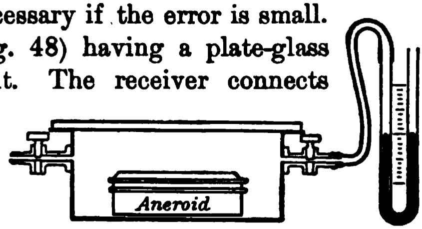


FIG. 48. TESTING ANEROID

Partially exhaust the receiver till the aneroid indicates exactly an even centimeter (or half-inch) mark on its scale. It may be easier to exhaust a little too much, allowing air slowly to leak in

till the pressure is that desired. Read the difference in the level of the mercury columns in the manometer with a scale, or, better, with a cathetometer. The actual pressure on the aneroid is measured by the external barometric pressure less the manometer pressure. Both of the quantities should be corrected for the temperature of the mercury; but if the barometer reading is corrected at the beginning, usually appreciable errors will not be introduced by neglecting the temperature correction to the manometer.

Reduce the pressure till the aneroid indicates the next point on its scale for which the correction is desired, and repeat the observations. Continue until each point has been tested.

If the cover of the receiver is firmly held in place, it will be possible slightly to compress the air and then to test the aneroid at pressures greater than the external atmospheric pressure.

The corrections are the quantities which must be added to the aneroid indications to make them equal to the actual pressure. Record the work in tabular form as indicated below, and plot a curve of corrections. The table shows the comparison of an aneroid having an inch scale with a barometer having a centimeter scale.

The temperature corrections to an aneroid must be determined empirically; therefore the table of corrections can only be relied upon for the temperature of the trial, which should be recorded.

CALIBRATION OF ANEROID No. 1 R

February 27, 1902

Barometric reading, 74.30 cm; $t = 21°.6$; $B = 74.04$ cm				
ANEROID inches	MANOM. cm	B—MANOM. cm	B—MANOM. inches	CORRECTIONS inches
30.00				
29.50				
29.16	0.00	74.04	29.15	– 0.01
29.00	0.43	73.61	28.98	– 0.02
28.50				

XXXV. COEFFICIENT AND LAWS OF FRICTION BY SEVERAL METHODS

Determine the coefficients of friction, both static and kinetic, for pine wood sliding on a horizontal cast-iron plane; determine the kinetic friction when the pressure is increased by one and by two kilograms. Determine the kinetic friction for the same body on an inclined cast-iron plane; determine the kinetic friction for a brass block sliding on each of its unequal faces upon the inclined plane. Determine the kinetic rolling friction for a wood and a brass cylinder.

89. Coefficient of Friction. — Adjust the friction table so that its surface is horizontal, and place upon it the body to be tested, adding weights to make the total pressure three or more kilograms. By means of a cord and weights arranged to pull the block in a horizontal direction (Fig. 49) determine what force

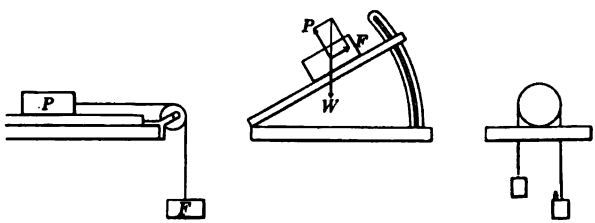


FIG. 49. FRICTION APPARATUS

is necessary to start the body from rest. The ratio of this force to the total pressure is the *coefficient of static friction.* Determine also the force necessary to keep the body in uniform motion after it is started, and the ratio of this to the pressure is the *coefficient of kinetic friction.* A correction to these observations may be made for the friction of the pulley over which the cord passes, determined as described below.

Make a number of experiments with different substances, as pine, oak, iron, and brass, for the sliding body. Also make a series of observations to determine the manner in which the friction increases with the increase in pressure between any two given surfaces, by repeating the observations with the pressure

increased by one, two, and three kilograms. Represent the results graphically.

The work may be simplified by using a spring balance attached to the cord and drawn horizontally. When used in this position a correction must be applied for the index error of the spring balance. The body can be weighed with the spring balance.

Friction on an Inclined Plane.—Place the blocks used in the preceding experiment, loaded as before, upon an inclined plane (Fig. 49) whose surface is of the same material as that of the horizontal plane. So adjust the angle of inclination that the body starts to slide downward. Then the ratio of the height of the inclined plane to its base, that is the tangent of the angle of inclination, is the coefficient of static friction, which should agree with the result already obtained. The tangent of the angle of inclination necessary to maintain the body in uniform motion after it is started is the coefficient of kinetic friction.

Determine the coefficient of kinetic friction of a brass block the dimensions of which may be $2.5 \times 5 \times 10$ cm, sliding in turn upon each of the three unequal faces.

Rolling Friction.—Arrange a cylinder to roll upon a track of two horizontal and parallel bars (Fig. 49). Suspend two equal weights by a cord passing around the cylinder, and by applying additional weights to one side determine the coefficients of static and kinetic rolling friction. Vary the experiment by using different substances; also use tracks of several materials, and vary the loads in each case.

REFERENCE. — *Barker*, Physics, pp. 81–87.

XXXVI. SURFACE TENSION WITH CAPILLARY TUBES

Determine the surface tension of water, using three different capillary tubes.

90. Capillarity and Surface Tension. — If a tube is held vertically, dipping into a liquid which from the nature of the materials wets the tube, the liquid is drawn upward along the walls of the tube. Since liquids possess surface tension, this

effect alone would cause the liquid to rise and fill the tube to its top, regardless of length or diameter; but when the liquid rises above the normal surface, gravity begins to act to draw it downward. These two forces are oppositely directed, and tend to stretch the surface of the liquid. It will rise, therefore, only so high that the maximum tension which the surface can exert is counterbalanced by the weight of the liquid lifted. The tension exerted by the surface on a line one centimeter long is called the surface tension, and is designated by T.

For a tube of radius r the tension exerted along the circumference is $2\pi rT$. The weight (in dynes) of the liquid column is $\pi r^2 h\delta g$, h being its height and δ its density. These two forces are equal.

$$2\pi rT = \pi r^2 h\delta g,$$

from which $$T = \frac{rh\delta g}{2}.$$

To correct for the meniscus, h should be measured from the level of the liquid in the outer vessel to a point one third of the radius of the tube above the lowest surface of the meniscus, as indicated in Fig. 50.

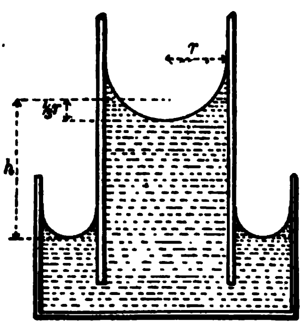


FIG. 50. CAPILLARITY

The tubes may have a diameter varying from 0.1 cm to 0.01 cm. These tubes may be readily drawn from larger ones if those of the desired size are not provided. The radius of the tube required is that at the top of the capillary column; the tube may be broken at this point after the experiment, and its diameter measured with the micrometer microscope. If the tube is of uniform bore, its radius may be conveniently determined by mercury calibration. The height of the liquid column may be measured with a glass scale or, better, with a cathetometer.

The surface tension varies with the temperature; therefore record the temperature of the liquid.

It is necessary that the tubes and vessel for the liquid should be chemically clean (Art. 91), and that the liquid should be pure. On account of the difficulty of cleaning tubes of small bore required in this experiment, it is desirable to use only tubes which have never become soiled. The tubing should be sealed at the time of its manufacture, and such pieces as are required are cut from the stock, and used for capillary measurements but once.

REFERENCE. — *Ostwald*, Physico-Chemical Measurements, pp. 163–168.

91. Cleaning Glassware. — Glass vessels and tubes are commonly best cleaned with a solution of caustic potash. The use of caustic soda should be avoided, because glass is dissolved more by it than by caustic potash. The entire surface to be cleaned must be scrubbed with a brush; after thorough rinsing with distilled water, the vessel may be allowed to drain and dry spontaneously. The surface will be left in better condition if it is dried by passing through the vessel a current of cold, dry air.

When the surface cannot be reached by a brush, small lead shot may be introduced with the caustic, and, by shaking, the entire surface should be scrubbed. After rinsing with water, wash with nitric acid to remove all trace of the lead; then rinse with distilled water and dry with a current of air.

Surfaces that cannot be treated as above may be washed with warm aqua regia or with chromic acid. The latter should remain in contact with the surface for several hours. Rinse and dry as before.

Warm air may be used for drying to save time, but it is not so good as cold air, as it usually leaves a stain on the surface.

If much grease is on the surface this should first be removed with carbon bisulphide, and then the cleaning may be continued by one of the above methods.

Instructions for cleaning optical glass surfaces are given in Art. 200.

XXXVII. SURFACE TENSION BY DIRECT MEASURE

Determine the surface tension of water and of soap solution.

92. Surface Tension. — A direct measurement of surface tension may be made with a beam balance in the manner described below. If a Jolly balance (Art. 102) is used, the counterbalancing of the surface tension is conveniently made by gradually increasing the tension of the spring. A second observation is then required to determine the weight in grams necessary to produce the same tension in the spring.

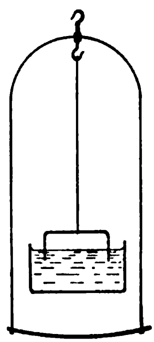


FIG. 51
SURFACE TENSION

Make of fine platinum or glass wire a $\sqcap$-shaped fork with prongs about 1.5 cm long and 3 cm apart. Suspend the fork by a fine wire from the hook of a balance (Fig. 51), and place a beaker containing the liquid on a bridge over the pan at such a height that when the beam is horizontal the prongs only dip into the liquid. Exactly counterbalance the fork and its suspension while it is in position. Now cause a film of the liquid to fill the space between the fork and the surface by tipping the beam to immerse the whole fork and then carefully raising it out. This film will tend to contract and pull the fork back to the surface. Determine by repeated trials the additional weight necessary to balance this tension; that is, the weight necessary to maintain the balance beam in horizontal position while the film is intact. If w is this weight, and l the width of the fork, the surface tension in dynes per centimeter of breadth of surface is

$$T = \frac{wg}{2\,l}.$$

Record the temperature of the liquid. Exercise great care to have the fork and beaker clean and the liquid pure. Care in manipulation is required to secure the liquid film. Excessive difficulty in obtaining it is probably caused by insufficient

cleaning of the fork. If the fork is of platinum, it may be cleaned by bringing to a red heat in a Bunsen flame. Methods for cleaning glass are described in Art. 91.

REFERENCE. — *Hastings and Beach,* General Physics, pp. 138–159.

XXXVIII. SPECIFIC VISCOSITY WITH TORSION PENDULUM

Determine the viscosity, at various temperatures, of lubricating and cylinder oils, compared with that of water.

93. Viscosity. — The resistance which a fluid offers to change of shape is its viscosity; it is ascribed to a kind of internal friction. *The coefficient of viscosity* is the tangential force per unit area of either of two horizontal planes, one of which is relatively fixed while the other moves with unit velocity, the space between being filled with the viscous material.

Viscosities are experimentally determined in two ways, from the resistance to flow through small tubes, and from the resistance to the motion of a solid body through the fluid.

If a torsion pendulum (Art. 82) is caused to vibrate while immersed in a liquid, part of its energy is expended in imparting motion to the surrounding liquid. The work done in imparting equal velocities to different liquids is proportional to their viscosities. The oscillations of the pendulum are isochronous, but their amplitudes continually diminish, and its motion is not simple harmonic.

When any other motion which would otherwise be simple harmonic has impressed upon it a retarding force which varies as the velocity, it is said to be *damped.* The arcs of successive swings of the pendulum diminish in a constant geometrical ratio; that is, the logarithms of these arcs are in arithmetical progression. The natural logarithm of the ratio of successive arcs of such a damped vibration is the *logarithmic decrement* (Art. 267). If R is the ratio of any amplitude of the vibration to the next succeeding amplitude, the logarithmic decrement is

$$\lambda = \log_e R.$$

It can be shown that the viscosity of a liquid in which the pendulum is vibrating is proportional to

$$K = \frac{\lambda}{T},$$

where T is the period of the pendulum.

A heavy disk has a short rod attached to it in the prolongation of its axis, and this is attached to a suspended wire, forming a torsion pendulum. The dimensions of the parts are such that the period is long, 30 seconds for instance. A pointer attached to the rod swings over a divided circle the center of which is in the axis of the wire, permitting the angular amplitude of vibration to be observed. The liquid to be investigated is contained in a suitable receptacle, preferably such that it may be placed in an oil bath while observations are being made. A gluepot is a suitable vessel. The vessel is then placed so that the disk of the pendulum is immersed, and the liquid is heated to the desired temperature.

The disk is displaced, 180° for instance, and caused to vibrate without lateral displacement. The period, T, is determined as explained in Art. 83; when the liquid is very thick it may be possible to use only the first part of this method, as the pendulum will soon come to rest.

Successive turning points, as indicated on the divided circle, are observed. From these are found the successive arcs of vibration, independent of direction. Let a_1 be any arc, and a_{1+n}, the nth following arc, which for accuracy may be about $\frac{1}{2}a_1$. The logarithmic decrement is

$$\lambda = \log_e R = \frac{2.3026}{n}(\log_{10} a_1 - \log_{10} a_{1+n}).$$

With these values of λ and T calculate K, the viscosity factor. Make three determinations with different mean amplitudes of vibration.

In this manner find the viscosity factor for water at about 20°; and for lubricating oil at the temperature at which it will be used in machinery, about 40°; and for cylinder oil at about 180°. The ratio of the factor for the oil to that for water is the specific viscosity.

CHAPTER V

DENSITY AND SPECIFIC GRAVITY

XXXIX. DENSITY BY HYDROSTATIC WEIGHING

Determine the density of aluminum.

94. Density and Specific Gravity. — The density of a homogeneous substance is the ratio of its mass to its volume at 0°; it is the mass per unit volume. The specific gravity of a substance is the ratio under specified conditions of its weight to the weight of an equal volume of water. In the centimeter-gram-second system, if the water is at 4°, density and specific gravity differ by 1 part in 20,000, specific gravity being the larger. (The density of water at 4° is found to be 0.99995: International Congress of Physics, Paris, 1900.) If the specific gravity is referred to water at 20°, it differs from the density by 1 part in 500. The term "density" should always have the meaning given, except that, in case it is not possible to reduce the volume of the body to 0°, the temperature of the determination must be specified.

In density determinations the mass of the body may be found by methods already described. The direct measurement of volume is usually difficult, and the method involved in the Principle of Archimedes may be employed. The loss in the weight of the body when immersed in water is the weight of the water displaced. The density of water at various temperatures having been determined with great precision, its volume per gram is known, and thus the volume of the displaced water is found.

The most useful methods for density determinations are described in connection with the hydrostatic balance and the

pyknometer. Other methods, the results of which are specific gravities rather than densities, are often more convenient, and give results which differ from true densities by amounts which for many purposes are insignificant.

95. Density by Hydrostatic Weighing. — The hydrostatic balance is one arranged to weigh a body while it is immersed in water, the operation best adapted for determining the volume in density investigations. Often a beaker of water is placed on a bridge over the pan, and the body whose volume is to be determined is suspended by a fine wire from a hook on the pan support; or the water may be placed below the pan. The latter arrangement, since it leaves the pan at all times accessible for the adjustment of weights, facilitates the operation. A pan with a short hanger may be provided, so that the dish of water may be placed in the balance case; or there may be a hole in the bottom of the case permitting the suspension of the body from the bottom of the ordinary pan in a dish of water placed underneath the balance.

The density of a substance is the ratio of its mass to its volume, the accurate determination of which requires corrections for the buoyancy of the air, the density of the water, weight of the harness, inequality of the balance arms, and variation in the sensibility of the balance. The method of weighing by substitution with a constant load eliminates several of these errors and makes it possible to correct for the others in the simplest manner.

Arrange a harness of wire to support the body, and attach this harness to the underside of the pan by a single wire about one hundredth of a centimeter in diameter. Submerge the harness in recently boiled distilled water in the position required for the hydrostatic weighing of the body. The harness is to be left in this position throughout the experiment. Place in the second pan any convenient load (weights from a second set) which is a trifle heavier than the body; this also is to remain unaltered during the weighings. Put precision weights in the pan to which the harness is attached, and adjust with the greatest care to bring the pointer to the middle of the scale. This

should be done to the nearest milligram, and then by vibrations and interpolation (Art. 42) fractions of a milligram may be determined. The hanger suspended in the water will dampen the vibrations and quickly bring the beam to rest; even though the vibrations are of little assistance, interpolation for the fractions of a milligram may be employed for loads of less than a hundred grams. The surface tension of the water around the wire will have a slight influence, but with wire of the smallest suitable diameter this may be neglected.

Remove the weights and put the body whose density is required in their place. Then add such weights as will exactly restore the balance to its equilibrium position, making the adjustment with the same care as before. The difference between the weights in the pan at the first and second adjustments is the apparent mass of the body in air.

Place the body in the harness in the water, removing all air bubbles from its surface. A camel's-hair brush or a piece of wire may be used for this. Stir the water, if necessary, and read its temperature. Add weights to the pan to restore equilibrium, adjusting with the greatest care. The weights added in this operation represent the apparent loss of weight of the body in water, and correspond approximately to its volume.

The observations are completed by reading the barometric height and the temperature of the air in the balance case.

The ratio of the weight of the body in air to the loss of weight in water is its approximate specific gravity. This may differ from its true density by as much as one part in three hundred. There must be applied corrections for the buoyancy of the air and for the density of the water of the experiment.

Since on one side of the balance containing the ballast load, the volume has not changed during the experiment, the buoyancy has been constant, and it may be considered as a part of the load, needing no further consideration. On the first side, however, the volumes have differed in each of the three weighings, and the result of each observation must be corrected by subtracting an amount equal to the weight of the air displaced. The volumes in the pan in the first and third weighings are the

volumes of the weights used. These will be known if the weights have been properly calibrated (Arts. 49 and 96); if not thus known, the volumes are to be calculated from the assumed density of the weights, — 8.4 for brass weights. In the second weighing the volume of air displaced is that of the weights in the pan plus the volume of the body whose density is being determined. The latter is approximately known from the uncorrected weighings, as already noted. The volumes in the pan multiplied by the mass of one cubic centimeter of air at the pressure and temperature of the experiment (Table 12) give the values of the required corrections.

The difference between the first and second corrected weighings is the true mass of the body, and the difference between the third and second is the mass of the water displaced. The temperature of the water being known, the volume of one gram of water is found (Table 3), and thus the true volume of the body at the temperature, $t°$, of the experiment. The ratio of mass to volume is the density of the body at $t°$.

If the coefficient of linear expansion, a, of the substance is known, the volume expansion factor is $1 + 3\ at$, and the density at $t°$ multiplied by this factor will be the true density at 0° of the substance.

A numerical example of the determination of the density of a piece of aluminum alloy is given.

A piece of copper wire 0.01 cm in diameter is attached to the left pan, and extends downward into a jar of distilled water beneath the balance. Under the water the wire ends in a harness adapted to support the piece of aluminum. A ballast load of 300 g is placed in the right pan. Weights from a set which had been calibrated and whose volumes had been determined are placed in the left pan to bring the pointer to the center of the scale, the adjustment being possible under this load only to the nearest milligram. The weights required are found to be 292.560 g, having a volume of 35.1 ccm. The piece of aluminum is placed in the left pan, and weights of only 88.480 g, having a volume of 10.6 ccm, are needed to bring the balance to the same position as before. The aluminum is now placed in the harness under water, the water is stirred, and its temperature is observed to be 22°.2. The weight required to restore equilibrium is 148.627 g of 17.8 ccm volume. The approximate volume of the aluminum is $148.6 - 88.5 = 60.1$ ccm. The corrected barometric pressure is 73.8 cm, and the temperature of the air in the balance case is 22°.7.

From tables the density of the air is found to be 0.00115, the density of the water 0.99772, and the coefficient of expansion of aluminum 0.000023. The complete record may be made as follows.

Density of Aluminum Alloy

January 15, 1902

Weighings by method of constant load, with Staudinger balance and Rueprecht weights.

Barometer, 74.07 cm at 22°.5 = 73.80 cm at 0°; air, 22°.7; water, 22°.2. 1 ccm air = 0.00115 g; density of water = 0.99772.

Observed Weighings		Vol. in Pan	Correction	Corrected Weighings	Masses
To counterbalance	292.560 g	35.1 ccm	0.040 g	292.520 g	Al. 204.121 g
Al. in air	+ 88.480	70.7	0.081	88.399	Water 60.208
Al. in water	+ 148.627	17.8	0.020	148.607	

$$\text{Density of alloy at } 22^\circ.2 = \frac{204.121 \times 0.99772}{60.208} = 3.3825.$$

Expansion factor, 1.00153; density of alloy at 0° = 3.3877.

96. Other Methods for Hydrostatic Weighing. — The dish of water may be placed on a bridge over the pan, and the body whose volume is to be determined is suspended by a fine wire from a hook on the pan support; or the dish of water may be placed on the pan, the body being suspended from a fixed support. In the latter case the increase in the apparent weight of the water is the weight of the water displaced. With these arrangements the details of making the weighings and corrections for the buoyancy of the air differ from those described, but the general principles are the same.

For substances that float in water a sinker of lead may be attached to the harness before beginning the observations, remaining in this position throughout the experiment.

If the substance is soluble in water, some other suitable liquid of known density may be substituted, the result obtained being multiplied by this density.

The volumes and densities of a set of weights for calibration purposes may be determined by a single hydrostatic weighing of each piece. Arrange a fine wire to support the weight in water, and counterbalance the wire by the rider or otherwise. Determine the weight of each piece when it is immersed in water. The difference between this weight and its nominal value is its volume; the ratio of its nominal mass to its volume is its density. The results will be sufficiently accurate for the purpose mentioned.

XL. DENSITY WITH THE PYKNOMETER

Determine the density of a liquid and of fragments of glass.

97. Density with the Pyknometer. — The specific-gravity bottle, or pyknometer, is a small flask designed to hold a definite volume of a liquid, and sometimes also to receive small solid bodies. It is made in a variety of shapes, the forms shown in Fig. 52 being the most useful. Form *a* may have either a solid stopper or one with a capillary tube, and it is to be completely filled with the liquid. Form *b* is provided with a thermometer for indicating the temperature of the liquid; it is to be filled to a mark on the slender stem. These two forms may be used for either solids or liquids. Form *c* is for liquids only. It has a thermometer sealed in and two tubular arms. One arm is provided with an extension tube to dip into the supply of liquid, which is caused to flow into the pyknometer by suction applied to the other arm. This latter tube is of such small diameter that when the bottle is filled the liquid rises by capillary action to the end of the tube. By applying a piece of filter paper to the end of this arm, a small amount of the liquid may be drawn

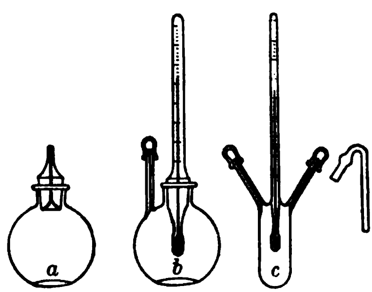


FIG. 52. PYKNOMETERS

out, so that at the given temperature the liquid shall stand at a mark placed on the larger arm. The unfilled portion of the latter permits a slight increase of temperature during the weighing without the overflowing of the liquid. The two arms are provided with caps to prevent loss by evaporation.

Density of a Liquid. — The pyknometer being clean and dry (Art. 91), place it on one pan of the balance and accurately weigh it or counterpoise it. Fill the bottle with distilled water, and determine the weight of the water. Fill the bottle with the liquid whose density is required and weigh again. These apparent weights are to be corrected for the buoyancy of the air. For making this correction, the volume of the bottle may be taken as equal to the weight of the water filling it, and the volumes of the weights are to be computed from the assumed density, — 8.4 for brass weights. The excess of the volume of the bottle over that of the weights used in each case, multiplied by the density of the air at the temperature and pressure of the experiment (0.0012 with sufficient approximation), is the correction which is to be added to the apparent weight to give the weight *in vacuo*. The ratio of the weight of the liquid to the weight of the water is the specific gravity as compared with water at the temperature of the experiment, and this result multiplied by the density of water at this temperature (Table 3) is the density of the liquid at the temperature of the experiment.

The neglect of the correction for the buoyancy of the air amounts, for liquids of density 0.5 or 2.0, to about one part in a thousand, while it vanishes for liquids of density 1.0. The correction for the density of the water is commonly about one part in five hundred.

The temperature of the bottle when filled with the liquid should be the same as when filled with water; otherwise the corrections required are more complicated than above outlined. The pyknometer may be filled while immersed in a water bath, and it should be handled only by the neck.

Density of a Solid in Fragments. — Weigh the fragments of the solid in air. Fill the pyknometer with distilled water and determine the weight of bottle and water. Put the fragments

into the bottle, displacing part of the water; replace the stopper, being sure that the bottle is filled properly with water, wipe dry, and weigh again. The difference between the sum of the first two weighings and the last is the weight of the water displaced. The ratio of the weight of the solid in air to the weight of the water displaced is its specific gravity.

The bottle used in this method should be as small as is convenient. If the solid is soluble in water, weigh it in some liquid of known density in which it is insoluble. Its specific gravity as compared to this liquid, multiplied by the density of the liquid, is its true specific gravity.

In work of precision corrections for the temperature of the water and the buoyancy of the air must be applied according to the principles given above and in Art. 95.

Other Methods. — In some cases it may be sufficiently accurate to assume that the capacity of the bottle has been adjusted by the maker to an amount marked upon the bottle. A "100 Grams" specific-gravity bottle may be assumed to hold 100 g of water, or to have a capacity of 100 ccm. Hence it will not be necessary to weigh the bottle filled with water; and if the bottle is accompanied by a counterpoise, the determination of the specific gravity of a liquid is reduced to a single weighing.

REFERENCE. — *Stewart and Gee*, Vol. I, p. 127.

XLI. DENSITY OF AIR BY WEIGHING AND EXHAUSTION

Determine the mass of one cubic centimeter of dry air at 0°, under a pressure of 76 cm.

98. Density of Air. — A convenient form of receiver for this experiment is a glass globe of the form shown in Fig. 53, having a capacity of one or more liters. If the globe contains any moisture, this must be removed by drawing dry air through it; moderate and careful heating will assist in the operation. Exhaust the receiver until the pressure is reduced to less than one centimeter, and let p be the pressure indicated by the gauge when the receiver is closed. Weigh the globe as accurately and

quickly as possible. Fill the receiver with dry air, by allowing the air slowly to flow into it through a drying bottle containing concentrated sulphuric acid. Again weigh the globe carefully. The difference between these two weighings will be the weight of the air exhausted. Repeat this operation twice, securing the same degree of exhaustion in each case, and let m be the mean of the three determinations. Let t be the temperature of the air with which the globe is filled, and P the corrected barometer reading. Fill the globe with water of known temperature, and weigh to the nearest tenth of a gram. The difference between this weight and that of the exhausted globe is the weight in grams of the water, whose volume is equal to that of the globe. This quantity divided by the density of water at the given temperature (Table 3) is the volume, v, of the receiver.

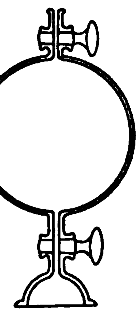


FIG. 53. GLOBE FOR WEIGHING AIR

The mass of 1 ccm of air at 0° and under a pressure of 76 cm, that is, its density, is

$$D = \frac{m}{v} \cdot \frac{76}{P - p} \cdot \frac{273 + t}{273}.$$

(Empty the receiver at the end of the experiment and place it so that all water can drain out.)

XLII. DENSITY OF A GAS BY WEIGHING AND COMPARISON WITH AIR

Determine the density of carbon dioxide.

99. Density of a Permanent Gas. — Fill a light glass globe provided with a stopcock with the gas. This may be done by displacing mercury over a mercury trough, or simply by displacing the air if there is sufficient gas for this. The globe is closed, and its weight, w_1, is determined. Displace the gas by air, and find the weight, w_2, of the globe open. Fill the globe with

mercury (or water), and let M be its weight. From the observed temperature and the barometric height, find the density of the air, l, with the aid of Table 12. The temperature and tables will give the density, Q, of the mercury (or water). If the gas and air with which the globe was filled were both at the same temperature and pressure, the density of the gas is

$$D = \frac{w_1 - w_2}{M - w_2} \cdot \frac{Q - l}{l} + 1.$$

If the temperature and pressure of the gas were t_1 and p_1, and of the air t_2 and p_2, then the value obtained from the above formula must be multiplied by

$$\frac{p_2}{p_1} \cdot \frac{1 + 0.00367\, t_1}{1 + 0.00367\, t_2}.$$

REFERENCES. — *Kohlrausch*, Physical Measurements, pp. 56–63; *Ostwald*, Physico-Chemical Measurements, pp. 99–107.

XLIII. DENSITY OF LIQUIDS WITH A U-TUBE

Determine the density of mercury with the U-tube. Determine the density of sulphuric acid by Hare's Method.

100. Density with a U-Tube. — Provide a U-tube with arms about a meter long. Pour mercury into the tube until it stands at a height of five or six centimeters in each arm; into one arm now pour distilled water nearly to fill it. The heights of the free surfaces of the liquids above their common surface are inversely proportional to the densities. Determine these heights carefully with a cathetometer, setting the cross wires tangent to the meniscus, and calculate the relative density of the mercury. This result is the density of mercury at its temperature, $t°$, compared with water at $t°$. Multiply this by the density of water at $t°$ (Table 3), which will give the density of mercury at $t°$.

This method can be used for comparing the densities of any two liquids which do not mix. For liquids which will mix, the following method is often convenient.

101. Relative Density by Hare's Method.—Two upright tubes, each about a meter long, dip into separate vessels containing the liquids to be compared,—for example, distilled water and an acid. The tops of these tubes are connected to a single tube provided with a stopcock, which leads to a small air pump or other suction apparatus. The withdrawal of air will cause the liquids to rise in the tubes to heights inversely proportional to their densities, neglecting capillary action. Determine these heights with a cathetometer, and calculate the relative densities. This result when multiplied by the density of water at the temperature of the experiment, $t°$, will give the density of the liquid at $t°$.

Instead of the cathetometer, any convenient scale may be placed behind the tubes for measuring the heights of the liquid columns.

XLIV. SPECIFIC GRAVITY WITH THE JOLLY BALANCE

Find the specific gravity of five solids and one liquid, making three determinations of each.

102. The Jolly Balance.—A spring balance is often a convenient substitute for the beam balance; it is especially suitable when relative weight only is required, as in specific gravity determinations. In one form the Jolly balance consists of a long spiral spring suspended in front of a mirror millimeter scale. Attached to the lower end of the spring is a small weight pan, and hanging from this by a fine wire is a second pan which is always to be immersed in water. In reading the elongations any convenient point at the bottom of the spring may be taken as the index, which point should always be made to coincide with its image as seen in the mirror scale, to avoid parallax.

The necessity for continual adjustment of the position of the dish of water is somewhat troublesome; this is avoided by the form of construction shown in Fig. 54. The pans are suspended at a constant level by raising or lowering the upper end of the spring, which is accomplished by turning the milled head S;

the amount of extension is indicated by a scale graduated on the telescoping tubes. The pans are prevented from slipping away when the load is changed by the short glass tube and stops shown at *T*. Marks on the wire and tube are to be brought to the same level when making a setting.

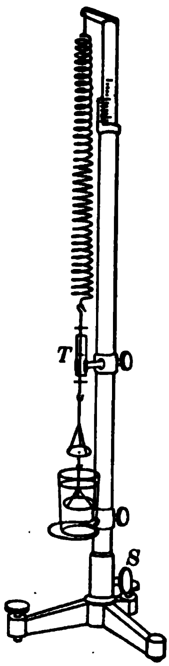


FIG. 54
JOLLY BALANCE

Upon the assumption that the stretch of the spring is proportional to the load it carries, this apparatus permits a very convenient determination of specific gravity with moderate precision.

Specific Gravity of a Solid. — Ascertain the elongation of the spring produced when the solid is supported by the upper pan, the lower one being in water; and also the elongation produced when the body is placed upon the lower pan in water. The ratio of the first elongation to the difference between the two elongations is the specific gravity required.

A method perhaps more accurate, but involving more trouble, is to determine the elongations as above described, and to find by trial in each case what weights alone placed in the pan will produce the same extension of the spring. The ratio of the first weight to the difference between the two is the specific gravity.

The relation between the stiffness of the spring employed and the weight of the body experimented with should be such that a considerable elongation of the spring is produced.

Specific Gravity of a Liquid. — Suspend from the spring, in place of the pans, any solid which is not affected by the liquid. Read the index when the solid is suspended in the air, when immersed in water, and also when immersed in the given liquid. The ratio of the difference between the first and third of these readings to the difference between the first and second, is the specific gravity of the liquid.

XLV. SPECIFIC GRAVITY WITH HYDROMETERS

Determine the specific gravity of several solids and liquids, using various forms of hydrometers.

103. The Constant Immersion Hydrometer. — A very simple instrument for measuring the specific gravity of a solid or liquid is the constant immersion hydrometer, which consists of a glass or metal elongated bulb so constructed as to float in water in an upright position. At its lower end is a small basket, and a slender stem on the upper end carries a small weight pan above the water.

To determine the specific gravity of a solid, ascertain what weight, W_1, on the pan will cause the instrument to sink to a mark on the stem; with the solid to be tested on the pan, find what weight, W_2, must be added to produce the same result; place the solid in the basket in the water and determine again the weight, W_3, required to sink the bulb as before. The ratio of the weight of the solid in air to its loss of weight in water, that is, $\frac{W_1 - W_2}{W_3 - W_2}$, is the required specific gravity. Bodies that float in water may be tied to the basket or held in place otherwise.

If the hydrometer is made of glass, it may be used to determine the specific gravity of a liquid. If m is the weight of the hydrometer itself, W_1 the weight required to sink it in water to the index mark, and W_2 the weight to sink it in the given liquid, then $\frac{m + W_2}{m + W_1}$ is the specific gravity of the liquid.

104. The Variable Immersion Hydrometer. — For indicating directly the density of a liquid, variable immersion hydrometers are used. These are bulbs of glass with elongated stems so constructed that they will float in liquid, sinking to a depth depending upon the density of the liquid. An empirical scale is provided, the reading of which, at the surface of the liquid, indicates directly the required density. The scale should be read *through* the liquid, the eye being level with the surface. By using properly constructed hydrometers, and applying

corrections for the temperature of the liquid, a fair degree of accuracy can be obtained, while the method is the simplest possible. It however requires an amount of liquid sufficient to float the instrument.

These hydrometers are constructed with scales of great variety, adapted to special uses, indicating the percentage of alcohol, sugar, or of a salt in a solution, and for other similar purposes.

REFERENCE. — *Stewart and Gee*, Practical Physics, Vol. I, pp. 146–154.

Part III — Sound

CHAPTER VI

FREQUENCY OF VIBRATION

I. FREQUENCY OF A SOUND BY BEATS

Determine the frequency of a tuning fork by the method of beats. Make a copy of a "Standard A" fork.

105. The Tuning Fork. — The most important of acoustical instruments is the tuning fork, which, when properly constructed and mounted, gives a sound of great purity and constancy of pitch. Forks should be carefully protected from rust and from rough usage. They should never be struck with metal. They may be sounded with a violin bow, or with a hammer of wood or ivory, or with one made of a rubber cork on a spring metal rod for a handle. A felt piano hammer attached to a spring handle is excellent for striking forks. When the fork is provided with an electromagnetic vibrator, the vibrations may be started with the felt hammer.

The number of vibrations given by a steel tuning fork varies slightly with the temperature. The standard temperature at which forks are usually rated is 20°. The temperature coefficient has been found nearly constant for forks of any pitch, and to have the value -0.00011. A fork giving 256 vibrations per second at 20° will have its frequency diminished by 0.0282 for one degree increase in its temperature.

A fork can be adjusted in pitch by filing. Filing the prongs at the ends will raise the pitch, while filing them near the yoke will lower it. *The student should not alter any fork except upon definite orders.*

106. Frequency of Sound with a Tuning Fork. — Two sounds nearly in unison produce beats the number of which per second is equal to the difference of their vibration frequencies. Sets of standard forks are often provided such that one may be selected which, when sounded with an unknown fork, will give a number of beats that can be easily counted. The frequency of the given sound is, therefore, equal to that of the standard fork increased or diminished by the observed number of beats. To determine whether the unknown fork is higher or lower than the standard, load one prong of it with a piece of wax; if the beats are now fewer, the fork altered was originally the higher in pitch, and vice versa.

A fork may be brought into unison with a standard, that is, the standard may be copied, by adjusting the fork to make the beats fewer, and finally to cease. But to count the beats when they are fewer than one per second is difficult. A convenient number for counting is four per second. One can count the number, at about this rate, made in ten seconds with ease and accuracy. Hence, to tune a fork to a standard, an auxiliary fork which is exactly four beats per second sharper than the standard is provided. The given fork can now be adjusted till it is four beats per second flatter than the auxiliary fork, in which case it must be exactly in unison with the standard.

LI. FREQUENCY OF A SOUND WITH THE SIREN

Determine the number of vibrations of two organ pipes and a tuning fork.

107. Frequency of Sound with the Siren.—The siren (Fig. 55) consists of a metal disk, pierced with several series of holes in concentric circles, which is caused to rotate and periodically to interrupt jets of compressed air. This rotation may be produced by the impact of the jets upon the oblique sides of the

holes, the rapidity of rotation being controlled by varying the air pressure. A better form has the holes bored through the disk perpendicularly, the rotation being produced by an electric motor controlled with a rheostat. A counting device is arranged to record the number of rotations. Often the siren has two disks, each with several series of holes, any one or any combination of which may be used as desired.

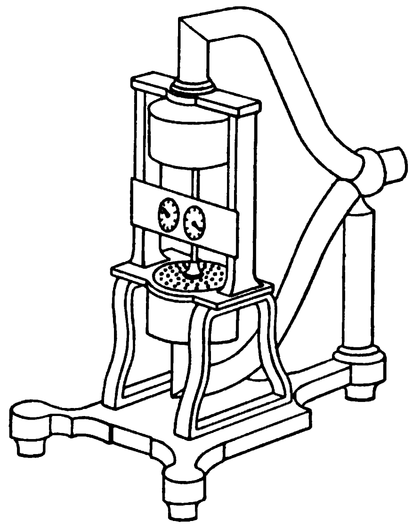


Fig. 55. The Siren

The interruption of the air jets will, in general, give rise to a musical tone, the pitch of which depends upon the rapidity of these interruptions. To determine the number of vibrations of a given sound, so adjust the speed of the siren disk that its tone is in unison with the given one. If sufficient variation in air pressure cannot be obtained, a different number of holes in the disk must be used. It is necessary to adjust with care, that the sounds may remain in unison during the counting. When the sounds differ slightly beats may be heard. If these are caused by increased speed of the disk, the unison may be restored by a very light touch of the finger on the disk. If perchance this causes the beats to increase in frequency, the disk was before rotating too slowly; a slight pressure with the hand on the wind chest bellows may restore it to the proper speed. When the unison has become constant determine the number of rotations of the disk in one minute. This number multiplied by the number of holes in the series used, and divided by sixty, is the number of puffs of air per second producing the siren tone, and is the number of vibrations of the given sound with which it is in unison. It may be more convenient, using a

stop watch, to determine the time required for the disk to make exactly 1000 or 2000 rotations.

If it is inconvenient to secure exact unison, the number of beats per second may be counted, and their number subtracted from the frequency determined as above, when the disk rotates too rapidly, or added, if the rotation of the disk is too slow.

LII. FREQUENCY OF VIBRATION BY GRAPHIC METHODS

Determine the frequency of a tuning fork.

108. Frequency of Vibration by Graphic Methods. — A graphic method will often be the most convenient for comparing a tuning fork or other vibrating body with a standard fork or with a standard clock.

The fork must be provided with a style for marking on smoked paper; if the style is permanent, it may be of thin sheet brass cut to a point and so attached as to swing edgewise; if it is temporary, it may be a stiff bristle, a piece of quill or of sheet celluloid, attached with wax.

The smoked paper may be carried on the cylinder of a chronograph if this has sufficient speed, or a special cylinder may be arranged; perhaps a ribbon of smoked paper on reels (Art. 72) is the most efficient device. The fork is firmly supported so that the style will mark lightly on the paper when the fork is in vibration and the paper is moving underneath. If a drum is used, either the fork or the drum should have a motion in the direction of the axis making the trace in a spiral form.

A break-circuit clock is connected through a relay (Fig. 37) to the primary circuit of an induction coil; the secondary of the coil is connected to the metallic drum carrying the paper and to the fork, or to a metallic point placed beside the fork style. As the clock interrupts the current through the coil, a spark will pass from the point through the paper, producing a small spot in the smoke, thus marking the clock beats. By counting the number of fork waves between two such marks, or, better, between the marks corresponding to an even number of seconds

to eliminate eccentricity in the break, the frequency of the fork is directly determined. A varying rate of moving the paper introduces no error, as the frequency depends upon the number of waves between two points and not upon their size.

The addition of the style may appreciably change the period. By determining the change in frequency caused by a known added load, a correction for the style may be computed.

Two forks are compared by having their traces made side by side simultaneously.

LIII. RATIO OF VIBRATION FREQUENCIES BY LISSAJOUS'S FIGURES

Determine the number of vibrations of several vibrating bodies by means of Lissajous's optical comparator.

109. Frequency of Vibration by Optical Method.—The optical comparator of Lissajous depends upon the principles of the composition of simple harmonic motions. If the periods of two rectangular simple harmonic motions are in certain simple ratios, the results of their composition will be characteristic curves which are easily recognized. If the ratio of the periods is exact, the curve will have a fixed form, the shape of which will depend upon the relative phases of the two vibrations. When the periods are such that they differ slightly from the exact ratio, their relative phases will change continuously and slowly pass through all possible values, with the result that the curve of their combination will successively exhibit all forms belonging to the given ratio. The curve will pass through a complete cycle of changes in the time required for one vibrating body to gain or lose one whole vibration on the number which it would have made in the same time if the ratio of the frequencies were exactly that corresponding to the observed curve.

In the most convenient form of the optical comparator the objective of a microscope is carried by one prong of a standard tuning fork, the body tube being on a separate support and perpendicular to the plane of vibration of the fork. (See Fig. 56.)

The body whose vibration frequency is to be determined is adjusted so that its line of vibration is at right angles to that of the fork, and so that some distinct point of it can be seen through the microscope. Usually it will not be difficult to find a suitable point as the object, — a speck of adhering chalk dust will often suffice; or the surface may be slightly smoked and a point made in the smoke with a needle. It is convenient to maintain the vibration of the standard fork by an electromagnetic attachment. For universal comparisons a number of forks, or forks whose frequencies can be altered by sliding weights, will be necessary.

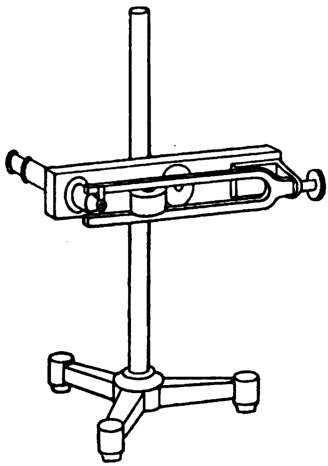


FIG. 56. LISSAJOUS'S TUNING FORK COMPARATOR

A standard fork is selected for the comparator such that its motion compounded with that of the given body will produce a figure which may be recognized as representing a certain ratio of frequency. The time required for the figure to pass through a complete cycle of phase changes is observed. Thence the frequency of the given body may be obtained with great accuracy. If the standard fork has a frequency S, and the figure corresponds to the ratio $1:n$, and if the cycle of changes occurs in c seconds, the unknown fork has a frequency

$$N = nS \pm \frac{1}{c}.$$

Often it may be necessary to observe the effect of loading the fork with a small piece of wax to determine whether it gains or loses a vibration in the time of the phase cycle.

The method is applicable to vibrating strings as well as to forks.

REFERENCE. — *Zahm*, Sound and Music, p. 416.

110. Copying a Standard Fork. — Two independent forks may be easily compared with such a comparator as has just been described, or a standard fork may be copied. So load the comparator fork in any manner that it gives a known figure with the standard fork, and adjust the second fork to give the same figure. The pitch of a fork may be raised by filing the ends of its prongs, and lowered by filing the prongs near the yoke.

111. Projection of Lissajous's Figures. — Instead of the microscope method described, it is sometimes more convenient, especially for demonstration purposes, to use forks with mirrors attached to their ends, a pencil of light being reflected from one mirror to the other, and finally into a telescope or upon a screen. The pencil of light will move with the compounded motion of the two forks, and will describe Lissajous's figures.

LIV. FREQUENCY OF A FORK BY OPTICAL COMPARISON

Determine the frequency of a tuning fork, including the measuring of the period of the auxiliary pendulum.

112. Reed's Method for rating a Tuning Fork. — Often it is desired to know the absolute frequency, or to determine slight changes in the frequency, of a fork with great precision. Lissajous's optical method (Art. 109) will give the ratio of two frequencies with precision only when this ratio is very approximately represented by a small integer. Of the various methods for accurately comparing any fork with a standard clock, Reed's method is probably the simplest that has been devised. The feature of the method is a flashing apparatus which may be adjusted to give flashes of any desired frequency, of very short duration (0.00001 second) and of great regularity. The frequency of the fork being investigated must be known to the nearest integer (Art. 108); this frequency should not be more than 300, and the satisfactory application of the method requires that a mirror be attached to one prong of the fork.

The flashing apparatus consists of an auxiliary pendulum, about a meter long, whose period is adjustable by movable

weights. Near the middle of the pendulum rod and parallel to it is attached a fine needle, n (Fig. 57). A short vertical axis, a, is pivoted in a stationary support; attached normally to this axis is a second fine needle or glass tube, t, a fraction of a millimeter in diameter, which rests against the needle n. A light mirror, m (a galvanometer mirror 2 cm in diameter), is fastened to the axis a. A small drop of oil joins the two needles, so that as the pendulum swings the mirror rotates through an arc of considerable magnitude. This arc may be made larger or smaller by varying the lengths of the several levers involved.

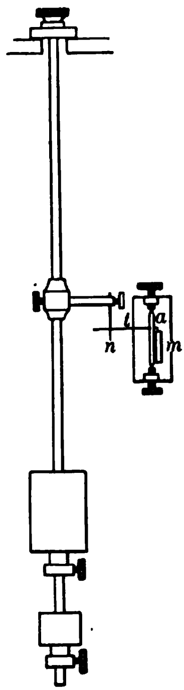


FIG. 57
REED'S FLASHING PENDULUM

For observation the apparatus is arranged as shown in Fig. 58. E is a bright source of light which is condensed by the lens L_1 on a vertical slit S_1. The light from the slit is reflected by the flashing mirror, when the pendulum is at rest, to the observing instrument which is placed three or more meters from the pendulum. For a fork of high frequency this distance and the intensity of the light have both to be increased. The observing instrument consists of a collimator and a view telescope; a spectroscope or spectrometer serves for this purpose, though an Abbe-Zeiss autocollimating telescope, as described by Reed, is simpler. The light from the flashing mirror is thrown upon the collimator, the image of the slit S_1 being focused on the collimator slit S_2 by the lens L_2. The light passing through the collimator falls upon the mirror attached to the tuning fork to be rated, and from this it is reflected to the view telescope. The various parts are to be so adjusted that a bright, well-defined narrow image of the slit is seen.

113. The Stroboscopic Method. — If the pendulum is caused to vibrate, the slit will flash into view once in each swing. When the fork is also caused to vibrate, the flash will still be

seen at every swing of the pendulum, but its position in the field of view will vary unless the fork makes an exact integral number of vibrations in one swing of the pendulum. Suppose, for illustration, that the fork makes $30\frac{1}{8}$ vibrations in one swing

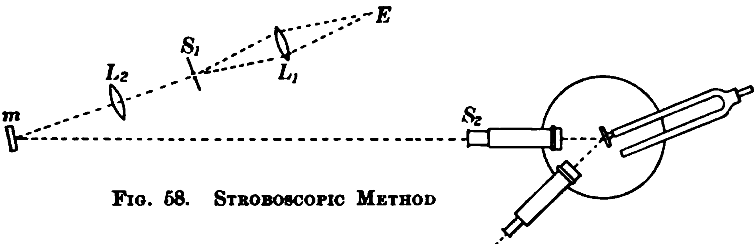


FIG. 58. STROBOSCOPIC METHOD

of the pendulum. If the first flash is observed in the center of the field, at 0 in Fig. 59, when the second flash occurs the fork will have made $30\frac{1}{8}$ vibrations, and the flash will appear at the point 1, $\frac{1}{8}$ of a vibration past the center; at the end of the next swing the fork will have made $60\frac{2}{8}$ vibrations, the flash appearing at point 2; the flash will thus appear successively at the points 3, 4, 5, 6, 7, until, at the end of the eighth swing, when the fork has vibrated $240\frac{8}{8}$ times, the flash again occurs at the starting point. Since the fork vibrates in a straight line and not in an ellipse, the various points all lie in one line; whether a given flash corresponds to point 4 or 8 is determined by the direction in which the flashes approach this point. Such a series of images from an intermittently illuminated vibrating body constitutes a *stroboscopic cycle.*

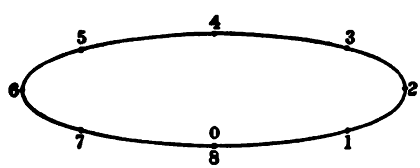


FIG. 59. STROBOSCOPIC MOTION

Usually one cycle will not take place in an integral number of swings of the pendulum. If the flash occurs once at the center, and then after a time occurs again at the center, by counting the number of flashes between the two coincidences and dividing by the number of cycles, the time of one cycle in pendulum swings is determined.

The fraction of a cycle between two successive flashes is the fraction of a vibration which added to some integer gives the frequency of the fork referred to the period of the flash. This integer must be determined from an independent source, this method giving only the fraction.

If the number of vibrations per pendulum beat is a whole number minus a small fraction, a similar stroboscopic cycle will occur, taking place in the opposite direction. With a straight-line motion this direction cannot be discovered, and some subsidiary method is necessary to determine whether the fraction is to be added or subtracted.

A pendulum swinging freely will slowly diminish in amplitude, and the period will slightly decrease at the same time. A study of the conditions of the stroboscopic cycle will show that, as the period of the pendulum slowly decreases, the number of flashes per cycle will increase if the number of vibrations is a whole number plus a fraction, and that the number of flashes per cycle will decrease if the number of vibrations is a whole number minus a fraction.

For convenience of observation the period of the pendulum should be so adjusted that with the given fork the number of flashes per cycle is about ten.

If the flash image of the slit is too wide, it may be made narrower by increasing the distance of the collimator from the flashing mirror, by narrowing either one of the slits or by increasing the arc through which the mirror swings.

In general, if c is the number of swings per cycle, and n, to the nearest integer, is the number of complete vibrations of the fork in one swing, then the exact number of vibrations per swing is $n \pm \frac{1}{c}$; and if the exact period of the pendulum is t, the absolute frequency of the fork at the temperature of the experiment is

$$N = \frac{n \pm \frac{1}{c}}{t}.$$

This value should be reduced to 20° by means of the temperature coefficient for tuning forks explained in Art. 105.

A numerical example follows.

Frequency of an Adjustable Koenig Fork; Reed's Method

Time by an Auxiliary Pendulum from Standard Self-winding Clock

July 26, 1902

To find the frequency of a fork known to give 32+ vibrations per second. Preliminary observations showed that there were 8+ flashes per cycle.

Cycles	Flashes	Swings per Cycle
6	51	8.500
10	84	8.400
8	67	8.375
11	92	8.364
	Mean, $c =$	8.410

Temperature of fork, 23°.6.

Since the observed values of c decrease as the amplitude of the pendulum diminishes, c has the negative sign.

The period of the flashes, t, was determined by a subsidiary experiment (see example under Art. 69) to be 0.92858 seconds. In this time a fork of frequency about 32 would make about 30 vibrations. Hence the frequency of the fork is

$$N_{23^\circ.6} = \frac{n - \frac{1}{c}}{t} = \frac{30 - 0.1189}{0.92858} = 32.179.$$

To reduce to 20°, the correction is $32.2 \times 0.00011 \times 3.6 = 0.013$; therefore

$$N_{20^\circ} = 32.179 + 0.013 = 32.194.$$

Reference. — *Reed*, Physical Review, Vol. 12, pp. 279–291, 1901.

CHAPTER VII

VELOCITY AND WAVE LENGTH OF SOUND

LV. VELOCITY OF SOUND BY RESONANCE

Determine the velocity of sound in air, using a resonance tube and a fork of known vibration number.

114. Velocity of Sound in Air. — Theory shows that the velocity of sound in a gas is unaffected by changes in pressure, but that it varies with temperature and hygrometric state. If v_0 is the velocity at 0°, then the velocity at a temperature t is

$$v_t = v_0 \sqrt{1 + at}.$$

The velocity in dry air at 0° is found by experiment (International Congress of Physics, Paris, 1900) to be

$$v_0 = 33136 \text{ cm per sec.};$$

whence

$$v_t = 33136 \sqrt{1 + at} \text{ cm per sec.};$$

or with sufficient approximation,

$$v_t = 33136 + 60.7\, t \text{ cm per sec.}$$

115. Wave Length of Sound by Resonance. — The resonance tube may be of glass about 100 cm long and 5 cm in diameter, arranged with a scale and water reservoir, as shown in Fig. 60. A rubber tube connects the reservoir with the bottom of the glass tube, so that the latter may be filled to any desired height with water. A fork of known vibration number, for instance 256, is firmly fastened above the tube as shown. The fork is made to vibrate by means of a felt piano hammer or by a violin bow, and the height of the water is adjusted till the sound is

most strongly reënforced. Several positions of maximum reënforcement may be found. Determine each by at least ten observations. The difference between successive positions of the water surface is one half of the wave length in air of the sound of the given vibration number. From this determine the velocity at the temperature of the air, and reduce it to 0° (Art. 114).

Repeat the experiment with other forks of higher pitch. Find the wave length for a fork of unknown frequency, and from the velocity in air determine the frequency.

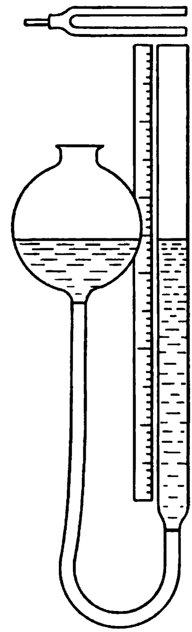


FIG. 60. RESONANCE APPARATUS

LVI. VELOCITY OF SOUND AND YOUNG'S MODULUS BY DUST FIGURES

Determine the velocity of sound in a gas, and in brass, steel, and wood. Compute Young's modulus for the solids.

116. Relative Velocity of Sound by Stationary Waves. — Kundt's apparatus for determining the relative velocity of sound in solids and fluids consists of a glass tube about 150 cm long and 3 cm, or more, in diameter, provided with a brass fitting at each end. On each fitting is a stopcock. Near one end of the tube is a closely fitting piston which may be moved a few centimeters by a handle passing though one end cap. The other end is provided with a clamp to grasp a rod about a meter long at its center. The end of the rod which projects into the tube carries a cork diaphragm which fits the tube loosely. (See Fig. 61.) Inside the tube between the piston and the diaphragm is placed cork dust, amorphous silica, or lycopodium powder.

Rub the free end of the rod with a piece of rosined leather if it is of wood or metal, or with a wet cloth if it is of glass. It will vibrate longitudinally and give a clear, shrill tone. A slow

motion of the rubber with light pressure is most effective. If the cork diaphragm on the rod rubs too heavily upon the glass tube, or if the rod slips so that it is not held at its middle part, it may be difficult to produce the tone. By properly adjusting the movable piston, the air column in the tube can be tuned

FIG. 61. DUST FIGURE APPARATUS

to resonance with this tone, when the positions of the nodes and segments will be clearly shown by the cork dust. After the adjustment appears to be perfect, lift one side of the apparatus and jar it, so that the dust will lie in a line part way up on the side of the tube; then give the rod a single stroke, making a clear sound. If the adjustment is perfect, the dust in the segments of the waves will be lifted into the air in flutings and will fall to the bottom, lying in loops like festoons hung from the points corresponding to the nodes. These nodal cusps where the dust has not been disturbed should be obtained as sharp as possible. By measuring from a well-formed node near one end to one near the other end, determine the average distance between the nodes. Shake the dust back against the side of the tube, and repeat the observations. The distance between the nodes is half the wave length of the sound in air at the temperature of the experiment. The length of the rod is half the wave length of the same sound in the material of the rod. The velocities in the two substances are proportional to the wave lengths. Knowing the velocity in air (Art. 114), calculate the velocity in the material of the rod.

Fill the tube with a gas and determine the wave length of the sound in the gas. Since the frequency of the sound is the same whether the tube is filled with gas or air, the velocities in the two media are proportional to the wave lengths. Calculate the velocity in the gas.

REFERENCE. — *Kohlrausch*, Physical Measurements, p. 140.

117. Young's Modulus by the Velocity of Sound.—The velocity of sound in a solid is

$$V=\sqrt{\frac{M}{D}},$$

M being Young's modulus (Art. 73) and D the density. If the material is in such form that the velocity of sound in it can be determined by the method of the preceding article, and its density is known, Young's modulus may be computed at once from this relation. The method is a valuable one.

LVII. VELOCITY AND WAVE LENGTH OF SOUND BY INTERFERENCE

Determine the velocity of sound in air and in a gas, and find the wave length and frequency of the sound of an organ pipe.

118. Interference of Sound.—An apparatus for studying sound interference consists essentially of a tube which between its extremities is divided into two branches, one of which can

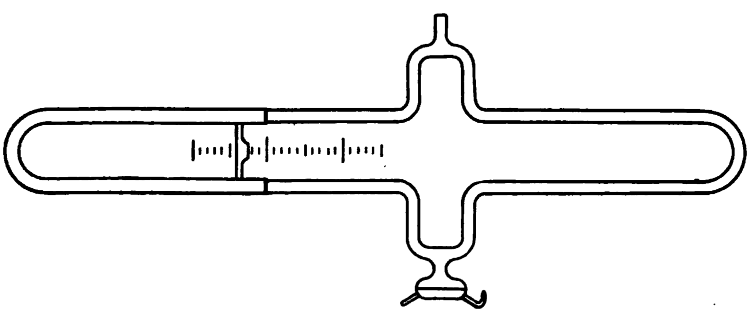


FIG. 62. APPARATUS FOR SOUND INTERFERENCE

be lengthened a known amount by a slide. A sound produced at one end will be divided into two parts, which, after traversing the two branches, will again be united. If one branch is longer than the other by an odd number of half wave lengths of the

sound, the two parts will meet in opposite phases, and interference will result. The intensity of the combined vibrations may be clearly shown by placing a manometric capsule on the united tube (Fig. 62) and observing the small flame of the gas jet in a revolving mirror.

Sound a fork of known frequency before the open end of the tube. To reënforce the sound a resonator may be placed in the tube, or the open end of the resonant box of the fork may be held to the tube. If the branches of the tube are of the same length, the jet will be strongly agitated, being affected by the sum of the two parts of the original sound. Now gradually lengthen one branch till the jet ceases to vibrate or until a minimum is found. The quiescence of the jet signifies that the two parts of the original sound, having traversed different paths, reach the ends of the tubes in opposite phases and interfere; hence one branch of the tube has been lengthened by a half wave length of the sound. The velocity of sound is equal to the product of the wave length and frequency of vibration. Determine the velocity of sound in air at the temperature of the experiment, and reduce the result to 0°.

Place a thin rubber diaphragm over the open end of the tube and fill it with gas. Determine the velocity of the same sound in the gas.

Remove the diaphragm and gas from the tube and, using the velocity of sound in air just determined, find the wave length of the sound of a given organ pipe, and thence its frequency.

Instead of uniting the two branches, a manometric capsule is sometimes attached to the end of each, the gas jets of which will indicate the state of vibration in the tube. A third gas jet is connected to both capsules and is under the influence of the combined effect of the vibrations in both branches, indicating the condition of interference.

Reference. — *Koenig*, Expériences d'Acostique, p. 76.

CHAPTER VIII

VIBRATING STRINGS AND AIR COLUMNS, AND SOUND ANALYSIS

LVIII. LAWS OF VIBRATING STRINGS WITH THE SONOMETER

Prove the laws of transverse vibrations of musical strings.

119. Laws of Vibrating Strings.— For studying the phenomena of vibrating strings the *sonometer* is often used. It consists of a resonant box upon which may be stretched several musical strings, whose tensions and lengths can be varied at will. Commonly there are two strings: one permanently attached to the sonometer, its tension being adjustable by a thumb screw; while the other string may be readily attached to the sonometer or removed from it, and at one end it passes over a pulley, so that it may be stretched by weights. The permanent wire may be called the comparison wire. The wires usually pass over fixed bridges one meter apart; this is therefore their maximum sounding length, while their effectual lengths may be made less by movable bridges. The sound is produced with a violin bow or by plucking with the finger.

The laws of the transverse vibration of strings are contained in the formula

$$n = \frac{1}{2\,rl}\sqrt{\frac{t}{\pi d}},$$

n being the number of vibrations per second given by a string of length l, radius r, density d, and under a tension of t dynes.

(a) Calculate the number of vibrations per second given by a string, using the above formula; also determine its frequency by comparison with a standard tuning fork.

(b) The number of vibrations of a string is inversely as its length. Tune two strings in unison. When they are nearly in unison beats may be heard, which diminish in frequency as unison is approached. Shorten the effectual length of one string till it gives the octave of its original tone as compared with the unaltered string. The number of vibrations has been doubled and the length halved. Show that the law is true for tones two and three octaves higher than the original tone.

Prove the law by comparing the lengths of a string which gives tones in unison with two tuning forks of known frequency.

(c) The number of vibrations is inversely as the radius of the string. Select two wires of the same material but of different diameters. Attach the heavier wire to the sonometer, stretching it with a weight sufficient to cause it to give a good tone. Tune the comparison wire in unison with this. Substitute the smaller wire for the first, stretching it with the same weight. Determine the effectual length of the comparison wire, which gives a tone of this pitch, using the movable bridge. The effectual lengths of the comparison wire should be proportional to the radii of the two given wires.

(d) The number of vibrations is inversely as the square root of the density of the string. Select two strings of the same diameter but of materials differing greatly in density, as, for instance, wires of steel and platinum, or aluminum and copper, or catgut and brass. Stretch the denser string upon the sonometer, and tune the comparison wire in unison. Replace the dense wire with the less dense, using the same load for stretching; and with the movable bridge find the effectual length of the comparison wire when in unison, and thus prove the law.

(e) The number of vibrations of a string is directly as the square root of the tension. Place a string upon the sonometer and stretch it with a load of one kilogram. Tune the comparison wire in unison with it. Make the load four kilograms, and the pitch should be varied one octave. By finding the effectual lengths of the comparison wire when it is in unison with the given wire under tensions of two or three kilograms, prove the law.

Reference. — *Daniell*, Physics, pp. 434–440.

120. Melde's Experiments with Vibrating Strings. — The various phenomena of transversely vibrating strings may be beautifully shown by attaching one end of a string to the prong of an electrically vibrated tuning fork, and so adjusting the length, tension, and density of the string that it will vibrate with one or more visible loops and nodes. Often vibrations of one and more segments can be seen superposed. The complex vibrations of a string attached to two forks are very instructive.

LIX. FORMS OF VIBRATION IN ORGAN PIPES BY MANOMETRIC FLAMES

Determine the relation of the nodes and ventral segments of the vibrating air column in open and closed organ pipes. Study the combinations of the sounds of two or more organ pipes.

121. Nodes in Organ Pipes. — Using an open organ pipe provided with manometric capsules, show, when the pipe is sounding a fundamental tone, that there is but one node, this being at the center. Show that there are two nodes when the first overtone is sounded, and three nodes when the second overtone is produced.

Experiment similarly with a closed pipe, finding the locations of the nodes and ventral segments.

Represent the results of the various experiments by sketches.

122. Combined Sounds. — Study the composition of the sounds of various pairs of organ pipes by connecting their manometric capsules to a single gas flame, which therefore vibrates under the combined effects of the two pipes. Use pipes nearly in unison, to show the effects of beats, and pipes in various ratios, such as 3 : 4, 2 : 3, 8 : 15, etc.

123. Organ Pipe with Water Trough. — A stopped pipe is sometimes used in which a manometric capsule connection can be placed at any point of its length through a longitudinal slot, the slot being closed air tight by laying the pipe in a trough of water. With such a pipe study the condition of vibration of the entire air column.

REFERENCE. — *Koenig*, Expériences d'Acoustique, pp. 47–56 and 206–217.

LX. COMPOSITION OF A SOUND WITH RESONATORS

Determine the number and relative intensities of the overtones present in the sound of an open and a closed organ pipe.

124. Sound Analysis. — The sound analyzer consists of a series of adjustable resonators which may be connected to a number of manometric capsules, and the vibrations of the gas flames of the capsules can be observed in a rotating mirror.

Adjust the resonators connected to the capsules so that they are in tune with the first eight possible partial tones of a sound whose pitch is Ut_2. Sound a pipe of this pitch in front of the resonators; careful observation of the series of flames will determine what partial tones are present and what are their relative intensities.

It is necessary that the resonators should be exactly tuned. For a tone to which they have not been adjusted by the maker, the following method may be used. Tune a sonometer string in unison with the given fundamental. Then tune the resonators to the fundamental and harmonics of this string, which latter can be easily produced. The proper adjustment of a resonator can be determined quickly by detaching the rubber tube from the capsule and placing the end in the ear.

Reference. — *Koenig*, Expériences d'Acoustique, p. 70.

PART IV — HEAT

CHAPTER IX

EXPANSION

LXX. LINEAR EXPANSION OF A SOLID WITH THE COMPARATOR

Determine the coefficient of linear expansion of a glass tube.

125. Coefficient of Linear Expansion. — The increase in unit length of a substance resulting from one degree increase in its temperature is its coefficient of linear expansion. As this value varies with the temperature it is necessary to specify the temperature corresponding to any given numerical value. If a rod at two temperatures t_1 and t_2, has the lengths l_1 and l_2, the mean coefficient of expansion between these temperatures is

$$a = \frac{l_2 - l_1}{l_1(t_2 - t_1)}.$$

To measure the coefficient of expansion with precision requires elaborate apparatus, both for determining the increase in length, which is relatively a small quantity, and for measuring the temperature change. When the substance can be had in the form of a tube or rod about a meter long, the following simple method is usually sufficient.

A glass tube about 130 cm long and 4 or 5 cm in diameter has two openings blown in one side near the ends, at a distance apart somewhat less than the length of the material to be

investigated. Corks are fitted to the large tube to support the rod in the manner shown in Fig. 63. These corks are inserted under the holes in the glass tube, and the cork is cut away so that fine marks placed on the specimen may be observed with microscopes. These corks are perforated with short tubes. The ends of the large tube are closed by other corks also perforated with short tubes. A mineral wool jacket covers the large tube to protect it from outside temperature changes. By

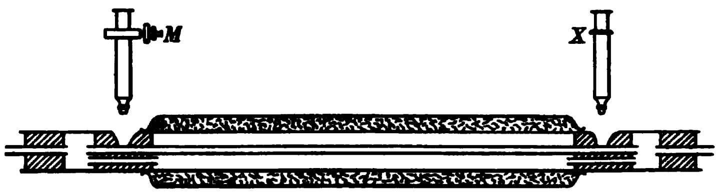


FIG. 63. LINEAR EXPANSION APPARATUS

this arrangement the inner rod may be surrounded with a current of water or steam while it is observed. A metal frame of the form sketched in Fig. 40 conveniently supports the microscopes and the expansion apparatus.

Pass a current of water through the tube till the temperature becomes constant. Place the tube so that the mark at one end is under the index microscope, *X*, and then set the micrometer microscope, *M*, to the other mark. Empty the water from the tube and connect a steam supply to the apparatus. Allow the steam to pass freely till the temperature is again constant, that is till the rod ceases to expand. Set one end to the index microscope, and with the micrometer determine the increase in length, $l_2 - l_1$ of the formula. Do not disturb the microscopes in any way between the first and second observations, not even for focusing; adjust the tube by its supports for this purpose. The large tube may be moved as convenience requires.

Remove the expansion apparatus from the bed of the comparator, place a meter bar under the microscopes, and focus by adjusting the supports of the bar, not by moving the microscopes. Determine the length of the rod when it was cold, to

the nearest half millimeter; this is the quantity l_1 of the formula. The temperature of the water is directly measured, and that of the steam is found with the aid of the barometric pressure.

LXXI. CUBICAL EXPANSION OF GLASS WITH THE WEIGHT THERMOMETER

Determine the mean coefficient of cubical expansion of glass.

126. The Weight Thermometer. — In experiments on the volumetric expansion of liquids and solids, an instrument called the weight thermometer is often used. It consists of a cylindrical bulb of about 15 ccm capacity, with a stem of about 0.5 mm bore, open at the end, and bent in the form shown in Fig. 64. The bulb is held upright in a small stand, which also supports a cup under the end of the stem. If the weight of mercury which fills the bulb at 0° and 100° has been determined, any other temperature may be determined by finding the weight which fills it at this temperature. Weighings can be performed with great precision, and the method obviates the necessity for calibrating the stem and scale of a thermometer; but the operations of weighing are too slow for ordinary temperature observations. The instrument may be used to determine the relative expansions of liquids, and if the absolute expansion of mercury is assumed, the volumetric expansion of the glass envelope may be measured. The present exercise is that of the latter operation.

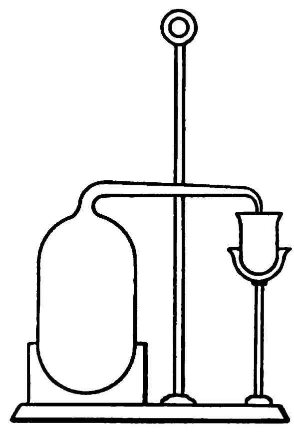


FIG. 64. WEIGHT THERMOMETER

The thermometer, including the stand and cup, empty, clean and dry, is carefully weighed. The bulb is to be filled with mercury: gently warm it, dip the stem into mercury, and upon cooling some mercury will be drawn in; repeat the operations

till the bulb and stem are completely filled. A better method, which, however, requires care and skill to prevent breaking the glass, is to boil the mercury which partially fills the bulb, when upon dipping the stem into mercury the vapor will be condensed and the bulb will be entirely filled. With the stem still in mercury, place the bulb in a bath of crushed ice, and cool it to 0°. Remove the mercury supply, being sure that the stem is full to the end. Holding the empty cup under the stem to catch all the mercury that may run out, take the bulb from the ice, dry it and place it in its support. Weigh again to determine the weight of mercury that filled it at 0°; the mercury will run out as the bulb warms, but as this is caught by the cup it does not interfere with the weighing. Place the bulb in a thermometer boiler or other convenient steam bath, leaving it until its temperature is that of the steam. Again place the bulb in its support and weigh it, the cup being empty.

Let m_1 be the mass of the mercury filling the bulb at 0°, and m_2 that which fills it at the temperature of the steam, $t°$. The volume at 0° of the mass of mercury m_1 is $\frac{m_1}{\delta}$, δ being the density of mercury; this is also the volume of the bulb at 0°. If the coefficient of cubical expansion of the bulb is 3β, its volume at $t°$ is $\frac{m_1}{\delta}(1 + 3\beta t)$. The mass of mercury which fills the bulb at $t°$ is m_2; the volume of this mass of mercury at 0° is $\frac{m_2}{\delta}$, but at $t°$ it has the volume $\frac{m_2}{\delta}(1 + 3at)$, $3a$ being the coefficient of cubical expansion of mercury. These two volumes are equal; that is,

$$\frac{m_1}{\delta}(1 + 3\beta t) = \frac{m_2}{\delta}(1 + 3at),$$

from which it follows that

$$3\beta = 3a\frac{m_2}{m_1} - \frac{1}{t}\cdot\frac{m_1 - m_2}{m_1}.$$

The mean coefficient of cubical expansion of mercury, $3a$, between 0° and 100° is 0.000182.

LXXII. EXPANSION OF LIQUIDS WITH THE DILATOMETER

Determine the coefficient of cubical expansion of water.

127. Coefficient of Expansion of Liquids. — The dilatometer is a simple glass bulb with an open stem, used for measuring the expansion of liquids after the manner of the ordinary thermometer. Two convenient forms are shown in Fig. 65. The capillary tube attached to the bottom of the bulb and bent upward facilitates filling by suction, after which the end is closed with sealing wax or by a stopcock. The other form is filled by gently heating, placing some liquid in the enlarged top, cooling the bulb to draw the liquid into it, and repeating the operation till the bulb is full. The stem may be graduated, or an auxiliary scale may be used to measure the apparent expansion.

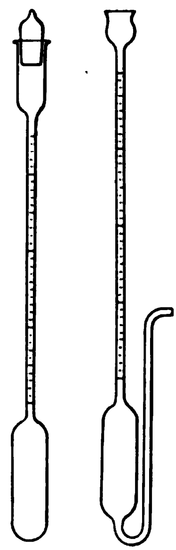


FIG. 65
DILATOMETERS

The constants of the dilatometer are to be determined by preliminary experiment. These are the weight of the instrument when empty, clean, and dry; the volume of the bulb at 0° to the zero line of the graduated scale, and the volume of the bore of the stem per unit of the divided scale; and the coefficient of the voluminal expansion of the glass bulb. If the observations for the coefficient of expansion of the bulb are carried out as described in the preceding exercise (Art. 126), one additional weighing, that of the weight of a measured length of a thread of mercury in the stem (Art. 52), suffices for the determination of all these constants.

The dilatometer is filled with the liquid to be investigated, so that when the temperature is that of the ice bath, 0°, the column of liquid in the stem rises only a small distance above the zero line. Let V_1 be the volume of the liquid when the temperature has become constant, equal to the volume of the bulb to the

zero line plus the volume of that portion of the stem above this line which is occupied by liquid.

Transfer the dilatometer to the warm bath (steam) and ascertain the apparent volume of the liquid, V_2, that is, the standard volume of the bulb plus the volume of that portion of the stem now occupied by liquid.

If $3\,a$ is the coefficient of cubical expansion of the liquid, by heating it from $0°$ to $t°$, its volume has been increased by an amount represented by $V_1 3\,at$. It now occupies a volume of the glass which at $0°$ is V_2, but the coefficient of cubical expansion of the glass bulb being $3\,\beta$, the increase in this volume at $t°$ is $V_2 3\,\beta t$. The observed apparent increase in volume, $V_2 - V_1$, is equal to the difference between the actual increases in the volumes of the liquid and the envelope; that is,

$$V_2 - V_1 = V_1\, 3\, at - V_2\, 3\, \beta t,$$

from which the mean coefficient of cubical expansion of the liquid is found to be

$$3\,a = 3\,\beta \frac{V_2}{V_1} + \frac{V_2 - V_1}{V_1 t}$$

Reference. — *Ostwald*, Physico-Chemical Measurements, p. 108.

CHAPTER X

THERMOMETRY

LXXIII. CONSTANTS AND ERRORS OF A THERMOMETER BY CALIBRATION

(a) Determine the fundamental interval of a thermometer.
(b) Calibrate the scale of a thermometer at ten equidistant points.
(c) Form a complete table of corrections to a thermometer.

128. Thermometry. — The mercury-in-glass thermometer is the almost universal laboratory instrument for the measurement of temperature. Such skill has been attained in the manufacture of thermometers that the indications of those now obtainable from reliable makers may be depended upon to a few hundredths of a degree. When greater precision is needed the thermometer may be submitted to one of the several national standardizing bureaus, who will furnish certificates of such constants and corrections as are desired. Still it will often be necessary for the user himself to make the many and somewhat difficult observations for standardizing the thermometer.

The mercury thermometer depends upon observations of the relative expansion of mercury and glass. The properties of glass are such that the bulb of a thermometer, having been expanded by exposure to a high temperature, upon being transferred to a low one contracts quickly for the greater part of the possible contraction; while there remains a small residual contraction which proceeds very slowly over a period of days or weeks. The amount of this residual contraction depends mainly upon the nature of the glass and the difference between the two temperatures. Hence for the same temperature a thermometer may give many different indications, depending upon its previous

treatment. For thermometers of Jena Normal Glass the error of this kind, due to a change of temperature of 100°, is about 0°.1. To obviate this difficulty, in precise thermometry, a temperature is always referred to that zero point which is found, or would be found, immediately after the temperature observation.

The measurement of temperatures by the mercury-in-glass thermometer is founded upon the following procedure. The thermometer is subjected to the temperature of steam under standard conditions until the indication is constant; the point corresponding to what would be the end of the mercury column, if the steam were under a pressure of 76 cm of mercury, is 100°. The thermometer is quickly transferred to melting ice; the *lowest* point reached by the mercury is 0°. A one-hundredth part of the volume between the two points is one degree. The thermometer is subjected to the unknown temperature till its indication is constant; it is then quickly transferred to melting ice, and the lowest indication is observed, which may be different from the former ice point. The temperature in degrees is the ratio of the volume between these last two points to the degree volume as defined.

To standardize a thermometer its fundamental interval must be found by locating the fixed points; then the relation of the internal volume of the tube to the scale which has been provided is to be determined, that is the scale is to be calibrated. In addition to these corrections, which should never be neglected, there are several others of less importance which are due to the following causes: the internal pressure of the mercury column which varies in height with the temperature, the external pressure due to varying barometric pressure, and the varying depth to which the thermometer may be immersed in a liquid.

129. Calibration of a Thermometer. — The unavoidable variations in the internal bore of the tube, and the errors of graduation, make it necessary to calibrate a thermometer in order that its readings may be reduced to the ideal scale.

The calibration of volume requires the separation of a thread of mercury from the column the length of which shall be approximately an aliquot part of the total length of the scale, — for

example 10°, on a thermometer scale from 0° to 100°. The thermometer is inverted, and by gently tapping the end on the hand the mercury is caused to run down into the tube. Sometimes a portion is separated by this operation alone; if not, a sudden turning of the thermometer with the proper jerking or tapping will often cause a separation at the neck of the bulb. This separated column is moved in the tube till its lower end is at some observed convenient mark, — for instance 20°; the thermometer is then placed horizontally, and the bulb is gently heated with the hand or otherwise till the mercury in the bulb expands and joins the column in the tube. The heat is withdrawn, and as the temperature falls the entire column of mercury is slowly withdrawn. When the upper end of the column is at that point of the scale which is above the point of junction by the length of thread desired, — at 30° in this instance, — the thermometer is given a longitudinal shock, when the column will again separate at the previous point of junction, leaving a thread of the desired length. Patience and repeated trials are sometimes necessary to success in this operation. The separation results from the presence of a small bubble of gas which adheres to the glass at the place of junction.

If the mercury thread is very fine, the separation in this manner is not practicable, and it may be necessary to heat the tube very carefully in the tip of a minute flame, continually turning the tube on its axis till the mercury is vaporized at the desired point of separation. Further heating will distill mercury to the separated thread, or from it, to make its length exactly that required. This method is rather dangerous, and requires skill; even when it has been applied successfully the thermometer, if of thick-walled tubing, may at some future time break where the heat was applied. If several threads are required after one has been produced by heating, the others may usually be obtained by the first method.

Place the thread of mercury between 0° and 10°, a cooling mixture being placed on the bulb to prevent the loss of the thread. To avoid the use of the cooling mixture, some mercury might have been placed in the bulb at the top of the thermometer

before the ten-degree thread was separated. Read the length of the thread, in degrees, as accurately as possible, using a low-power microscope. Place the thread between 10° and 20°, and read its length again; place it between 20° and 30°, and so on to the end. Find the average of the ten lengths observed; the amount that must be added algebraically to each observed length to make it equal to the average, is the correction to the corresponding portion of the thermometer. The successive sums of these corrections are the corrections to the volumes included between the zero mark and the successive ten-degree marks.

The most complete calibration of a thermometer at ten points requires the making of fifty-four observations with mercury threads 10°, 20°, 30°, . . ., 90° in length. The making of the observations and their reduction is fully described in Art. 30. A calibration in other aliquot parts may be made in the manner described for ten parts.

Somewhat greater precision may be secured by making a double calibration with the dividing engine, as explained in Art. 27. The scale is first calibrated as there illustrated. Then mercury threads are measured with the dividing engine in the same manner as was the scale; this will give the corrections to the volumes referred to an ideal scale of lengths in millimeters. The differences between the several volume corrections and the corresponding scale corrections, each referred to the ideal scale, are the corrections of the volumes referred to the actual thermometer scale. These corrections, expressed in millimeters, are reduced to degrees by the aid of the measured length of a degree in millimeters.

130. Fundamental Interval of a Thermometer. — To determine the boiling point the thermometer is placed in a steam bath obtained by means of the well-known form of boiling-point apparatus. The entire mercury column should be in the steam, and the bulb must always be above the water. The steam should be generated copiously, and it must have a free exit from the boiler. If the steam does not escape freely, the excess of pressure produced in the boiler may be measured and added to the barometer pressure. The reading is best made with a

telescope, the thermometer being withdrawn from the steam just sufficiently to allow the mercury to be seen. If the corrected barometric pressure, including the excess of pressure in the boiler, is B, then the temperature of the steam is

$$t = 100° - 0°.375\,(76.0 - B).$$

It may be more convenient to take the temperature of the boiling point from tables (Table 16).

The thermometer is to be transferred from the steam to the ice as quickly as is consistent with safety. It may without danger be plunged in ice from a temperature of 40°. To facilitate this transfer, two dishes of water may be prepared, — one at a temperature of about 60°, and the other at about 30°; the thermometer is dipped into these successively, and finally into the ice.

The ice bath is made of pure, clear ice, finely crushed, and closely packed into a receptacle of several liters capacity, the interstices being filled with distilled water. The thermometer should be immersed so that only the end of the mercury column is visible in a reading telescope whose line of sight is on a level with the top of the dish. The thermometer should be gently tapped on the end, and the *lowest* indication is the zero point desired. This will occur in perhaps two minutes after removal from the steam, and will be followed by a slow rising of the mercury.

The correction to the boiling point is the quantity which must be added to the interval in scale degrees between the observed steam and ice points to make this equal to the degrees of the true boiling point. This correction for the error in the fundamental interval affects the 100° point by its full value. There is no correction because of this error at 0°. All other readings require corrections, which are in the same proportion to the boiling-point correction as the given reading is to 100°.

131. Table of Corrections for a Thermometer. — The explanation of the method of forming a table of corrections for a thermometer can best be made by means of a complete numerical example.

Calibration of Jena Normal Glass Thermometer, Gerhardt 4234, at the Ten-Degree Points from 0° to 100°

Scale from −6° to +106°, divided in $\frac{1}{10}$°

July 24, 1902

A thread of mercury about ten degrees long was obtained by heating; by putting a little cotton wet with ether on the bulb, this was placed between 0° and 10°, and then without the ether it was placed between 10° and 20°, between 20° and 30°, etc., and finally between 90° and 100°. Three sets of such observations were made.

Position	Length			Mean	Correction	Σ Correction
0°						0°.000
0°–10	9°.88	9°.88	9°.88	9°.880	+0°.0648	+ .065
10 –20	.90	.90	.90	.900	+ .0448	+ .110
20 –30	.93	.93	.92	.937	+ .0078	+ .117
30 –40	.94	.95	.95	.947	− .0022	+ .115
40 –50	.96	.96	.96	.960	− .0152	+ .100
50 –60	.96	.96	.96	.960	− .0152	+ .084
60 –70	.96	.96	.96	.960	− .0152	+ .069
70 –80	.95	.96	.96	.957	− .0122	+ .057
80 –90	.98	.98	.98	.980	− .0352	+ .022
90 –100	.97	.97	.96	.967	− .0222	.000
	Average 9°.9448					

The thermometer was placed in a steam bath, the reduced barometer pressure being 74.61 cm, corresponding to a boiling point of 99°.48. The thermometer read 99°.44. It was immediately transferred to an ice bath, in which the lowest indication was −0°.01. The fundamental interval of the scale is thus 99°.97, requiring a correction at the 100° point of +0°.03.

The sum of the calibration correction and that for the fundamental interval gives the correction which when added to the reading at any part of the scale reduces this to the true mercury-in-glass temperature. If, further, we add the correction to reduce to the hydrogen thermometer scale (Art. 133), the result is the correction which when added to the scale reading of any temperature reduces this to the true temperature according to the normal hydrogen scale. The following table shows all these corrections.

Readings	Calibration	Fundamental Interval	Mercury-in-Glass Scale	H. Scale	Total Corrections to Readings for H. Scale
0°	0°.000	0°.000	0°.000	0°.000	0°.000
10	+ .065	+ .003	+ .068	− .055	+ .013
20	+ .110	+ .006	+ .116	− .090	+ .026
30	+ .117	+ .009	+ .126	− .109	+ .017
40	+ .115	+ .012	+ .127	− .115	+ .012
50	+ .100	+ .015	+ .115	− .109	+ .006
60	+ .084	+ .018	+ .102	− .096	+ .006
70	+ .069	+ .021	+ .090	− .076	+ .014
80	+ .057	+ .024	+ .081	− .053	+ .028
90	+ .022	+ .027	+ .049	− .027	+ .022
100	.000	+ .030	+ .030	.000	+ .030

These corrections are founded upon the assumption that the zero mark of the scale is the true ice point. According to the principles explained above, the ice point is to be determined immediately after each temperature measurement. Then the true temperature in every case is the scale reading plus the corresponding tabular correction, *plus the reading of the ice point with its sign changed.* The ice point will be found of such constancy that it will need only occasional determination.

It appears from the calibration that the maker has purposely constructed the scale of the thermometer to indicate temperatures on the hydrogen normal scale instead of mercury-in-glass temperatures; and that within the ordinary range of laboratory work the indications of the thermometer without any correction will never be in error more than 0°.02.

To test the calibration, the temperature of a large jar of water was measured with this and another standard thermometer. The thermometers were wholly immersed, the water was frequently stirred, and after some time the results were:

	No. 1721	No. 4234
Reading	24°.30	24°.12
Calibration correction	− 0 .003	+ 0 .12
Zero correction	− 0 .06	− 0 .01
Corrected reading	24 .24	24 .23

132. Exposed Column of Mercury. — The method described for measuring temperature requires that all the mercury be at one temperature. It often happens that a part of the column is necessarily exposed to a temperature different from the temperature of the bulb. Let l be the length in degrees of the exposed

column, t_1 its mean temperature determined by an auxiliary thermometer, and t_2 the temperature of the bulb; then for a Jena Normal Glass thermometer the correction to the reading is

$$+ 0.000157\, l\, (t_2 - t_1).$$

133. Comparison of Mercury-in-Glass and Hydrogen Thermometers. — The scale of the hydrogen thermometer is now accepted as the scientific standard for all temperature measurements. The indications of a mercury thermometer made of Jena Normal Glass (16 III) require the following corrections to correspond to the hydrogen thermometer (Chappuis, International Congress of Physics, Paris, 1900).

Readings	0°	10	20	30	40	50	60	70	80	90	100
Corrections	0°.000	−0.055	−0.090	−0.109	−0.115	−0.109	−0.096	−0.076	−0.053	−0.027	0.000

LXXIV. TEMPERATURE AND EXPANSION OF GASES WITH THE AIR THERMOMETER

Standardize an air thermometer and measure an unknown temperature. Calculate the coefficient of expansion of the gas.

134. The Air Thermometer. — The normal temperature scale is that of the hydrogen gas thermometer, founded upon the principles described below. Hydrogen is best suited for low-temperature measurements, while for temperatures above 180° it attacks some kinds of glass and permeates metals. For general use the best practical standard thermometer uses nitrogen as the thermometric material.

The gas, or air, thermometer may consist of a bulb of 50 ccm or more capacity, filled with dry air or hydrogen, or other suitable gas, communicating with a mercury manometer by means of a tube of small bore. Sometimes the bulb is rigidly attached to the manometer as shown in Fig. 66, and sometimes it is connected by a long, flexible metal tube. The manometer consists of a long and a short glass tube (the diameter being a centimeter or more) which are joined together and to a mercury cistern. A screw plunger in the cistern regulates the height to which the

mercury rises in the manometer. Inside the short manometer tube, near its junction with the capillary tube, is a small index, to which the height of the mercury is always to be adjusted, causing the gas in the bulb to occupy a constant volume.

The use of the apparatus as a thermometer depends upon the fact that the pressure of a gas at constant volume varies directly as its absolute temperature. The pressure of the inclosed gas is always equal to the barometric height plus the difference between the levels of the mercury in the open and closed arms of the manometer. Corrections must be applied to reduce the barometer readings to 0°, and also for the temperature of the manometer (Art. 54).

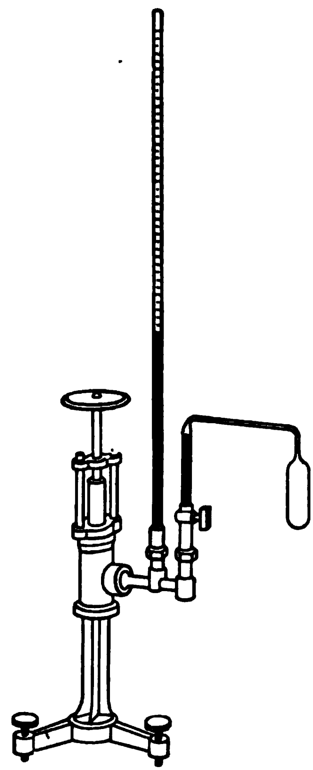


FIG. 66. AIR THERMOMETER

To calibrate the thermometer, place the bulb in a steam bath and adjust the manometer till the surface of the mercury touches the index. Determine the difference between the mercury levels by reference to the glass scale or, better, with a cathetometer. Let P_s be the sum of the manometer and barometer pressures (reduced to 0°) at the temperature, t_s, of the steam (Art. 130). Place the bulb in an ice bath, and let P_0 be the total pressure of the gas at the same volume as before, and at the temperature, 0°, of the melting ice.

Now place the bulb in the region the temperature of which, t_x, is to be determined, and observe P_x, the total pressure. (Read the barometer at each observation, to note changes in the atmospheric pressure.) Then approximately

$$t_x = t_s \frac{P_x - P_0}{P_s - P_0}.$$

In this formula it is assumed that the space occupied by the air in the connecting and manometer tubes, which is not varied in temperature, is comparatively insignificant; and the expansion of the glass bulb is neglected. If v is the volume of the air space which is heated, and v' the volume which is not heated (determined by mercury calibration), 3β the coefficient of cubical expansion of the glass, and t_r the temperature of the air in the room, then

$$t_x = t_s\left(\frac{P_x - P_0}{P_s - P_0}\right)\left[1 - \frac{P_s - P_x}{P_0}\left(\frac{3\beta}{0.00367} + \frac{v'}{v}\cdot\frac{1}{1 + 0.00367\,t_r}\right)\right],$$

in which 3β may be taken as equal to 0.000025.

The Coefficient of Expansion of Gas. — From the above measurements the coefficient of expansion of the gas, referred to its volume at 0°, is

$$a = \frac{P_s - P_0}{P_0 t_s},$$

the symbols having the meanings given above.

REFERENCES. — *Kohlrausch*, Physical Measurements, pp. 93–97; *Guillaume*, Thermométrie, Chap. V.

LXXV. HIGH TEMPERATURES WITH RESISTANCE AND THERMO-ELECTRIC PYROMETERS

Determine the melting point of zinc.

135. Measurement of High Temperatures. — An instrument for measuring temperatures, especially those which are above the range of the ordinary mercury thermometer, is called a *pyrometer*. A mercury thermometer with a vacuous space is useless for temperatures above 300°, and even above 100° the mercury begins to distill from the heated column to the cold part of the stem, causing inaccuracies. Mercury thermometers made of special hard Jena glass to withstand the heat, and containing carbon dioxide under a pressure of twelve or more atmospheres above the mercury to prevent its vaporization, are capable of measuring temperatures up to 550°.

For high temperatures, as well as for ordinary and low temperatures, the air thermometer is the standard; but for general use either one of two other instruments has been found more practicable. For precise work the platinum resistance thermometer of Callendar and Griffiths is best adapted. It will measure temperatures as high as 1200°, though its precision diminishes above 700°. The thermo-electric thermometer of Le Chatelier is simpler in construction and manipulation, and it can be used by workmen in technical industrial operations. It measures temperatures up to 1500°, or even to the melting point of platinum, 1775°, but with somewhat less precision than is attained with the platinum thermometer.


FIG. 67. PLATINUM THERMOMETER

136. The Platinum Thermometer. — The variations of the electrical resistance of a coil of platinum wire, as its temperature changes, furnishes a valuable means of measuring temperature. The platinum thermometer (Fig. 67) consists of a coil of pure platinum wire wound on a mica frame, which is protected by a glass or porcelain tube; copper or platinum wires pass from the coil to binding posts on the end of the tube so far removed from the coil that they are uninjured when the coil is heated. The coil may have a resistance of about 2.6 ohms at 0°, which increases about one ohm for each hundred degrees increase in temperature. The resistance is measured by a Wheatstone's bridge, which, if especially designed for the purpose, will indicate variations in the resistance of 0.00001 ohm, corresponding to 0°.001 in the temperature.

To avoid breaking, the thermometer should be gradually heated to the temperature to which it is to be subjected. This may be done in a muffle, or by placing it in the furnace or crucible before the heat is applied. The platinum thermometer is so fragile and requires such care in making the measurements that it is not adapted to industrial operations.

To measure a temperature, the fundamental interval is first determined, much as for a mercury thermometer (Art. 130), by finding the resistances, R_0 and R_{100}, when the coil is at the temperature of melting ice and of steam at 76 centimeters pressure. The change in resistance per degree is, then, $\frac{R_{100} - R_0}{100}$; and if the resistance at some unknown temperature is R_t the *platinum temperature* is

$$t_p = (R_t - R_0)\,\frac{100}{R_{100} - R_0}.$$

This platinum temperature differs from that of the air thermometer; for temperatures up to 600°, the relation between the true temperature, t, and the platinum temperature, t_p, is given by the following equation.

$$t = t_p + 1.54\left[\left(\frac{t}{100}\right)^2 - \frac{t}{100}\right].$$

137. The Thermo-Electric Pyrometer. — The electromotive force of a thermo-electric couple varies with its temperature in such a manner that it may be used as a trustworthy measure of temperature. The thermo-electric pyrometer consists of a thermo-electric couple made of pure platinum and an alloy of platinum with ten per cent of iridium, fused together. The junction is protected by a porcelain tube and is to be placed in the high temperature to be measured. Commonly the cold junction does not exist, though for research work it may be used, being placed in steam or ice. A thermo-electromotive force is generated which is approximately proportional to the temperature. If one junction is at 0° and the other at 300°, the electromotive force is 2.26 millivolts; at 600°, 5.14 millivolts; at 900°, 8.33 millivolts; at 1200°, 11.83 millivolts; and at 1500°, 15.64 millivolts.

The thermo-electromotive force is very conveniently measured with a portable D'Arsonval galvanometer (Art. 209), in which the direct deflections are observed, the scale being an arbitrary one, or corresponding approximately to the temperatures. The method possesses the great advantage that the

galvanometer may be permanently placed, while the thermo-element is taken to any desired distance for use. The lead wires should be protected from any variation in temperature. The indications of the pyrometer are almost instantaneous, and the tube with the couple may be small in size; in some cases the tube may be dispensed with, only the junction being heated.

No simple equation has been found connecting temperature and thermo-electromotive force in general, and the best procedure is to calibrate a given thermo-electric couple by reference to well-established *fixed points* (Art. 138), interpolating graphically for the unknown temperature. Two or more fixed points should be chosen which are as near as possible to the temperature being measured. The error of such a measurement need not be greater than one per cent.

For research work the electromotive force, e, should be accurately measured by comparison with a standard cell by the compensation method described in Arts. 250 and 251. Then any temperature, t, measured with a given thermo-couple is determined by the equation

$$\log e = a \log t + b,$$

the constants a and b having been determined by measuring the electromotive force for two known temperatures. Le Chatelier found the constants to give the equation

$$\log e = 1.2196 \log t + 0.302.$$

With this method the error of a determination may not be greater than one part in five hundred.

138. Fixed Points for High Temperatures. — The following well-determined high temperatures are suitable for standardizing any form of pyrometer.

Ebullition of water . .	100°	Fusion of aluminum . .	657°
Ebullition of naphthalene	218°	Ebullition of zinc . . .	930°
Fusion of tin	232°	Fusion of gold	1064°
Fusion of zinc	419°	Fusion of copper . . .	1084°
Ebullition of sulphur . .	445°.2	Fusion of platinum . .	1775°

When measuring the fusion point of metals which are obtainable in any desired quantities the metal is melted in a crucible, and then the pyrometer is inserted. The crucible is allowed to cool, and the stationary temperature of solidification is easily observed. With small quantities of metal, and for the ebullition points, special methods are required which are described in the references.

REFERENCES. — *Le Chatelier*, High Temperature Measurements, pp. 83–128 and 200-208; *Chappuis*, and *Barus*, Rapports au Congrès International de Physique, Paris, 1900, Tome I, pp. 144–177; *Chappuis*, Travaux et Mémoires du Bureau International des Poids et Mesures, Tome XII, Part 3.

CHAPTER XI

CALORIMETRY

LXXVI. CALORIMETER CONSTANTS BY CALCULATION AND EXPERIMENT

Determine the heat capacity of a calorimeter and the correction for radiation. (This problem is to be performed as part of some calorimetric measurement.)

139. Calorimetry.— The measurement of the quantity of heat which appears or disappears during a physical or chemical change is the object of calorimetry. The changes usually considered are fusion, vaporization, combustion, solution, and change of temperature. The unit of heat is $\frac{1}{100}$ of the quantity of heat required to change the temperature of 1 gram of water from 0° to 100°; it is called the mean *calorie.* The calorie represents the actual quantity of heat required to change the temperature of 1 gram of water from 17° to 18°, or from 72° to 73°. At all other temperatures the quantity required is either slightly more or less than 1 calorie, having a maximum value of 1.008 calories at 0°, and a minimum of 0.997 at 40°. In terms of the mean calorie, the calorie at 17°, the heat equivalent of fusion of ice is 80.1 cal_{17}, and the mechanical equivalent of the mean calorie is 4.184×10^7 ergs.

A calorimeter often consists of a cup of about 500 ccm capacity, for containing water or other liquid, which may be provided with a cover, a stirrer, and a thermometer. The cup is supported on nonconducting points inside another cup, somewhat larger in size, forming an air jacket around the inner cup. Both cups should have bright, polished surfaces. The qualities desired in the material for a calorimeter cup are that it should be thin,

bright, a good conductor of heat, and unaffected by the liquids experimented with. Platinum or silver are the best materials, while for ordinary purposes nickel-plated copper or brass is suitable. Glass is less suitable because of its low conductivity.

The stirrer may be disk shaped, to be moved up and down through the liquid by means of a handle. A turbine-wheel stirrer is more efficient, and it may be kept in continuous operation by connection with a small motor; this is almost a necessity if many measurements are to be made.

Other forms of calorimeters are used according to the requirements of the method, such as the ice and cooling calorimeters described in Arts. 142, 143, and 146.

Thermometers for calorimetry measure only changes of temperature, which rarely exceed 10°. Such need not have the boiling point on their scales; the scales may cover only a few degrees, and can therefore be open, the graduations being in $\frac{1}{10}°$, $\frac{1}{50}°$, or $\frac{1}{100}°$. Such a thermometer needs calibration as to its scale, and must be compared with a standard thermometer to determine the value of the degree of its scale. To overcome capillary resistance, the thermometer may be gently tapped on the end before a reading is taken. Since the observed changes are small, the readings should be made with the greatest possible care, the tenths of the smallest division of the scale being estimated or read with a micrometer microscope.

140. Heat Capacity of a Calorimeter. — The number of calories required to raise the temperature of the entire calorimeter and contents one degree is its heat capacity. It is the sum of the heat capacity of the water (or other liquid) and the *water equivalent* of the other parts that are subject to the temperature changes. These parts are commonly the cup, the stirrer, and the thermometer. A direct experiment may be made to determine the water equivalent of these parts, but it is more often calculated from known constants.

The cup and stirrer are usually of the same material; the product of their combined mass and the specific heat of the substance is the water equivalent. It is assumed that the entire apparatus is equally heated. Since the mass is small, — and the

material is always a good conductor of small specific heat, — and the mass of the water is relatively large, this assumption cannot cause an appreciable error.

In calculating the heat equivalent of the thermometer, glass being a poor conductor of heat, only that part is to be considered which is immersed. This is of glass and mercury combined in uncertain proportions. Since the heat capacities for equal volumes of glass and mercury are nearly the same, 0.46 calories per cubic centimeter, the water equivalent is most conveniently determined from the volume of the immersed portion. This volume may be determined by dipping the thermometer into water contained in a tube with volumetric graduations; or it may be easily determined while the calorimeter cup and water are on the balance for the determination of their mass. When the cup has been balanced the thermometer is dipped into the water to the same depth as it will be in the heat experiment; the apparent increase in the weight of the water, in grams, is the volume in cubic centimeters of the immersed portion of the thermometer.

If, then, m_1 is the mass of the liquid in the calorimeter whose specific heat is c_1, m_2 and m_3 are the masses of the cup and stirrer of specific heats c_2 and c_3, and v is the volume of the immersed portion of the thermometer, the heat capacity of the calorimeter is

$$K = m_1c_1 + m_2c_2 + m_3c_3 + 0.46\,v.$$

If water is the liquid used, c_1 may be assumed to be 1, and if the cup and stirrer are of the same material, the formula becomes

$$K = m_1 + (m_2 + m_3)\,c_2 + 0.46\,v.$$

If the body tested is contained in a receptacle which is introduced into the calorimeter with it, the heat capacity of the receptacle must be considered in a manner which will be obvious from the conditions of the experiment.

141. Radiation Corrections. — In some methods of calorimetric measures the errors due to radiation or absorption are obviated by surrounding the calorimeter with a protecting jacket. This is illustrated by the ice calorimeter and by the cooling

calorimeter. In the method of heating, the radiation effects are negligible. In the other methods described, these errors may be of considerable magnitude, and precaution must be taken to diminish and to determine their effects. It is often assumed that the radiation errors will be compensated by starting with the calorimeter at a temperature as much below that of the room as at the end of the experiment the temperature will be above that of the room. The assumption that the calorimeter will gain as much heat during the first half of the experiment as it will lose during the second half is not justified unless the rise in temperature of the calorimeter is proportional to the time. It rarely happens that this is true; hence it is necessary in precise work to determine the exchange of heat for each observation. The methods employed depend upon Newton's law of cooling, — that the exchange of heat by a body with its surroundings is proportional to the difference in temperature and to the time. When the difference in temperature is small, not exceeding 10°, this law may be applied for the purposes of correction; but for large temperature differences it is not sufficiently exact.

It is necessary to determine the radiation errors for two methods of measurement. In one, the method of mixtures, the change in temperature of the calorimeter is very rapid at first, and very slow towards the end of the observation. In the second case, as in determining the heat equivalent of vaporization, the heat is supplied to the calorimeter at a nearly constant rate, causing a more uniform rise in temperature.

Radiation Correction for Method of Mixtures. — In calorimeter observations by the method of mixtures (Art. 144) the temperature changes very rapidly at first, and as the two substances come nearer to the final common temperature the exchange of heat between them takes place very slowly. It should therefore be arranged that this final temperature shall be about that of the surroundings. The calorimeter should be cooled below the room temperature by an amount equal to the expected temperature change. This change is found either by calculation from the assumed specific heats or by a preliminary experiment. Observations are made of the temperature of the calorimeter at

intervals of a minute, to determine the amount of warming per minute per degree difference in temperature. The heated body is then placed in the calorimeter at a noted time, and the temperature of the calorimeter is read every fifteen or twenty seconds till it has become constant, or until the temperature change is uniform. The average temperature during the mixture operation is calculated. The difference between the average and the room temperature, multiplied by the duration of this part of the experiment and the rate of warming, is the temperature gained by absorption. The correction is this quantity with the negative sign.

A complete numerical example is given to illustrate this method. It shows that the error due to absorption is small when the operations are carried out as described, being only one part in two hundred in this particular case.

Specific Heat of Iron

METHOD OF MIXTURES

April 15, 1903

Weight of copper cup and stirrer, 53.3 g; specific heat of copper, 0.093; volume of the immersed thermometer, 2.6 ccm; weight of water, 452.3 g; from which

$$K = 53.3 \times 0.093 + 2.6 \times 0.46 + 452.3 = 458.5.$$

Weight of iron which is in the form of a hollow cylinder, 243.2 g, heated to 99°.42; temperature of room, 18°.5; thermometer No. 1721, divided to $\frac{1}{10}°$.

Time	Temperature		
$2^h\ 35^m\ 00^s$	12°.34		
36 00	12 .37		
37 00	12 .40		
38 00	(12 .43)	Iron introduced	Average 16°.31
38 20	16 .80		
38 40	17 .09		
39 00	17 .17		
39 20	17 .18		
39 40	17 .18	Highest temperature	
40 00	17 .18		
41 00	17 .18		

The temperature of the calorimeter at the instant the iron was introduced was calculated from the rate of warming.

During the first observations the calorimeter gained heat at the rate of 0°.03 per minute, with a mean temperature difference from that of the room of 6°.2, a gain of 0°.005 per degree per minute. After the iron was introduced the average temperature for the two-minute duration of the experiment was 16°.31, differing from that of the room by 2°.2. Therefore the gain in temperature during the experiment because of absorption is 0°.005 × 2.2 × 2 = 0°.022.

The observed temperature change of the calorimeter was 17°.18 − 12°.43 = 4°.75; while the iron changed 99°.42 − 17°.18 = 82°.24.

The specific heat of the iron is

$$c = \frac{458.5 \times (4.75 - 0.02)}{243.2 \times 82.24} = 0.1084.$$

Radiation Correction when Temperature Change of Calorimeter is Uniform. — In some calorimetric observations the calorimeter changes temperature at a nearly uniform rate (Arts. 147, 148, 149, and 150). If the temperature of the calorimeter at the beginning is as much below the room temperature as at the end it will be above that of the room, the calorimeter will gain about as much heat during the first half of the experiment as it will lose during the second half. When the highest precision is required it may be desirable to verify or correct this assumption by determining the heat exchanges for each experiment.

Observations are made, as described in the preceding article, to determine the increase in temperature because of absorption. Similar observations made during the time the calorimeter is above room temperature will permit the loss because of radiation to be computed. The algebraic sum of the two effects is the error; the correction has the same magnitude but the opposite algebraic sign. This method is illustrated in the numerical example given in Art. 150.

LXXVII. SPECIFIC HEAT BY BLACK'S METHOD

Determine the specific heats of iron, zinc, lead, and brass.

142. Black's Ice Calorimeter. — If the heat given out by a body is caused to melt ice, the quantity of heat may be determined from the mass melted. The calorimeter consists of a

block of ice with a cavity to contain the warmed body and the water produced in cooling, and a cover to prevent loss of heat. The method is both simple and accurate when solid ice, such as manufactured ice, is used.

Weigh the body to be tested, which may conveniently be in the shape of a sphere of four centimeters diameter, with a small hook attached by which to lift it. Heat the body to a known temperature in a water or steam bath.

A piece of solid ice, selected to be free from air bubbles and cracks, is shaped so that its upper surface is slightly convex. In the middle of this surface make a cavity large enough to contain any one of the bodies whose specific heat is to be determined. The cavity may be made of the proper size and with a smooth surface, by causing one of the warmed spheres to melt its way into the ice. Provide a smooth slab of ice, slightly concave on its under surface, to serve as a cover. The shapes of surfaces suggested are to prevent the water produced by surface melting from running into the cavity.

Select a sponge of a size convenient to wipe out the cavity, moisten it, squeeze it dry, place it in a small beaker, cover with a watch glass, and weigh carefully. Wipe the ice cavity with another sponge, transfer the warmed body to the cavity with the least possible loss of heat, and quickly cover it with the ice slab. As soon as the body has cooled to 0°, which may be in ten minutes under the circumstances described, remove the cover and body, wipe up all the water in the cavity with the weighed sponge, and weigh again to determine the amount of ice melted.

If m is the mass of the body, t its temperature when placed in the ice, and M the mass of the ice melted, the specific heat of the substance is

$$c = \frac{80.1\,M}{mt}.$$

LXXVIII. SPECIFIC HEAT WITH BUNSEN'S ICE CALORIMETER

Prepare a Bunsen ice calorimeter and use it to determine the specific heat of copper.

143. Bunsen's Ice Calorimeter. — If the heat given out by a body is caused to melt ice, the quantity of heat may be determined from the change of volume which results. One gram of ice has the volume 1.0908 ccm, while one gram of water at 0° has the volume 1.0002 ccm. The quantity of ice that must be melted to cause a change of 1 ccm in volume is $\frac{1}{0.0906} = 11.04$ g; and the heat required to melt it is $11.04 \times 80.1 = 884$ calories.

When properly applied this method is not subject to the errors of radiation, and it may be used to measure small and slow thermal changes. It is suitable for determining the specific heat of solids and liquids, and especially when only a small quantity of the substance to be experimented upon is available. The disadvantage of the method is the trouble of preparation, and its accuracy is not greater than that of other common methods.

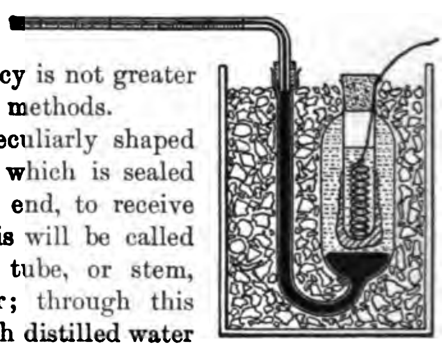


Fig. 68. Bunsen's Ice Calorimeter

The calorimeter is a peculiarly shaped glass vessel (Fig. 68) into which is sealed a tube closed at its inner end, to receive the body to be tested; this will be called the test tube. Another tube, or stem, leads into the calorimeter; through this the calorimeter is filled with distilled water which has been recently boiled to remove air. Mercury is poured into the stem, displacing some of the water, till the stem is wholly filled with mercury, and the lower portion of the body is partially filled, as indicated.

Place the calorimeter carefully in crushed ice, which may be conveniently contained in a wooden pail. Pack the ice closely and fill the interstices with ice-cold water.

A tube about 30 cm long, the bore of which is uniform and has a diameter of 0.5 mm, is fitted by a rubber cork or a ground joint

into the upper end of the stem, so as to stand in a horizontal position. Calibrate this tube with mercury by the method of Art. 52, determining the volume of the bore of the tube per centimeter of length. Insert the tube in the stem and push it down till it is filled with mercury, taking care that no air remains in the stem between the cork and mercury.

When the calorimeter has been packed long enough for the whole to have become cooled to 0°, pour a little ether or carbon bisulphide into the test tube, and rapidly evaporate it by forcing air through it with a foot blower. This will cause some of the water in the body to freeze around the test tube. Since ice has a greater volume than the water from which it is formed, the progress of freezing can be inferred by the flow of mercury from the calorimeter. Continue the freezing until about a cubic centimeter or more of mercury has been forced out. When the carbon bisulphide or ether has all been evaporated or removed, pour into the test tube sufficient water at 0° to half fill it, and place a cork in the mouth.

The body whose specific heat is to be determined may be in the form of a wire spiral. Weigh it and heat it in a water bath to the boiling point.

If the mercury in the fine tube is moving outward, the water in the body of the calorimeter is freezing, which may be caused by impurities in the packing ice producing a temperature below 0°; if the mercury is moving inward, the packing is probably imperfect and must be improved; if the mercury is stationary for fifteen minutes or more, the calorimeter is ready for use. It is often impossible to secure a stationary condition, in which case the rate of motion may be observed and a correction applied for the error thus introduced into the observations. Taking great care to avoid loss of heat, transfer the warmed body to the inside of the test tube; care is also necessary to prevent breaking the tube. The heat given up by the body as it cools to 0° will melt some of the ice around the test tube, causing a contraction of volume, which is indicated by the inward movement of the end of the mercury column in the fine tube. Measure the motion which has thus taken place when the mercury

has become stationary, which may require an hour's time. A scale is often etched upon the tube for making this measurement, but a separate scale is equally convenient.

If the motion of the mercury column produced by the cooling of the body to 0° is l, and v is the volume of unit length of the bore of the tube, the heat given out by the body is 884 lv calories; and if m is the mass of the body, and t its temperature when put into the calorimeter, its specific heat is

$$c = \frac{884\, lv}{mt}.$$

References. — *Ostwald*, Physico-Chemical Measurements, p. 136; *Kohlrausch*, Physical Measurements, p. 121; *Preston*, Theory of Heat, p. 217; *Nichols*, A Laboratory Manual of Physics, Vol. II, p. 237.

LXXIX. SPECIFIC HEAT OF A SOLID BY THE METHOD OF MIXTURES

Determine the specific heat of aluminum and of iron.

144. Specific Heat by Method of Mixtures. — One of the most useful methods for determining the specific heat of solids and liquids is that known as the method of mixtures. In this method the body to be tested has its heat mixed with the heat of the calorimeter; usually there is no physical mixture of the substances.

In Regnault's apparatus the heating of the body is accomplished without wetting it. A large vessel to hold steam or hot water has a tubular hole through its middle, passing in either a vertical or inclined direction. The body to be heated is held in the middle of the tube by a string; there are contrivances for stopping the ends of the tube, and a thermometer is passed through the upper stopper, so that its bulb is near the body. The heating is to be continued until the water has been boiling for some time, and the indication of the thermometer is constant.

The calorimeter, usually of the form described in Art. 139, is mounted on a small carriage running on a track so that it

may be quickly brought under the lower end of the tube through the heater, to receive the warmed body with little loss of heat. A protecting screen should be placed between the heater and the calorimeter.

The measurement of a specific heat, with the determination of the heat capacity of the calorimeter and the radiation correction, as described in Arts. 140 and 141, requires the following observations.

Weigh the body to be tested and place it in the heater. Weigh the dry calorimeter cup and stirrer, and measure the volume of that part of the thermometer to be used. Pour into the cup sufficient water to cover the body tested when the latter lies on the bottom of the cup, and determine the mass of the water.

In order that the radiation correction may be small, the temperature of the calorimeter when the body is introduced should be so much below that of the surroundings that the final temperature shall be very nearly the same as that of the room. A preliminary trial of the experiment may be made to determine approximately the temperature change that will occur; or the change to be expected may be calculated from assumed values of the quantities involved.

After the body has become thoroughly heated, cool the calorimeter a little below the temperature desired for the beginning of the test, and make observations of the temperature at intervals of a minute to determine the rate of warming. At a noted time observe the temperature of the calorimeter, that of the heated body, and transfer the latter to the calorimeter as quickly as possible and without splashing the water. Remove the calorimeter from the heater. Stir the water and note the temperature at intervals of 20 seconds till it becomes constant, or until it changes uniformly because of radiation or absorption. Continue the temperature observations for several minutes at intervals of a minute, to determine the rate of change. Calculate the radiation correction as explained in Art. 141.

If K is the heat capacity of the calorimeter, t_1 its temperature when the body is introduced, t_2 the highest temperature attained

by the calorimeter, R the radiation correction, M the mass of the body being tested, and t_3 its temperature when introduced into the calorimeter,

$$K[(t_2 - t_1) + R] = Mc(t_3 - t_2),$$

from which c, the specific heat of the substance, is to be calculated.

LXXX. SPECIFIC HEAT OF A LIQUID BY THE METHOD OF MIXTURES

Make two determinations of the specific heat of glycerin.

145. Specific Heat of Liquids by Method of Mixtures. — The method of mixtures described in the preceding article may be used for determining the specific heat of a liquid, the variations in the details of construction and operation of the calorimeter being mentioned below.

The liquid to be tested is placed in a heater, H (Fig. 69), which is supported in a larger vessel in such a manner that it may be entirely surrounded by water. Heating arrangements and a stirrer for the water bath are provided, while a thermometer with its bulb in the heater will indicate the temperature of the liquid. If the liquid is not too volatile, the water bath may be boiled; for very volatile liquids an ice bath or a freezing mixture may be substituted for the hot water, and the variations in procedure required will not need description. In either case the liquid must be left in the bath till the indication of the thermometer is constant.

A tube having a suitable stopcock leads from the bottom of the heater through the water bath, and through a double wooden protecting screen, into the calorimeter. The calorimeter cup is supported, inside the usual protecting case, on an adjustable stand. In the interior of the cup, fastened so as to be entirely surrounded by the water of the calorimeter, is a receptacle, R, for the liquid. The tube from the heater leads into the upper part of this receiver; while above it, and connected with it by a short tube, is a smaller vessel, C, which serves to condense

any vapor that may rise from the warm liquid. The condenser opens to the air through a tube. The calorimeter is provided with a finely divided thermometer and a stirrer.

The calorimeter cup, with the receptacle, stirrer, etc., are weighed when clean and dry. Sufficient water is then placed in the cup to cover the receptacle and condenser, the mass of the water being determined. To minimize the errors of radiation the calorimeter should be cooled so much below the temperature of the room that the final temperature shall be nearly the same as that of the room. If the liquid to be tested is cooled instead of heated, the calorimeter must be warmed at the beginning of the experiment. The change in the temperature of the calorimeter to be expected is determined by calculation from known constants or by a preliminary trial of the experiment.

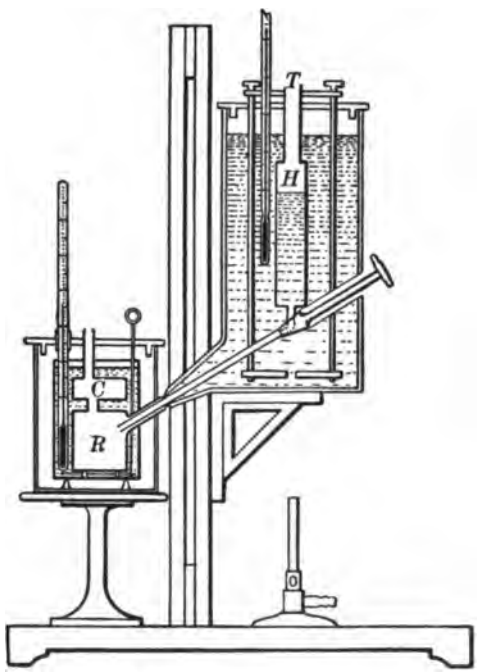


FIG. 69. APPARATUS FOR SPECIFIC HEAT OF LIQUIDS

When the liquid has become thoroughly heated, cool the calorimeter a little below the temperature desired for the beginning of the test, and make observations of the temperature at intervals of a minute to determine the rate of warming. Place the calorimeter in position to receive the liquid from the heater. At a noted time observe the temperature of the calorimeter and of the liquid, open the stopcock, and, by air pressure applied to the tube T of the heater,

force the liquid quickly into the calorimeter. Remove the calorimeter from the heater. Stir the water and note the temperature at intervals of 20 seconds till it becomes constant or until it changes uniformly because of radiation or absorption. Continue the temperature observations for several minutes at intervals of a minute to determine the rate of change. Calculate the radiation correction as explained in Art. 141.

Weigh the calorimeter and contents to determine the mass of the liquid which has been tested.

If K is the heat capacity of the calorimeter (Art. 140), t_1 its temperature when the liquid is introduced, t_2 the highest temperature attained by the calorimeter, R the radiation correction, M the mass of liquid tested, and t_3 its temperature when introduced into the calorimeter,

$$K[(t_2 - t_1) + R] = Mc(t_3 - t_2),$$

from which c, the specific heat of the liquid, is to be calculated.

LXXXI. SPECIFIC HEAT OF A LIQUID BY COOLING

Determine the specific heat of turpentine.

146. Specific Heat by Method of Cooling. — The determination of specific heats by the method of cooling is based upon this principle, that when a body cools in a given inclosure the quantity of heat radiated depends only upon the difference of temperature between the body and the inclosure, and upon the nature of the surface of the body. In other words, the times required for equal masses of different substances, under identical conditions, to cool through a given temperature range, are proportional to the quantities of heat they hold; that is, to their specific heats.

If a substance of mass m_1 and specific heat c_1 cools through a given temperature range in the time t_1, and a second substance with an identical radiating surface, in the same cooling chamber, having a mass m_2 and a specific heat c_2, requires a time t_2

for the same temperature change; and if K is the heat capacity of the receptacle and thermometer (Art. 140), then

$$\frac{m_1c_1 + K}{m_2c_2 + K} = \frac{t_1}{t_2}.$$

From this equation one of the specific heats may be calculated if the other quantities are known.

This method is useful for liquids, and especially for those which can be obtained in small quantities only, since they may be given the identical radiating surfaces required, by placing equal volumes successively in the same containing vessel.

The calorimeter consists of a containing vessel and a cooling chamber, as represented in Fig. 70. The small flask for containing the liquid is made of silver, platinum, or glass, and the thermometer serves as stopper and handle. The cooling chamber may be filled with water at the room temperature, though crushed ice is preferable.

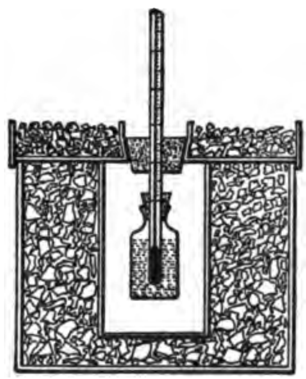


FIG. 70. COOLING CALORIMETER

The flask is filled with a known mass of the liquid investigated, which is then heated to a temperature of 50° or more; the flask is transferred to the cooling chamber, and times are noted at which the thermometer indicates 40°, 35°, 30°, etc., down to a temperature near that of the cooling chamber. The same flask is then filled with water to the same depth as before, and the mass of the water is determined. This is then heated and cooled, and the time observations are made as before. The rates of cooling through the same temperature range are thus determined; a graphic solution may be the most convenient. Substitution of the observed quantities in the formula will enable the determination of the specific heat required. No corrections are needed for water equivalent of the calorimeter, loss by radiation, etc.

LXXXII. SPECIFIC HEAT OF A LIQUID BY THE METHOD OF HEATING

Determine the specific heat of a solution of 1 gram-molecule of cane sugar (molecular weight 342) in 50 gram-molecules of water (molecular weight 18); find the molecular heat of sugar in solution.

147. Specific Heat of Liquids by Method of Heating. — The specific heat of a liquid contained in one calorimeter may be compared with that of a standard liquid contained in a similar calorimeter, as proposed by Pfaundler, by observing the increases in temperature produced when equal quantities of heat are imparted to the calorimeters. These equal quantities of heat are generated by sending a current of electricity through two equal resistance coils connected in series, one of the coils being placed in each calorimeter. If the quantities of liquids employed are so proportioned that the rise in temperature is the same, or nearly the same, in each calorimeter, the corrections for the water equivalent of the cups, etc., and for radiation are negligible.

Magie's form of this apparatus consists of two exactly similar and equal calorimeters, the details of which are shown in Fig. 71. Through insulating bushings in the cover pass two heavy copper wires, between which beneath the surface of the liquid is the resistance coil of about 4 ohms. This coil is preferably made of a German silver alloy which has a zero temperature coefficient of resistance. The spiral is stretched downward and the middle is held in place by a glass rod attached to the cover. The coil and lead wires are covered with an insulating varnish which is not affected by the liquids.

A turbine stirrer and thermometer are arranged as shown. The turbines of the two calorimeters are turned by the same belt of rubber cord, as any inequality in the stirring produces appreciable error. The thermometers are provided with automatic electric tappers.

The two calorimeters must be similarly placed with respect to the surroundings, and they may be inclosed in a box or in a water tank, for further protection.

One cup contains the standard liquid, for example 700 g of water, and the other cup as much of the liquid to be tested as will give an equal rise in temperature. This amount is determined either by trial or by computation from the assumed specific heats. Cool the two cups to a temperature about 5° below that of the room, place them in position, and stir the liquids. If the temperatures are unequal, warm the cold cup with the hands. When the two are approximately equal read the thermometers, turn on the current, about 5 amperes for the conditions described, and start the stirring. Continue the current till the temperature is as much above that of the room as at the beginning it was below this temperature. Continue the stirring and observe the highest uniform temperature attained.

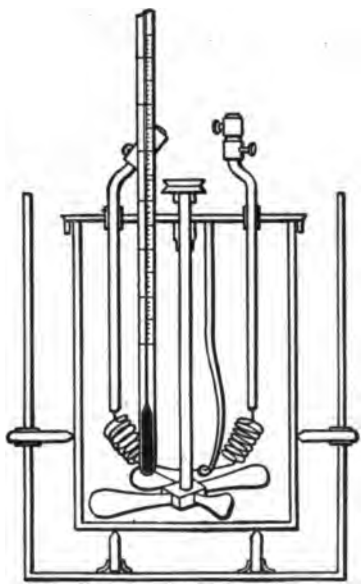


FIG. 71. CALORIMETER FOR ELECTRIC HEATING

From the observations determine the rise in temperature of each liquid; let θ_1 and θ_2 be these increases, m_1 and m_2 the masses of the two liquids, whose specific heats are c_1 and c_2; then

$$m_1c_1\theta_1 = m_2c_2\theta_2,$$

from which the unknown specific heat c_2 may be calculated. If the standard liquid is water,

$$c_2 = \frac{m_1\theta_1}{m_2\theta_2}.$$

For those investigations to which it is applicable, this method is probably the most precise known. Details as to the comparison of thermometers and resistances, and of precautions necessary for extreme precision are given in the references.

For a solid which has such poor conductivity that the usual methods are not suitable, the following may be used for determining its specific heat. The solid in a finely powdered state is placed in one calorimeter with some liquid which does not dissolve it. If the specific gravity of the solid is not too great, when the liquid is stirred the powder travels through the whole cup, and takes the temperature quickly. The second cup contains some of the same liquid without the solid. The comparison of the specific heats as described will permit that of the powder to be determined.

Professor Magie gives the following example of the use of this calorimeter.

Specific Heat of Milk Sugar Solution

METHOD OF HEATING

April 1, 1902

A solution of 1 gram-molecule of milk sugar in 200 gram-molecules of water was prepared. The molecular weights of these substances being 342 and 18, the masses were in the proportion of 342 to 3600. In cup A was 700 g of water, and in cup B 736.57 g of solution.

	Cup A	Cup B
Temperature before heating	15°.130	15°.115
Temperature after heating	25 .324	25 .309
Rise of temperature	10 .294	10 .294

$$\text{Specific heat of solution} = \frac{700 \times 10.294}{736.57 \times 10.294} = 0.9504.$$

Hence the heat capacity of 3942 g of solution would be 3746.4 calories; of this amount of solution 3600 g are water, the heat capacity of which is 3600 calories. Therefore the apparent heat capacity of the 342 g, or 1 gram-molecule, of the milk sugar in solution is 146.4 calories, which is then the apparent molecular heat.

Reference. — *Magie*, Physical Review, Vol. 9, p. 72, 1899; Vol. 14, pp. 193–203, 1902; Vol. 16, p. 381, 1903.

LXXXIII. HEAT EQUIVALENT OF FUSION OF ICE

Determine, by at least three trials, the heat equivalent of fusion of ice.

148. Heat Equivalent of Fusion. — The quantity of energy in the form of heat required to change a unit mass of the substance from the solid to the liquid state without changing its temperature is its heat equivalent of fusion. The details of its experimental determination in the case of ice may be arranged as follows.

Besides the usual errors of calorimetry there may be, in this experiment, the serious one of the carrying of water into the calorimeter by the ice.

The calorimeter cup may have a capacity of about a liter; determine its water equivalent, and that of the stirrer and thermometer (Art. 140). Pour water into the cup till it is two thirds full, and determine the mass of the water. The radiation error is made inappreciable by the following procedure. Warm the water till its temperature is about 10° above that of the surroundings, note the temperature of the calorimeter, and then introduce dry ice. The ice may be dried with a cloth or filter paper just before being used. Stir the water, keeping the ice beneath the surface by means of the stirrer, which may be a wire ring covered with gauze. Continue the cooling till the temperature is as much below that of the room as at the beginning it was above this temperature. More ice may be added if needed, or, if too much has been placed in the cup, the remaining pieces may be removed. Note the temperature when the last particle of ice is melted; that is, the lowest temperature attained. A final weighing will determine the mass of ice melted.

If K is the heat capacity of the calorimeter, t_1 the temperature of the water when the ice is introduced, t_2 the temperature when the ice is all melted, and M the mass of ice melted, then

$$K(t_1 - t_2) = Mx + Mt_2,$$

from which x, the heat equivalent of fusion, is to be calculated.

When the observations are made as described above, the combined errors of absorption and radiation are inappreciable.

Instead of following this method the calorimeter may be cooled only to the room temperature, and the correction for radiation may be determined as explained in Art. 141.

REFERENCE. — *Preston*, Theory of Heat, pp. 283-287.

LXXXIV. HEAT EQUIVALENT OF VAPORIZATION OF WATER

Determine, by at least three trials, the heat equivalent of vaporization of water.

149. Heat Equivalent of Vaporization. — The quantity of energy in the form of heat required to change a unit mass of the substance from the liquid to the gaseous state without changing its temperature is its heat equivalent of vaporization. For water it may be determined experimentally as described below.

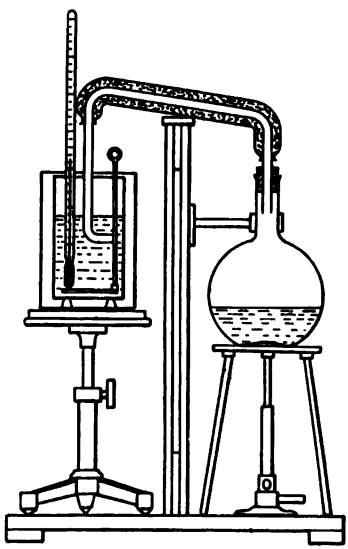


FIG. 72. APPARATUS FOR HEAT OF VAPORIZATION

The apparatus is to be arranged as indicated in Fig. 72, the boiler being a three-liter flask, and the calorimeter having a capacity of one liter. Between the boiler and the calorimeter is an asbestos or double wooden protecting screen. The steam delivery tube should, as far as possible, incline towards the boiler and be covered with a nonconductor. A water trap is placed in this tube just above the calorimeter, and the tube is bent in a horizontal direction about five centimeters from the end.

Besides the errors common to calorimetry, evaporation, radiation, and absorption, there is in this experiment another of relatively large amount, — the entering into the calorimeter of water from steam already condensed. The latter is reduced to a minimum by the construction described,

while the following method of procedure will reduce the effects of the other errors.

Weigh the dry calorimeter cup and stirrer, and determine the water equivalents of these and also of the thermometer (Art. 140). Nearly fill the cup with water and reduce its temperature 10° or 12° below that of the surroundings. Weigh to determine the mass of the water. When steam is coming abundantly from the boiler, and the delivery tube is hot so that little condensation takes place, thoroughly stir the water in the calorimeter, observe its temperature, and quickly place it under the delivery tube, so that the horizontal portion of the latter shall be a few centimeters below the surface of the water. Stir the water continually while the steam is condensing, and continue the condensation till the temperature of the calorimeter is as much above the room temperature as it was below this temperature at the beginning. Remove the calorimeter, stir the water, and note the highest temperature indicated by the thermometer. Again weigh the calorimeter cup, as quickly as possible to avoid evaporation, determining the mass of the steam condensed.

If K is the heat capacity of the calorimeter, t_1 the temperature when the steam is first admitted, t_2 the highest temperature of the calorimeter, t_3 the temperature of the boiling point corresponding to the barometric pressure at the time of the experiment, and M the mass of steam condensed, then

$$K(t_2 - t_1) = Mx + M(t_3 - t_2),$$

from which x, the heat equivalent of vaporization, is to be calculated.

When the observations are made as described above, the combined errors of absorption and radiation are inappreciable. Instead of following this method the calorimeter may be heated only to the room temperature, and the correction for absorption may be determined as explained in Art. 141.

Reference. — *Preston*, Theory of Heat, pp. 304–314.

LXXXV. MECHANICAL EQUIVALENT OF HEAT BY PULUJ'S METHOD

Determine the mechanical equivalent of heat.

150. Mechanical Equivalent of Heat. — The number of ergs which when transformed into heat will equal one heat unit, the calorie at 17°, is the mechanical equivalent of heat. The exact measurement of the work done and of the resulting heat, while theoretically simple, is much complicated in practice by numerous corrections of relatively large magnitudes. The method to be described, in which friction between metals produces the heat, has been found to be both convenient and of moderate precision.

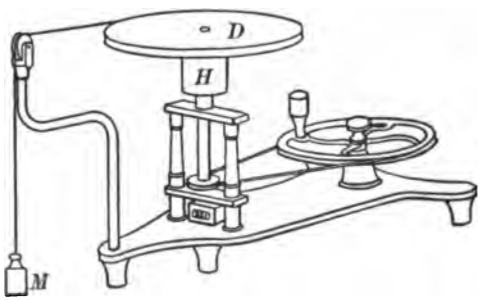


Fig. 73. Apparatus for Mechanical Equivalent of Heat

Two metallic cones, one fitting in the other, are supported by non-conducting rings in a cup, H (Fig. 73), so that the outer cone may be rotated, while the inner one is prevented from rotating by the action of a mass, M, attached to a wooden disk, D. The inner cone is hollow and is nearly filled with water. A thermometer passes through the disk into the water, and a small stirrer may be added. There is a counting device to register the number of rotations of the outer cone which are caused by turning the hand wheel. To secure the proper amount of friction between the cones, weights are placed on the disk.

By properly regulating the speed of the rotating cone, the work represented by the friction between the cones may be just enough to keep the mass M suspended, so that its weight, the force Mg, acts on the circumference of the disk. If to keep this force acting during the experiment it is necessary to turn the support n times, the result is the same as though the

support had remained stationary and the disk had turned n times in the opposite direction. Thus, to turn the disk the force Mg attached to its circumference would have had to move through the distance nc, where c is the circumference of the disk, and the work done would have been $ncMg$. This amount of work in the experiment is transformed into heat, most of which is absorbed by the cones and their contents and can be measured; some of the heat will be lost by passing to the supports and disk and by radiation; the latter part may be estimated and taken into account.

If K is the heat capacity of the cones, stirrer, and thermometer (Art. 140), t_1 the initial temperature of the water, t_2 the temperature at the end of the experiment, and R radiation correction (Art. 141), the heat produced is $K[(t_2 - t_1) + R]$, and the work per unit heat, that is, the mechanical equivalent of heat, in ergs, is

$$J = \frac{ncMg}{K[(t_2 - t_1) + R]}.$$

The weighings and temperature observations required are the same as for specific heat determinations, already described. While making the experiment, one observer should give attention to stirring the water in the calorimeter and to the temperature readings. It will require some care on the part of the second observer to maintain the proper rotation.

Before beginning an experiment, clean the rubbing surfaces and put a very little oil on them to prevent injury. A temperature change of from 10° to 15° is convenient, the room temperature being midway between the extremes. A uniform rate of turning is not required, but it is necessary that the mass should be kept suspended by the friction alone. After the cones are in place and before the temperature readings are made, a few turns may be given to the cone to make sure that the proper friction is secured to lift the mass. The pressure weights on the disk may be changed during the experiment to accommodate changes in friction.

The details of such a determination are illustrated by the following numerical example.

Mechanical Equivalent of Heat

METHOD OF METAL FRICTION

July 25, 1902

Weight of brass cones and stirrer	114.00 g
Weight of water in cone	24.02 g
Volume of thermometer immersed (No. 1721) . . .	1.5 ccm
Specific heat of brass	0.093
Heat capacity of calorimeter	35.25

Time	Temperature	
$3^h\ 41^m.0$	$17^\circ.78$	
42 .0	18 .08	
43 .0	18 .38	Turning begun. Counter 2180
45 .8	24 .50	Room temperature
49 .5	———	Turning ceased. Counter 4630
50 .0	31 .70	Highest temperature
51 .0	31 .55	
52 .0	31 .40	

From the first observations the rate of warming is

$$0^\circ.30 \div (24.50-18.08) = 0^\circ.047;$$

and the gain in temperature during the first part of the experiment is

$$0^\circ.047 \times (45.8-43.0) \times \tfrac{1}{2}(24.50-18.38) = 0^\circ.40.$$

From the final observations the rate of cooling is

$$0^\circ.15 + (31.55-24.50) = 0^\circ.021;$$

and the loss in temperature during the second portion of the experiment is

$$0^\circ.021 \times (50.0-45.8) \times \tfrac{1}{2}(31.70-24.50) = 0^\circ.32.$$

The corrected temperature change is

$$31^\circ.70-18^\circ.38-0^\circ.08 = 13^\circ.24.$$

The circumference of the disk is 80.7 cm; the suspended mass is 100 g; the number of turns is 2450. Therefore

$$J = \frac{80.7 \times 2450 \times 100 \times 980}{35.25 \times 13.24} = 4.153 \times 10^7 \text{ ergs}.$$

CHAPTER XII

HYGROMETRY

LXXXVI. HYGROMETRIC STATE BY VARIOUS METHODS

Determine the dew-point, and the absolute and relative humidity of the air by four methods.

151. Hygrometry. — The determination of the absolute and relative amount of vapor of water present in the atmosphere is the purpose of hygrometry. Such determinations are of value in many scientific and technical operations, as well as in weather prognostications.

The density of the water vapor in the air is the *absolute humidity;* it is the mass in grams of the water contained in one cubic centimeter. For convenience, hygrometric tables usually give the quantity of water in grams per cubic meter. The *tension* of the water vapor is the pressure which it alone would produce if measured by a mercury gauge. The *dew-point* is the temperature at which the amount of water actually present in the air is sufficient to produce saturation; it is the temperature to which the air must be reduced to cause the deposition of water just to begin. The *relative humidity* is the ratio of the amount of water actually present in the air to the amount that would saturate it, the temperature remaining unchanged. Since the vapor of water follows Boyle's Law, the relative humidity is also equal to the ratio of the actual pressure, f, of the vapor in the air, to the maximum possible pressure, F, at the same temperature; that is, the relative humidity is equal to $\frac{f}{F}$.

The four most useful forms of hygrometers are described below. For the convenient use of any one of these forms, hygrometric tables are necessary (Appendix, Tables 20, 21, 23).

Absorption Hygrometer. — By means of an aspirator (Fig. 74) a known volume of air is drawn through a drying bottle. If the aspirator contains just five liters of water, and it is allowed to operate twice, ten liters of air will have been drawn through the dryer. The rate of flow of the water should be such that this will require about an hour's time. If the flow is too rapid, the moisture in the air may not all be absorbed by the dryer.

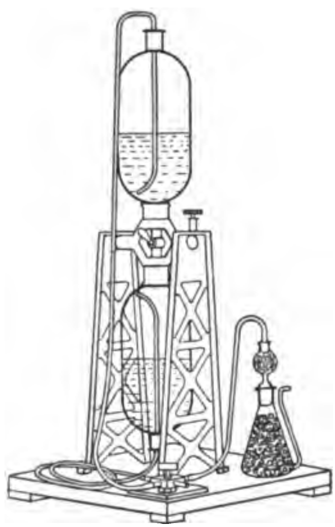


FIG. 74. ASPIRATOR FOR HYGROMETRY

The dryer may consist of a flask of about 250 ccm capacity, filled with small fragments of pumice stone saturated with sulphuric acid. The air is allowed to enter at the bottom and is drawn out at the top. This flask must be carefully weighed to the nearest milligram, both before and after the aspirator operation. The mass of water per cubic centimeter of air, which is the absolute humidity, is determined from the increase in weight. From the temperature of the air, by means of the tables, the mass of the water required to produce saturation is found. From these quantities calculate the relative humidity. From the absolute humidity and the tables find the dew-point.

This method may be made the most precise of all, in which case the influence of temperature and water in the aspirator must be considered.

REFERENCE. — *Preston*, Theory of Heat, p. 361.

Dew-Point Hygrometer (Daniell, Regnault). — In this form of instrument a polished surface has its temperature reduced till moisture from the air is condensed upon it. This temperature, t_d, is the dew-point. From the tables find the tension of the vapor corresponding to this dew-point, and also to the

temperature of the air; the ratio of the former to the latter is the relative humidity.

The absolute humidity is found by taking from the tables the density of vapor corresponding to the dew-point. This value of the density needs correction, because the air close to the instrument has been cooled. The tabular value multiplied by

$$\frac{273 + t_d}{273 + t},$$

where t_d is the dew-point, and t the air temperature, is the absolute humidity.

The dew-point hygrometer usually consists of two thermometers, one designed to measure the temperature of the air and the other the temperature of a polished surface which is cooled to the dew-point by the evaporation of ether. In one form one thermometer has a hollow, cup-shaped bulb, in which ether is poured. The evaporation is facilitated by blowing air into the ether. In another form the thermometer is inclosed in a bulb containing ether; this bulb is connected with a second on the outside of which is a muslin cover. This cover is saturated with ether, which is then evaporated by blowing upon it; this cools the second bulb, causing condensation of ether vapor inside, which in turn causes the evaporation of the ether in the first bulb, reducing its temperature to the dew-point.

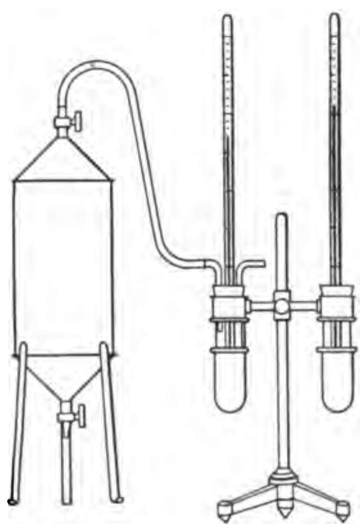


FIG. 75. DEW-POINT HYGROMETER

It is preferable to have the two thermometers of the same size and shape, and symmetrically placed with respect to the other parts of the apparatus, as shown in Fig. 75. The thermometer bulbs are inclosed in similar glass cylinders, whose ends are

closed by polished silver cups. One cup contains ether, and is connected to an aspirator. By allowing water to run out of the aspirator, air is drawn through the ether, evaporating and cooling it.

Observing the polished surface carefully, cool it till the dew just appears, and note the temperature; allow it to warm till the dew just clears away, and note the temperature. By repeated trials it will be possible to locate these two temperatures within a degree of each other; or even to adjust the rate of flow of water to keep the temperature of the bulb exactly at the dew-point. Make several determinations. Care should be taken that air currents and the heat of the body do not affect the instrument. A telescope may assist in reading the thermometer from a distance.

Wet- and Dry-Bulb Hygrometer, — *Psychrometer* (Auguste). — This instrument consists of two similar thermometers, one having its bulb covered with muslin arranged to be kept wet with water. If the air is not saturated, the water will evaporate from this muslin cover and cool the thermometer; the rapidity of evaporation, and therefore the amount of cooling, depends upon the humidity and the relative motion of the thermometer and the surrounding air. By means of empirical formulæ and tables, the absolute and relative humidity and the dew-point may be determined from the readings of the wet- and dry-bulb thermometers.

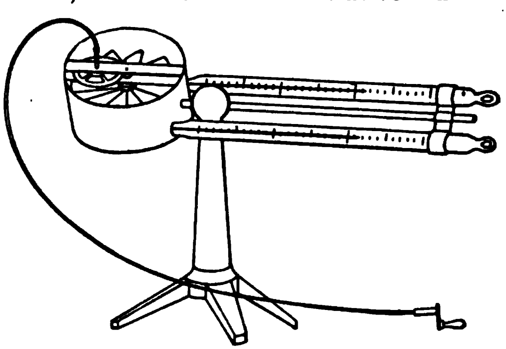


Fig. 76. Psychrometer

The formulæ here given assume that the relative motion of the thermometers and air is three meters per second or more, this velocity producing the greatest depression of the wet-bulb temperature. This condition may be secured by swinging the instrument as a pendulum or by

blowing the air against the thermometer with a fan. Such a ventilating fan is often made a part of the apparatus, as shown in Fig. 76.

If t is the temperature of the air as indicated by the dry bulb, t_w the temperature of the wet bulb, B the barometer reading, and e_w the tension of water vapor for the temperature t_w, as taken from the tables, then the actual tension is

$$e = e_w - 0.0066\,B\,(t - t_w)\,[1 + 0.00115\,(t - t_w)].$$

With tables such as the Smithsonian Meteorological Tables, this formula is both convenient and accurate, the value of e_w being found in one table (Table 23, Appendix), while the value of

$$0.0066\,B\,(t - t_w)\,[1 + 0.00115\,(t - t_w)]$$

is given in another table (Table 20). Having found e, the dew-point is the temperature taken from the table, for which this tension is sufficient to produce saturation. Still another table gives the relative humidity (Table 21), the temperature and dew-point being known. The absolute humidity is

$$f = 1.060\,\frac{e}{1 + 0.00367\,t}.$$

When the only available table is one of the tension of aqueous vapor, the following formulæ are more convenient, though less exact.

$$e = e_w - 0.0066\,B\,(t - t_w).$$

It will often be sufficient to assume the barometric pressure as 75 cm; then

$$e = e_w - 0.50\,(t - t_w).$$

Having found e, the temperature at which this tension is sufficient to produce saturation is the dew-point. The relative humidity is the ratio of e to the tension e_t found in the table, which would produce saturation at the temperature t of the dry-bulb thermometer; it is $\frac{e}{e_t}$.

The absolute humidity is

$$f = 1.060\,\frac{\quad}{1 + 0.00367\,t}.$$

If the temperature t_w is below the freezing point, the cover of the wet bulb should be freshly moistened with water before reading the temperature, and the formula should have the constant as given below.

$$e = e_w - 0.0058\,B\,(t - t_w).$$

REFERENCE. — Smithsonian Meteorological Tables.

Hair Hygrometer. — A very useful hygrometer consists of a prepared hair suspended over a wheel carrying a pointer. The hair changes length with change in the humidity, and by empirical adjustment a scale may be made to show the relative humidity. From specially prepared tables the absolute humidity and the dew-point may be found. The setting of the index should be tested occasionally by closing the case of the instrument and introducing a sheet of muslin which is wet with water. When the inclosed air has become saturated with water vapor, the reading should be 100.

Part V — Light

CHAPTER XIII

PHOTOMETRY

C. LUMINOUS INTENSITY WITH SIMPLE PHOTOMETERS

Show that the Law of Inverse Squares applies to light intensity. Find the mean horizontal candle power of various light sources (flat flame, Argand, and Welsbach mantle gaslights, and kerosene lamp) by means of several photometers.

152. Photometry. — The photometer is an apparatus for comparing the luminous intensities of two light sources, or for determining the distribution of light about a source. There is no practicable direct method for measuring light intensity; it then becomes necessary to compare a light with a chosen standard light. That feature of lights which is of most practical importance is their illuminating power. The light being investigated and the standard light are placed to illuminate the same or similar screens, and the relative distances of the sources from the screen are altered to make the two illuminations equal. The luminous intensities of the sources are then proportional to the squares of their distances from the screen.

This relation follows from the law that the intensity of illumination at different distances from a source of light varies inversely as the square of the distance. When the light is incident upon a surface at an oblique angle, the intensity furthermore varies as the cosine of the angle of incidence.

That the Law of Inverse Squares is applicable may be approximately shown by comparing the illuminations produced by different sources of known relative intensities; as, for instance, three and five candles.

The degree of precision with which two light intensities may be compared does not approach that attained in the comparison of most other physical quantities. Some of the difficulties are that no precise, constant standard has been devised, that the two lights are usually of different colors, and that the determination of the equality of illumination is a question of judgment, not of measurement.

153. Candle Power; Horizontal Candle Power. — The unit of intensity in which illuminating power is usually expressed is the candle. This unit is neither very definite nor very precise, and other units are sometimes used. Brief specifications for a few light units are given in Art. 155. Horizontal candle power expresses the intensity of the light emitted by the source in a horizontal line as compared with that of a standard candle; or it expresses the number of candles which must be concentrated to give an illumination equal to that under investigation. Since most sources emit light of varying intensities in different directions, it is desirable to measure the candle power in a specified number of lines in a horizontal plane; and the average of these is the mean horizontal candle power. Incandescent electric lamps may be rotated about a vertical axis while being tested so that the effective illumination in any one direction is the average of that emitted in all directions; a single measure then gives the mean horizontal candle power.

For the purposes of this exercise it will be sufficient to measure the candle power in four directions 90° apart, beginning, for a flat flame, with one edge toward the photometer screen. More complete methods are described in the next exercise.

In order that the measures may be of value, the style of burner, size of flame, rate of fuel consumption, etc., should be specified.

154. Simple Photometers. — Many devices for the comparison of luminous intensities have been made, but only a few of the

commonly useful forms will be mentioned. In all photometers the essential part is the screen, together with the arrangement for observing the illumination.

In some photometers the screen and the reference light are fixed in position, the unknown light being moved to secure the setting; in other forms the two lights are fixed at the opposite ends of the photometer bench, the screen being movable between them. Two kinds of scales are in use, — one showing the distances between the screen and the lights, the other indicating, for a given position of the adjustable part, the ratio of the intensity of the unknown light to the reference light. The former is usually the more accurate, while the latter is very convenient. The scale of distances may be used for computation in connection with tables giving the logarithms of the light ratios.

Rumford Photometer. — The two sources of light, a screen, and an opaque object — such as a rod of wood — are arranged so that two shadows of the rod are cast near each other upon the screen. That portion of the screen which is shielded from one light by the rod is illuminated by the other light. If, then, the lights are placed at such distances that the two shadows appear equally dark, the illuminations produced by the lights upon the screen are equal, and the intensities of the lights are proportional to the squares of their distances from the screen.

Foucault and Ritchie Photometers. — Instead of the opaque screen of the preceding form, Foucault employed a translucent screen arranged so that each light illuminated a different half of the screen. A sight tube opposite the lights excluded from view everything except these two portions of the screen. One of the lights is moved until the greatest possible uniformity of illumination is secured.

In Ritchie's modification the two lights are fixed at the ends of a photometer bench, while on the line between them are two mirrors inclined to throw the lights at right angles on the translucent screen. The screen and mirrors are moved towards one light or the other till the illuminations are equal.

Paraffin-Diffusion Photometer. — Two thick plates of paraffin are placed together, with an opaque layer — such as a sheet of tin

foil — between them. These plates are placed in the line joining the two lights, and perpendicular to this line. Each plate will then receive light from only one source, which will diffuse into the material of the plates. By looking at the edges of both plates of wax, one may judge as to the equality of the illumination which falls upon them. This simple device is very satisfactory, even for lights differing in color.

Bunsen Photometer. — This form of photometer is probably used more extensively than any other; it is as convenient as any, and is excelled in precision only by the Lummer-Brodhun form (Art. 159). A piece of white paper is rendered translucent over a part of its surface by saturation with paraffin or stearin. If light falls normally upon such a screen, a considerable part of it will be reflected from the opaque surface, while the waxed surface will transmit a large part of the light, thus appearing darker. If a second light be placed on the opposite side of the screen at such a distance as to produce equal illumination, this light will be transmitted and reflected from its side of the screen in the same proportions as that of the first light from the first side of the screen. Under ideal conditions the waxed spot should disappear when the illuminations are equal; in practice one must adjust the distances of the lights or the position of the screen between the two lights so that the contrast between the waxed and unwaxed portions of the screen is the same whether the screen is viewed from one side or the other. Or the screen may be moved till the spot most nearly disappears as seen from one side, and then be moved till the same condition is fulfilled as seen from the other; the point midway between these two positions is taken as the one from which to measure.

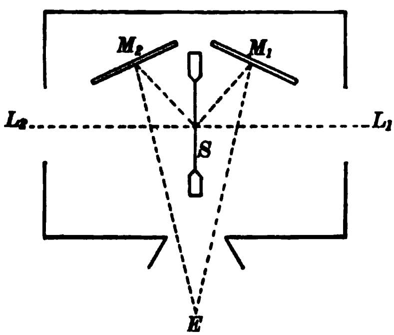


Fig. 77. Bunsen Photometer Screen

Often the Bunsen screen is provided with mirrors or prisms which permit both sides to be seen at once; this greatly facilitates its use. In Fig. 77 S represents the Bunsen screen, which is illuminated by light from L_1 and L_2. M_1 and M_2 are mirrors which permit both sides of the screen to be seen from E.

CI. INTENSITY OF LIGHT BY PRECISE METHODS

(a) Determine the mean spherical candle power of an incandescent electric light.
(b) Determine the gas consumed per candle power per hour by a flat flame, an Argand, and a Welsbach mantle gaslight.

155. Standards of Light. — No altogether satisfactory standard of illuminating power has been devised; the following are the ones commonly in use in this country. The specifications given are abridged; for more detailed descriptions of these and other standards, the treatises mentioned in the references may be consulted.

The British Candle is made of spermaceti whose melting point is 112° F.; the candle is of 0.85 inch average diameter; its wick is of 3 strands of cotton, each of 18 threads; it should burn 120 grains of grease per hour with a flame 1.77 inch high.

In complete photometers the candle is supported on a balance with which the amount of material consumed may be readily determined. If this amount does not vary from that specified by more than five per cent., the candle power of the light is taken as proportional to the rate of consumption.

The German Candle (Vereinskerze) is made of paraffin with a melting point of 55°; it is 20 mm in diameter; its wick is made of 25 threads of cotton; the standard flame height is 50 mm.

The Hefner-Alteneck Amyl Acetate Lamp burns pure amyl acetate from a wick 8 mm in diameter and with a flame 40 mm high.

This lamp, when made and used according to the specifications of the Physikalisch-Technische-Reichsanstalt, is considered

the most reliable available standard. Its intensity, often called the Hefner unit, is equal to 0.88 English Candles, and to 0.83 German Candles.

156. Mean Spherical Candle Power. — Sources of illumination do not radiate light of equal intensity in all directions, and hence the measure of the intensity in one or more directions may give only a very incomplete idea of the actual quantity of light radiated by the source. For purposes of general comparison the mean spherical candle power is a more satisfactory measure of the illuminating value of a light. The mean spherical candle power of a light source expresses what would be the intensity if the total quantity of light emitted were so redistributed that the illumination over the entire surface of a sphere concentric with the light were uniform. To find the mean spherical candle power practically, it is necessary to measure the intensity in a large number of directions, arranged systematically, and then to average these measures so as to conform to the definition. The following method is a convenient one.

Consider a sphere concentric with the light; pass a horizontal plane through the center, the light being in its normal vertical position; designate the intersection of the plane and sphere the equator. Let the surface of the sphere be divided into seven zones each 30° wide, an equatorial zone extending 15° each side of the equator, and zones above and below this, the centers of which are "parallels of latitude" 30°, 60°, and 90° from the equator. The areas of these zones, considered as portions of the entire sphere, are

Upper pole	0.0170
60° N.	0.1294
30° N.	0.2242
Equator	0.2588
30° S.	0.2242
60° S.	0.1294
Lower pole	0.0170

If the average intensity of the illumination on each zone is measured and is multiplied by the area of the zone, the sum of all the products so obtained will be the mean spherical candle power.

When investigating an incandescent electric lamp, it is convenient to have a lamp socket which can be continuously and rapidly rotated by a motor. A photometric measurement is taken when the axis is vertical and the lamp is rotating; the result is the average intensity in the equatorial direction, — that is, the mean horizontal intensity. Incline the top of the axis 30° toward the photometer screen, and the measurement taken with the lamp rotating in this position may be considered the average intensity for the 30° zone. Incline the axis 60° and then 90° toward the photometer, taking readings as before, and proceed similarly for the lower hemisphere. The intensity for the lower polar zone will be practically zero.

If it is not convenient to rotate the lamp continuously, the mean of a number of readings for each zone, eight or more, may be employed. Observe the intensity of the lamp, the axis being vertical; rotate the lamp on its axis 45°, and measure the intensity; rotate it 45° again, and read; continue rotating it 45° at a time till it has been turned through 360°; the average is the equatorial intensity. Incline the axis 30°, and repeat the observations; continue in this manner for the seven zones. Multiply the average for each zone by the zone area as given above, and the sum of the products is the mean spherical intensity.

In the photometry of an electric lamp an ammeter should be in series with the lamp, and a voltmeter should be connected to the terminals. Keep the voltage at the value for which the lamp is designed, using a rheostat to control this. From the readings of these instruments compute the efficiency of the lamp; that is, the watts required per candle power.

157. Gas Meter. — In using a standard wet gas meter the instrument must be level, and the water in it must stand at exactly the proper height, as indicated by the gauge.

In work of precision the temperature of the gas must be noted; also its pressure and the barometer pressure.

158. Photometry of Intense Lights. — When the intensity of a light exceeds 20 candle power it cannot be accurately compared with the standard candle or the Hefner lamp. It is then desirable to determine the true intensity of a nominal 16

candle-power incandescent lamp, and to use this as an intermediate or secondary standard.

When lights of greater intensity, such as the electric arc, are to be measured it may be necessary to reduce the effective intensity by interposing in the path of the light a dispersion lens. The methods to be employed for such investigations are described in the references.

References. — *Palaz-Patterson*, Industrial Photometry; *Stine*, Photometrical Measurements.

159. The Lummer-Brodhun Photometer. — In the Bunsen photometer (Art. 154) a field of view is presented, part of which is illuminated from one source by reflected light and part is illuminated from the other source by transmitted light. The two portions having been differently treated their accurate comparison is difficult. The Lummer-Brodhun photometer is a device for producing a mixed field of view by optical means, in which the two beams of light pass through equal optical paths.

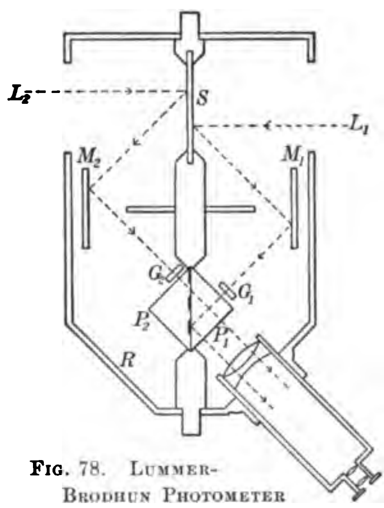


Fig. 78. Lummer-Brodhun Photometer

The two surfaces of an opaque white screen, S (Fig. 78), are illuminated by the two light sources L_1 and L_2. By means of the mirrors M_1 and M_2 and the two prisms P_1 and P_2 these two surfaces are observed through a single telescope in such a fashion as to permit great accuracy in the estimation of the equality of the two illuminations. The two right-angled prisms P_1 and P_2 have their hypothenuses ground so that they are in close contact over a circular area. After this the polish of the surface of P_2 is destroyed by a sand blast over portions of the shape shown black in Fig. 79. When the prisms

are put together in the sight box, light will reach the telescope from L_2 and M_2 only through the polished portion of the surface of P_2, where the contact is perfect. Light from M_1 falling upon these parts of the contact surface does not go to the telescope, but is transmitted in the direction of R; while light from M_1 falling upon those portions of the hypothenuse of P_1 which are not in contact with P_2 is totally reflected and enters the telescope. By this device the field of view appears elliptical in shape, with a vertical division into halves (Fig. 80), in each of which is a trapezoid. One source sends light to that portion of the semiellipse f_1, which surrounds the trapezoid t_2, and also to the trapezoid t_1; while the frame f_2 and the trapezoid t_2 receive light only from the second source. Each trapezoid is made darker in shade than the corresponding opposite semielliptical frame by interposing in the paths of the light which goes to the trapezoids extra pieces of glass, G_1 and G_2 (Fig. 78), the absorption of which produces a slight reduction in intensity. This feature is the contrast principle which aids in the photometry of lights differing in color. In Fig. 78 the paths of the light forming the trapezoids are indicated by the dotted lines.

FIG. 79. FACE OF LUMMER-BRODHUN PRISM

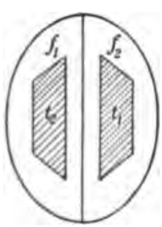


FIG. 80. FIELD OF VIEW OF LUMMER-BRODHUN PHOTOMETER

In setting the photometer the screen is to be moved to such a position between the two lights that the two trapezoids are both darker than the surrounding backgrounds, and so that, neglecting differences in color, the contrast between t_1 and f_2 is just the same as the contrast between t_2 and f_1. In this position the two backgrounds f_1 and f_2 are very nearly the same shade, and the dividing line between them nearly vanishes; but it will entirely disappear only when the two lights are of exactly the same color, which is very seldom.

The determination, then, is one of equality of contrasts rather than equality of illuminations.

160. The Rood-Whitman Flicker Photometer. — When lights differ in color it is difficult, or impossible, to compare their luminous intensities by any of the methods previously described. Even when the lights differ widely the flicker photometer permits of convenient and accurate comparison. In this device a screen, S (Fig. 81), is illuminated by one source, and is viewed near its edge through a sight tube, T. A second screen of the same material as the first, circular in form but with the alternate 90° sectors cut out, is mounted so as to be capable of rotation about an axis, F, through the center of the circle, and this is illuminated by the second source. The rotating screen is of such size that when the sectors come in front of the sight tube they eclipse the screen S and are themselves seen through the tube, while during the alternate quarter revolutions the screen S only is visible. The screen is rotated by hand or by a spring or electric motor. If the illuminations of the two screens are unequal in intensity, a flickering sensation is produced when they are observed through the tube. If the apparatus is so adjusted that the flicker disappears, the intensities of illumination are equal. The equality of luminous intensity is determined independently of difference in color.

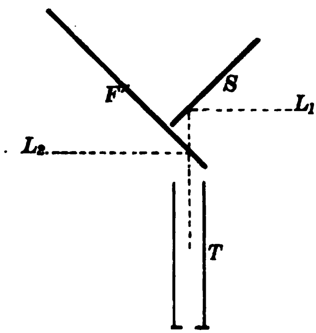


FIG. 81. FLICKER PHOTOMETER

REFERENCES. — *Whitman*, Physical Review, Vol. 3, p. 241, 1896; Science, Vol. 9, p. 734, 1899.

CHAPTER XIV

MIRRORS AND LENSES, MAGNIFYING POWER

CII. RADII AND FOCI OF MIRRORS BY VARIOUS METHODS

Determine the principal focus and radius of curvature of a concave mirror by the method of conjugate foci, by the method of parallax, and by the method of size of image. Determine the principal focus and radius of curvature of a convex mirror by the method of conjugate foci, and with sunlight.

161. Conjugate Foci.—The radius of curvature, R, the principal focus, F, and the conjugate foci, f_1 and f_2, of concave and convex mirrors have the relation

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F}=\frac{2}{R}.$$

A focal distance measured to a real focus (in front of the mirror) is positive, while one measured to a virtual focus (behind the mirror) is negative. If the resulting value of R is negative, it indicates that the center of curvature is behind the mirror,—that is, the mirror is convex; F will also be negative, indicating that the mirror renders convergent rays less convergent, or that it is in effect a diverging mirror.

If f_1 is the distance from the mirror of an object whose length is l_1, and f_2 is the distance from the mirror of the image the length of which is l_2, then

$$f_1 : f_2 = l_1 : l_2.$$

From these relations there arise various methods of determining the radii and focal lengths of mirrors.

In all the following experiments with mirrors and lenses, the foci should, as nearly as is practicable, lie on the line

perpendicular to the mirror or lens at its center point; otherwise the image will be more or less distorted and astigmatic. Also the definition of the image may often be much improved by covering the mirror or lens with a diaphragm permitting the use of only the central portion of its surface.

162. Concave Mirrors. — Place a bright object, as a gas jet, in front of a concave mirror, and adjust a screen to receive the image in its best defined position. Using the measured distances of the object and image from the mirror, f_1 and f_2 of the above formula, calculate the focus and radius of curvature. A fine wire cross supported in front of the light will serve a better purpose as an object, for the screen can be placed more accurately at the conjugate focus. Measure pairs of conjugate foci with the object at a considerable distance from the mirror, when it is about twice as far from the mirror as is the image, when the object and image are at equal distances from the mirror, and when the object is nearer than the image. Calculate the radius of curvature from each set of measurements.

Take note of the fact that when the conjugate foci are equal to each other each is equal to the radius of curvature, and that as one focal distance increases its conjugate approaches half the length of the radius of curvature, which is also the principal focal length.

Instead of receiving the image upon the screen, it is often more accurate to locate its position in the air by the method of parallax. Let the object be a brightly illuminated needle or pointed strip of paper. Place it in position in front of the mirror, when by looking towards the mirror from a distance the image can soon be found seemingly suspended in the air. By trial a second index needle may be placed so that its point is apparently just in contact with this image. If upon moving the eye from side to side the image and index needle appear to separate, or show parallax, it indicates that the image is nearer the mirror or farther from it than the needle. Adjust the index until the movement of the eye causes no relative displacement, that is, till there is no parallax; then either needle is at the conjugate focus of the other. When looking at the image and

index, keep the eyes distant about twenty-five centimeters; or, a magnifying glass may be used to aid in accurate setting.

The principal focus of a mirror may be approximately and quickly determined by measuring the distance at which parallel rays (sunlight) are brought to a focus.

Using for the object an illuminated scale and for the index a duplicate scale, adjust, as explained for the needles, till the index coincides with the image. Show, by measurements made for several positions of the object, that the sizes of the object and the image are in proportion to their distances from the mirror.

163. Convex Mirrors. — Virtual images only are produced by convex mirrors; these images cannot be projected upon a

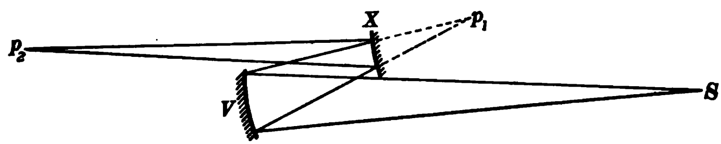


FIG. 82. CONJUGATE FOCI OF CONVEX MIRROR

screen, hence it is necessary to use some indirect method for obtaining conjugate focal points.

Let a convergent pencil of light produced from a source, S, by a concave mirror, V (Fig. 82), or by a convex lens, be intercepted by a convex mirror, X, so placed that the reflected beam shall still be convergent, and let the real image thus produced be received on a screen at p_2. This is one real focal point, while its conjugate virtual focus is at the point p_1 on the opposite side of the mirror where the light would have been brought to a focus by the concave mirror had it not been intercepted. The distances of the points p_1 and p_2 from the mirror are the conjugate focal distances f_1 and f_2, which when substituted in the formula given above, regard being had to the proper algebraic sign, will give the focus and radius of the convex mirror.

The simplest method for obtaining the focal length of a convex mirror is to allow a beam of sunlight to fall upon it through an aperture cut in a screen, and to place the screen at such a

distance from the mirror that the reflected pencil has, at the screen, twice the dimensions of the aperture. Then the distance between the screen and mirror is equal to the focal length of the mirror.

CIII. RADII AND FOCI OF LENSES BY VARIOUS METHODS

Determine the focus of a double convex lens by the method of conjugate foci, and by parallax. Determine the focus of a double concave lens by the method of conjugate foci, and with sunlight. Determine the focus of a combination of lenses from its focal position. Determine the focus, curvature, and index of refraction of a plano-convex and of a plano-concave flint-glass lens.

164. Lenses. — A lens may be composed of one or more pieces of glass of various optical densities and with surfaces of different curvatures. The complete formulæ connecting the index of refraction, thickness, curvatures, and foci of a lens are too complicated for our purpose.

For a single thin lens the principal focus, F, is related to the conjugate foci, f_1 and f_2, and to the index of refraction, n, and to the radii of curvature of its surfaces, r_1 and r_2, as follows:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

A focal distance measured to a real focus is positive, and one measured to a virtual focus is negative. The radius of curvature of a convex surface is positive, and the radius of curvature of a concave surface is negative. If the resultant value of the principal focus, F, is positive, the lens is in effect converging; if it is negative, the lens is in effect diverging.

165. Convex Lenses. — Place a light on one side of a wire cross and a convex lens on the other side. Arrange a screen to receive the image of the cross in its best defined position. Measure the distances of the cross and its image from the lens, f_1 and f_2, and calculate the focus from the above formula. Make three sets of measurements, one with the object at a considerable

distance from the lens, one with object and image about equally distant, and one with the object nearer than the image.

Instead of receiving the image upon a screen, it is more accurate to locate its position in space by the method of parallax. Place a brightly illuminated needle on one side of the lens for the object. Upon looking toward the lens from the other side, at a considerable distance, the inverted image of the needle may be found apparently suspended in the air. By trial a second index needle may be placed so that its point is exactly in contact with this image. This is determined by the absence of parallax, when the eye is moved from side to side, as explained more fully in connection with mirror measurements (Art. 162).

When a lens is midway between an object and its image, the object and image are of equal size, and their distance apart is four times the principal focal length of the lens.

Project an image of the sun upon a screen with the lens; the distance between the lens and screen when the image is best defined is the focal length of the lens.

With the object and screen at a fixed distance apart greater than four times the focal length of the lens, there may be found two positions of the lens between them which give distinct images. If l is the distance between object and screen, and d is the distance between the two positions of the lens, the focal length of the lens is

$$F = \frac{l^2 - d^2}{4\,l}.$$

This method is convenient when the lens is so mounted that accurate measurements to its center cannot be made, its displacement being measured instead.

166. Concave Lenses. — Virtual images only are produced by concave lenses; these images cannot be projected upon a screen, hence it is necessary to use some indirect method for obtaining conjugate focal points.

Let a convergent pencil of light produced from a source, S, by a convex lens, X (Fig. 83), be intercepted by a concave lens, V, so placed that the refracted beam shall still be convergent,

and let the image thus produced be received on a screen at p_2. This is one focal point for the concave lens, while its conjugate is the virtual focus p_1, where the light would have converged had it not been intercepted. Measure the distances f_1 and f_2, of the points p_1 and p_2, from the concave lens, and apply, with regard to the proper algebraic sign, in the formula given above to compute the principal focus of the concave lens.

Allow sunlight to fall upon the lens over which has been placed a diaphragm. Receive the emergent pencil of light

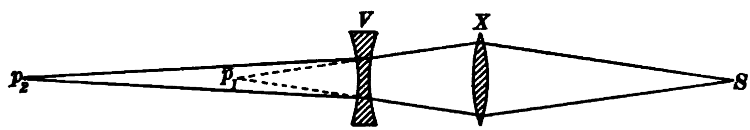


FIG. 83. CONJUGATE FOCI OF CONCAVE LENS

upon a screen placed at such a distance that the pencil at the screen has twice the dimensions of the aperture in the diaphragm. Then the distance between the lens and screen is the required focal length of the concave lens.

167. Radius of Curvature of a Lens. — If the focus and the radii of curvature of the surfaces of a lens are known, the index of refraction may be computed by the above formula. The curvature may always be accurately found with the spherometer, but for the purposes of this exercise the surfaces may be treated as mirrors, and the curvature found as described below. A plane surface has an infinite radius of curvature. Remember that in the formulæ for mirrors the radius of curvature of a convex surface is negative, while the curvature of this surface, considered as a lens, is positive; and that the curvature of a concave surface is positive as a mirror and negative as a lens.

For a concave surface, arrange cross wires in a small aperture in a white screen, place a light on one side of the screen and the lens on the other side so that its concave surface, acting as a mirror, projects a well-defined image of the cross wires on the screen near the aperture. The distance from the cross wires to the surface of the lens is the radius of curvature.

For a convex surface, arrange a convex lens, X (Fig. 84), to converge light from a source, S, towards the point p_1; the lens whose curvature is to be measured is placed so that its convex surface reflects the light to a focus at p_2. By the principles of Art. 163 compute the radius of curvature.

If a lens is made of crown glass, its index of refraction is approximately 1.5; and if one of its surfaces is plane, the corresponding radius of curvature is infinite. For this special case the formula given above reduces to

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{2R}.$$

Hence for a plano-convex or a plano-concave lens of crown glass the radius of curvature is approximately half the focal length, while for mirrors the radius of curvature is twice the focal length.

168. Focal Length of a System of Lenses. — If a lens has a thickness which is considerable as compared with its focal length, as is the case with microscope objectives and photographic combinations, the above methods are not sufficient. The following may be applied.

Arrange the lens combination in the manner commonly employed in lantern projections, to project an image of a strongly illuminated object of known size (a graduated glass scale, or a microscope stage micrometer) on a screen. Place the screen at

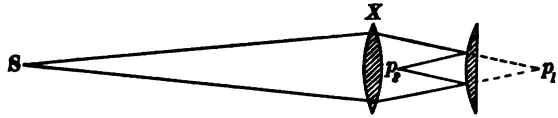


FIG. 84. RADIUS OF CURVATURE OF CONVEX SURFACE

such a distance that the image is highly magnified. Then if s is the length of a division of the scale whose image has the length S, and d is the distance of the screen from the assumed principal plane of the lens, the required focal length of the lens is

$$F=d\frac{s}{S+s}.$$

A graphic method may be useful for finding the focal length of a photographic combination. Allow sunlight to fall upon the front lens in the direction of its axis. A piece of tissue paper being placed against the back of the combination, measure carefully the diameter of the emergent beam. It often happens that the interior mounting of the lens is such that it cuts down the effective diametet of the incident light. In this case a diaphragm must be placed over the front lens, before the measures are made, the diameter of which is such that the full incident beam is emergent. Place the lens in the camera and accurately focus it for very distant objects, and measure the

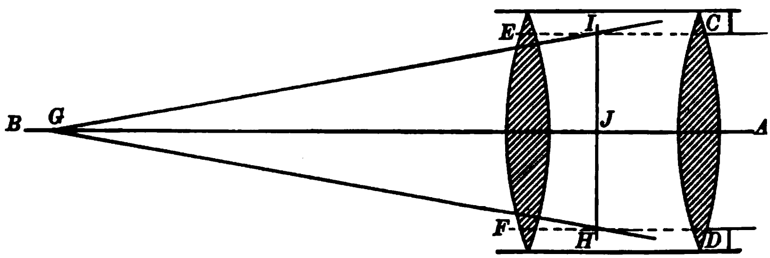


FIG. 85. FOCAL LENGTH OF COMBINATION OF LENSES

distance of the ground glass from the back surface of the lens. Then the graphic solution is as follows. Draw AB to represent the axis of the lens (Fig. 85). Plot CD, the diameter of the incident beam, on the front lens; and EF, the diameter of the emergent beam, on the back lens. G is the position of the focal plane, the ground glass. Draw lines through C and D parallel to the axis, and from G through E and F. At the intersections I and H, draw the line IH, perpendicular to the axis; this represents the principal plane of the lens. The distance from the focal plane to the principal plane, GJ, is the true focal length of the lens combination.

CIV. MAGNIFYING POWER

Find the magnifying power of a telescope when used for observing distant objects, and for objects distant two meters; find the power of an opera glass; find the power of a microscope using its several eye-pieces and objectives.

169. Magnifying Power. — The ratio of the angle subtended by the image of an object seen through an optical instrument to the angle subtended by the object when seen by unaided vision is the magnifying power.

Telescope. — The power of a telescope increases with the nearness of the object observed. When the power is stated without qualification, the amplification for an object at a very great distance, the astronomical magnifying power, is implied. This may be measured as follows.

Focus the telescope for an object at a great distance, and point it towards a bright surface, as the sky. Place the eye about twenty-five centimeters from the eyepiece, in the direction of the axis of the telescope, when there may be seen, outside the telescope, before the eyepiece, a small, real image of the objective and its mounting. The ratio of the diameter of the aperture in the cell of the object glass (this is taken because it is a well-defined object), to the diameter of its image, is the magnifying power. The small image is best measured by means of a fine scale ruled on glass, which is placed before the eyepiece. A hand magnifying glass may be used in the measurement. A specially constructed scale with magnifier attached, called a *dynameter*, is often provided. A diaphragm in the body of the telescope may reduce the cone of rays from the objective; this can be ascertained from an examination of the image with a magnifying glass. If there is such a diaphragm there should be placed over the object glass a screen having a reduced aperture, the whole of which may be seen from the eyepiece; this aperture and its image are then to be measured.

For measuring the power of a telescope as used in the laboratory, or of a Galilean telescope (opera glass), the following method is sufficient. Place a measuring rod, or other scale of

equal parts, at the distance for which the telescope is to be used, and focus the telescope upon the scale. Observe the scale through the telescope with one eye, and with the other eye unaided. Note that N divisions of the scale seen directly are equaled in length by n magnified divisions as seen through the telescope. Then the magnification is

$$M = \frac{N}{n}.$$

Instead of a divided scale, a masonry wall, a window frame, or other object with several well-marked, equal subdivisions may be used.

Microscope. — Place an object of known length under the microscope. A stage, or eyepiece, micrometer on glass with divisions of 0.1 mm is convenient. When one eye is looking at the object through the microscope, adjust a millimeter scale, held near the stage, to be seen distinctly by the other eye unaided. The scale is usually placed 25 cm from the eye, the normal distance of distinct vision. Seeing both scales simultaneously, determine how many magnified divisions, n centimeters, are equaled in length by N centimeters of the unmagnified scale. The magnification is

$$M = \frac{N}{n}.$$

Sometimes it is difficult to see both scales clearly. It may be more convenient, using a camera lucida, to draw the magnified scale on a piece of paper placed 25 cm from the eyepiece. This magnified image is then to be measured directly.

Reference. — *Kohlrausch*, Physical Measurements, pp. 184–188.

CHAPTER XV

GONIOMETRY

CV. ANGLE OF A CRYSTAL WITH THE GONIOMETER

Measure the angles between the several faces of crystals of calcite, galena, and garnet.

170. The Reflecting Goniometer. — Goniometry is the art of measuring solid angles or the inclination of planes, such as the angles between the faces of crystals and prisms. Wollaston's goniometer consists of a vertical divided circle, 10 cm or more in diameter, capable of rotation about a horizontal axis. Through this axis is a second one supporting the crystal. The crystal is attached to a small disk, *d* (Fig. 86), capable of rotation about three axes: the one, *a*, passing through the axis of the circle; a second axis, *b*, perpendicular to *a*; and a third one, *c*, which may have any inclination to the plane of the other two.

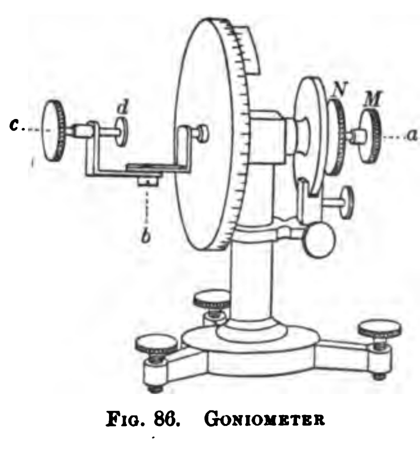


FIG. 86. GONIOMETER

It is necessary that the intersection of the two faces, the angle between which is to be measured, should be parallel to the axis of the circle, and it is desirable that it should coincide with this axis. This adjustment can be made approximately by varying the position of the crystal on the disk, to which it is usually attached with wax.

Place the instrument directly in front of a window at the distance of a meter or more, with the axis of the circle parallel to the horizontal bars of the window frame. Looking in the direction of the bottom of the window, the eye being very close to the crystal, rotate the crystal on the axis *a*, by means of the milled head *M*, till the top edge of the window seen reflected by the crystal face coincides with the bottom edge of the window seen directly.

Other more convenient marks may be selected, as the top of a chimney, a point on a roof, or any conspicuous feature in the landscape for one mark, and a mark on the floor or wall for the second. If necessary to make this coincidence exact, adjust the crystal on any of its three axes. Keeping the eye in the same position, turn the milled head *M* to get reflection from the second face, and secure coincidence between the two marks as before. When the edge of the crystal is parallel to the axis *a*, the crystal may be moved from the position of perfect coincidence for one face to perfect coincidence for the other face, by rotation on the axis *a* only; continue the adjustments until this condition is fulfilled.

By turning the milled head *N*, both the circle and crystal are rotated about the axis *a*; determine the angle between the two coincidence positions; this is the supplement of the angle between the faces of the crystal (Art. 176).

Make three or more determinations of each angle, repeating the entire adjustment of the edge of the crystal to parallelism with the axis of the circle each time.

If the circle is set to read 180°, and the crystal is turned by the milled head *M* to give coincidence from one face, and then the circle is turned by the head *N* to give the second coincidence, the circle reading may be the prism angle directly.

Often a telescope is attached to give definite direction to the line of sight, the cross wires taking the place of the lower window edge; and again a collimator may be provided to give a better object than the upper window edge. With these additions the instrument approaches the spectrometer in form, and should be used in the manner described in Exercise CVII.

CVI. ADJUSTMENT OF THE SPECTROMETER

Make the complete adjustments of a spectrometer.

171. The Spectrometer. — When a goniometer is constructed of large size, being adapted particularly to the measurement of prism angles and the deviation of light rays, it becomes a spectrometer. It has for its essential parts (Fig. 87) a horizontal table with a divided circle, a telescope, and a collimator. The telescope and table are capable of independent, measurable

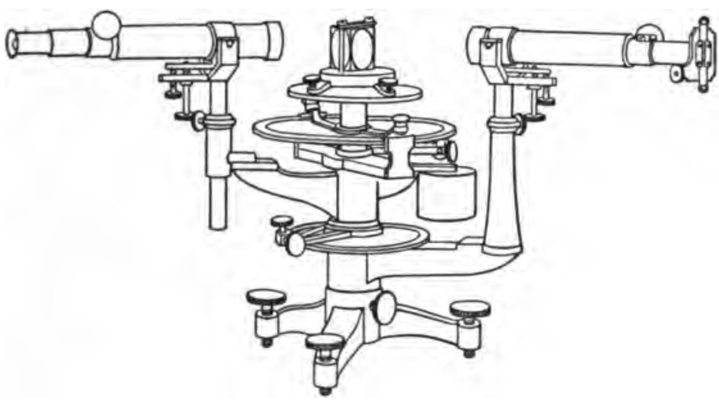


Fig. 87. Spectrometer

rotations about the principal vertical axis of the instrument, which passes through the center of the circle perpendicular to its plane; the collimator is commonly attached to the base of the instrument, and can be rotated about this axis only with the instrument as a whole. For most purposes it is required that the axes of the telescope and collimator should intersect the axis through the center of the circle, and should always be in the same plane perpendicular to it. In order that the spectrometer may be made to satisfy these conditions, and that it may be used for a variety of purposes, it is usually provided with many movements, adjusting devices, and graduated scales, which need not be described.

Before the spectrometer is used for any work of precision the accuracy of its adjustments should be verified. This may be done in various ways, but the following method for all the adjustments of a complete spectrometer is concise and accurate. It is important that the various operations be performed in the order described.

172. Adjustment of the Spectrometer. *Preliminary.* — Level the table of the instrument approximately, by means of the screws in the base. Make the axes of the telescope and collimator approximately horizontal.

Set the telescope and collimator (or the table) at such a height that they will be on the same level as the prism, or other apparatus to be used, when the latter is in its mounting and resting on the table.

Turn the telescope and collimator on their supports so that their axes shall intersect the vertical axis through the center of the circle. Often there are graduated scales to indicate these positions; if not, simple inspection will determine them sufficiently.

For making the precise adjustments, a mirror of plane-parallel glass, mounted on a support with leveling screws, is required. Preferably this mirror should be silvered on both sides, though it will serve silvered on one side or even if not silvered at all. Place this mirror on the spectrometer table and set it vertical by inspection.

To focus the Telescope for Infinity. — Place a collimating (Gauss) eyepiece in the telescope. This eyepiece is one having between the eye and the cross wires a clear, plane-parallel, glass reflector inclined at about 45° (Fig. 88), so as to throw light, received through an aperture in the side of the eyepiece, along the optic axis of the telescope. Place a light to illuminate this eyepiece. Looking through the telescope, slowly turn the spectrometer table one way or the other till, when the mirror is normal to the optic axis, a flash of light appears. If the flash cannot be seen, alter the position of the light with respect to the eyepiece, or repeat the preliminary adjustments with greater care. Continue the search until the flash is found,

when the light and mirror should be adjusted to cause the entire field of view to be illuminated.

Focus the eyepiece upon the cross wires by sliding it in its draw tube. By means of the rack and pinion slowly focus the telescope, observing the field of view carefully for the appearance of a second image of the cross wires. This may be faint, and at first appear only in part. When it is found, readjust the position of the light, rotate the eyepiece, focus the eyepiece, focus the telescope or alter the position of the eye, making any or all of these adjustments until both images of the cross wires appear distinct and motion of the eye discloses no parallax. When this is accomplished the telescope is focused for infinity, and its focus should not again be disturbed throughout the entire experiment.

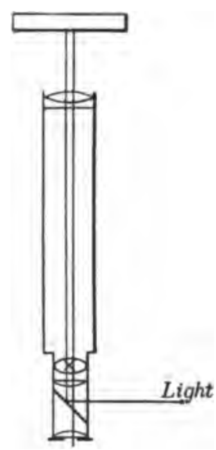


Fig. 88. Telescope with Collimating Eyepiece

To make the Optic Axis of the Telescope Perpendicular to the Axis of the Table.— Rotate the spectrometer table and alter the leveling screws of the mirror, to bring the reflected image of the cross wires into coincidence with the direct image.

Rotate the spectrometer table (not the mirror stand) till the second face of the mirror is toward the telescope, and find the image of the cross wires reflected from this face. Cause this image to coincide with the direct image by the following motions only. Rotate the table till the reflected image is vertically above or below the other; alter the inclination of the telescope axis in the vertical plane to move the reflected image *half way* towards the direct one; alter the leveling screws in the mirror stand till exact coincidence is obtained.

If the adjustments have been accurately made, coincidence should be obtained from the first face when it is turned to the telescope. If, upon trial, it is not exact, correct half the error by altering the inclination of the telescope, and the other half by tilting the mirror.

Continue in this manner till the reflected image from either face coincides with the direct image. When this is true the axis of the telescope is perpendicular to the axis of the table, and the surfaces of the mirror are parallel to this axis.

To make the Optic Axis of the Telescope Perpendicular to its Axis of Rotation. — The axis of rotation of the telescope should be coincident with the axis of the table. As the spectrometer is ordinarily constructed the relation of these two axes cannot be altered. If the two axes are not parallel, the optic axis can be made perpendicular to either one as desired, but of course not to both. Evidently it should be adjusted with respect to that axis about which rotation is to take place in the contemplated measurements.

The preceding adjustment having been completed, the accuracy of construction may be quickly tested. For the mirror is parallel to the axis of the table, and the telescope is perpendicular to it. If, now, the telescope is turned 180° about the central axis, it should be still perpendicular to the mirror if the axis on which it turns is parallel to that of the table.

If it is desired to set the optic axis perpendicular to the axis of rotation of the telescope, the method is similar to that of the preceding adjustment, except that the telescope is rotated from side to side of the mirror instead of the mirror being turned from side to side by the rotation of the table.

Instead of turning the telescope, which requires also the movement of the observer and light, the same result will be obtained by holding the telescope and revolving the other parts of the instrument on its base.

To test whether the Surface of the Table is Perpendicular to its Axis. — The telescope and mirror being adjusted for coincidence of the direct and reflected images of the cross wires, carefully lift the mirror above the table, rotate the table 180°, and replace the mirror. This operation should not have disturbed the coincidence in a vertical direction.

To adjust the Collimator. — Remove the mirror and sight the telescope, through the collimator, on the slit; cause the latter to be seen distinctly, and without parallax in respect to the

cross wires, by sliding the draw tube of the slit. Do not alter the focus of the telescope.

Turn the slit horizontal, and level the collimator by bringing the slit to the cross wires. Replace the slit in the vertical position, and do not again alter the adjustments.

If the spectrometer is not provided with a collimating eyepiece, the telescope may be focused on a distant object, the telescope, collimator, and table may be leveled by means of a spirit level, and the other adjustments can be made in ways that will suggest themselves. There are also other methods of adjustment which have not been mentioned, but in general they are less precise than the one described above.

173. Reading Divided Circles. — Both the axis of the spectrometer table and the axis about which the telescope rotates should pass exactly through the center of the divided circle. In actual construction this condition is difficult to fulfill; the error introduced in the circle indications is called the *eccentricity* of the circle. This error is wholly eliminated from the results by reading two opposite verniers (Art. 17) and taking the mean. Readings are sometimes made at three or four equidistant points to eliminate eccentricity and graduation errors.

One vernier may be designated as vernier A, and the other as vernier B. To avoid confusion the simple method may be adopted of calculating the angle from the readings of vernier A, and separately from vernier B, and taking the mean. If a vernier between two readings is rotated through the zero point, this must be taken into account in computing the angles. The readings for an angle reduced by this method would appear as follows:

	Vernier A	Vernier B
On first face	335° 5′ 20″	155° 2′ 20″
On second face	95 7 10	275 6 40
Difference	120° 1′ 50″	120° 4′ 20″
Angle	120° 3′ 5″	

A somewhat more concise plan is to record the degrees of vernier A only, but to use the mean of the minutes and seconds

of the two verniers. The above measurements, in this method, would appear as follows :

	On First Face			On Second Face		
Vernier A	335°	5′	20″	95°	7′	10″
Vernier B		2	20		6	40
Mean	335°	3′	50″	95°	6′	55″
Angle . . . . 120° 3′ 5″						

The method of averaging the entire readings of the two verniers for each setting should be avoided, as it is more cumbersome.

CVII. ANGLE OF A PRISM WITH THE SPECTROMETER

Measure the angle of a prism with the spectrometer by three methods.

174. Adjustment of Prism. *Mounting the Prism.* — Before beginning the work with the prism, the spectrometer should have been completely adjusted, as described in the preceding exercise. These adjustments must not be disturbed while placing the prism in its proper position.

Mount the prism on a support with three leveling screws, place it upon the spectrometer table with one face towards the telescope, and level the prism by inspection. For convenience it is desirable that the angle between the two lines joining one of the leveling screws in the base of the prism holder to the two other screws, should be the same as the angle between the two faces of the prisms. When this is true so place the prism on its support that each of its two edges shall be perpendicular to one of these lines joining the leveling screws. Then (Fig. 89) the face AB can be leveled by the screw D without disturbing the level of the face AC; and the face AC may be leveled by the screw F without disturbing the face AB.

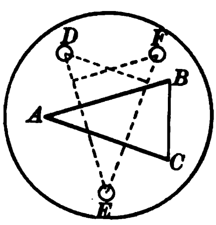


FIG. 89. POSITION OF PRISM ON STAND

Before beginning the adjustment of the prism, it will be well to turn the table and telescope into all the several positions

which they will later occupy in making the measurements, to observe whether it will be possible to read the two verniers of the divided circle for each of the settings. Sometimes one vernier may come under the collimator or telescope and be inaccessible. This difficulty may usually be obviated by altering the direction in which the prism stands on the table.

It should also be noticed whether in these positions the prism faces are in the most advantageous relations to the objectives of the collimator and telescope. The best position is not always that in which the prism holder is centrally placed on the spectrometer table.

To adjust the Prism. — The two prism faces must be parallel to the vertical axis of the spectrometer table (or of the telescope). Arrange the collimating eyepiece and light to observe the reflected image of the cross wires from the first face of the prism, as described in Art. 172. By turning the spectrometer table and by altering the leveling screws in the prism support only, bring the reflected cross into coincidence with the direct image. The first face is then in position. Turn the spectrometer table till the cross wires are seen reflected from the second face of the prism, and by the principle illustrated in Fig. 89 adjust the second face. This should not have disturbed the first face; but reobserve the cross in the first face, correcting any displacement which may have occurred. Continue thus until the reflected image from either face coincides with the direct image, when the prism will be in adjustment.

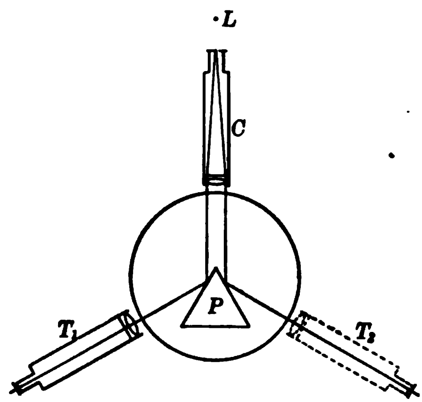


FIG. 90. ANGLE OF PRISM — METHOD I

175. Angle of Prism; First Method. *Without Collimating Eyepiece; Table fixed, Telescope movable.*—Place a light, L (Fig. 90),

to illuminate the slit. Turn the prism P with its refracting edge toward the collimator C, dividing the beam of light, part falling on one face of the prism and part on the other. The exact position of the prism is not important. Clamp all the movable parts of the spectrometer except the telescope. Set the telescope in the position T_1, on the image of the slit reflected from the first prism face. Read the two verniers of the divided circle, as explained in Art. 173. Turn the telescope to the position T_2, and set on the image of the slit reflected from the second face.

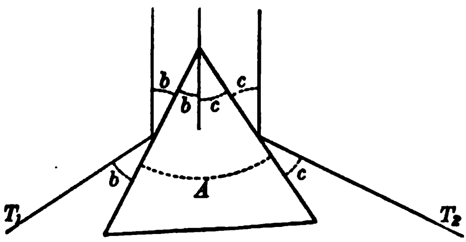


Fig. 91. Relation of Angles — Method I

The angle through which the telescope has been turned is twice the prism angle. For (Fig. 91) it has moved through the angle $b + A + c$. It is evident that the three angles b are all equal, and the same is true of the three angles c; also $b + c$ is equal to A; therefore $b + A + c = 2A$, twice the prism angle.

176. Angle of Prism; Second Method. *Without Collimating Eyepiece; Table movable, Telescope fixed.* — Turn the telescope of the spectrometer as near to the collimator as is convenient, and clamp all parts except the table. Illuminate the slit. Rotate the table until the image of the slit, reflected from one face of the prism, is set on the cross wires of the telescope (Fig. 92). Read the two verniers, as described in Art. 173. Rotate the table to bring the second prism face into position to reflect the image of the slit to the cross wires.

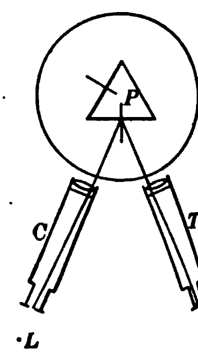


Fig. 92. Angle of Prism — Method II

The angle through which the table has been turned is the supplement of the prism angle; for in the first position the normal to one face bisects the angle between the axes of the telescope

and collimator, and in the second position the normal to the other face is turned to this same direction. The angle between the normals is the same as the angle between the two faces (Fig. 93), but the table has clearly been turned through the supplement of this angle.

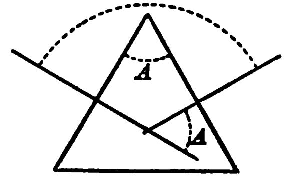


FIG. 93. RELATION OF ANGLES—METHODS II AND III

177. Angle of Prism; Third Method. *With Collimating Eyepiece; either Telescope or Table fixed, the other being movable.* — Clamp the telescope, and arrange a light, the collimating eyepiece, and the prism, to obtain the direct and reflected images of the cross wires (Fig. 94), as described in Art. 172. Secure coincidence between the two images by rotating the table only, which is possible if the adjustments have not been disturbed. Read the verniers; rotate the spectrometer table till coincidence between the direct and reflected images of the cross wires is secured with the second prism face, and again read the verniers.

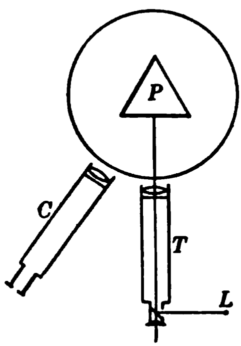


FIG. 94. ANGLE OF PRISM — METHOD III

From Fig. 93 it is clear that the table has been turned through an angle which is the supplement of the prism angle.

If the table is fixed, the telescope may be turned from one coincidence to the other, in which case the light must also be moved; or the spectrometer may be turned on its base, the telescope being held during the motion. This will avoid moving the light, and may keep the telescope in a more convenient position. This third method for measuring the angle between two reflecting surfaces is the most precise one.

CHAPTER XVI

INDEX OF REFRACTION

CVIII. INDEX OF REFRACTION BY MINIMUM DEVIATION

Find the index of refraction of a prism for sodium light.

178. Index of Refraction. — Light moves with different velocities through different media. The ratio of its velocity in vacuum, in which its velocity is greatest, to its velocity in any other medium is the *absolute index of refraction* of this medium. It is not practicable to compare these velocities directly. When light passes from one medium into a second, its direction of propagation in the second medium as compared with its direction in the first is a function of the angle of incidence upon the bounding surface and of the indices of refraction of the two media. If i is the angle of incidence, r the angle of refraction, n_1 and n_2 the indices of refraction of the two media, then

$$\sin r = \sin i \frac{n_1}{n_2}.$$

In laboratory measurements the first medium is usually air; if the index of refraction of air is taken as unity, the *relative index of refraction* of the second medium is

$$n = \frac{\sin i}{\sin r}.$$

In the particular case of light passing through a prism in the direction of minimum deviation, the relative index of refraction of the prism is

$$n = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A},$$

where D is the angle of minimum deviation (Art. 179) and A is the refracting angle of the prism (Art. 175).

The index of refraction of a liquid may be determined by this method, the liquid being contained in a hollow prism having sides of plane-parallel plates which will have no deviating effect.

If the index of refraction of a second medium relative to the first is multiplied by the absolute index of refraction of the first medium, the result is the absolute index of refraction of the second medium. The absolute index of refraction of air at 20° under normal pressure is 1.0002773.

179. Angle of Minimum Deviation. — The spectrometer and prism are to be completely adjusted by the methods described in Arts. 172–174. Illuminate the slit with the kind of light for which the index of refraction is desired, and so place the prism and telescope (Fig. 95) that the spectrum is visible in the telescope. For finding the spectrum, a luminous flame is desirable, as it gives a bright, continuous spectrum. Even when the sodium spectrum is to be examined it is not necessary to use the Bunsen flame; a borax bead, placed in the luminous gas flame, gives bright yellow sodium lines in the continuous spectrum, which are very convenient for observation.

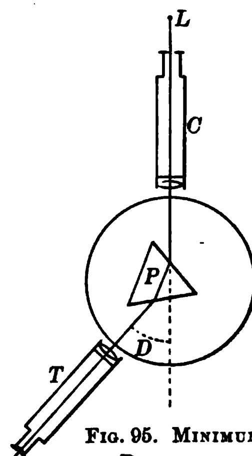


FIG. 95. MINIMUM DEVIATION

Having found the spectrum in the telescope, rotate the prism, following the motion of the spectrum with the telescope, in such a manner as to cause the deviation D, of the light by the prism, to have the least possible value. This position of the prism is a definite one, and is readily found by trial; a rotation of the prism in either direction from this point causes the deviation to increase. Set the telescope on the sodium line when the prism is at this turning point of minimum deviation.

Turn the prism (not the divided circle) on the table so that it produces deviation on the opposite side, as indicated in Fig. 96,

and set the telescope on the sodium line when the prism is adjusted for minimum deviation as before.

The angle between the two positions of the telescope is twice the angle of deviation D. In making the circle readings, use the two verniers as described in Art. 173.

Instead of finding the deviation on the second side, the prism might have been removed after the first setting, and the telescope sighted directly on the slit through the collimator. Then the angle between the two positions of the telescope is the angle of minimum deviation. The first method is preferable.

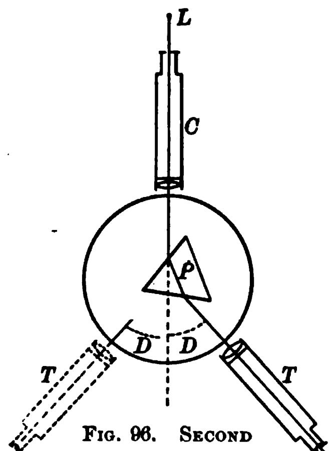


FIG. 96. SECOND POSITION FOR MINIMUM DEVIATION

CIX. INDEX OF REFRACTION WITH A MICROSCOPE

Determine the index of refraction of a piece of plate glass, and of water, alcohol, and carbon bisulphide, using a microscope.

180. Index of Refraction with a Microscope.—Any transparent substance which can be put in the form of a plane-parallel plate of suitable thickness may have its index of refraction determined with an ordinary microscope, with an accuracy giving two or three decimal places in the result. The objective of the microscope must have such a focal length (working distance) as to permit an object to be seen through the plate. The accuracy of the result is increased by using a plate of the greatest possible thickness together with a microscope objective of the shortest focal length that will permit observation through the thick plate.

Let c (Fig. 97) be any object, as a fine scratch on a piece of glass, over which is placed the plate of thickness t whose index of refraction is desired; and let cam be the path of a ray of light through the plate and air to the microscope when the

latter is accurately focused. The light enters the microscope as though it had come from the point b; that is, the plate has apparently elevated the object from c to b, represented by e. The angles of incidence and refraction are represented by i and r respectively. From the figure it is evident that, in the triangle abc,

$$\frac{ac}{ab} = \frac{\sin i}{\sin r} = n,$$

the desired index. But in actual microscopic observation the distance ao is very small as compared with ab and ac, and

$$\frac{ac}{ab} = \text{(approximately)}\ \frac{oc}{ob} = \frac{t}{t - e};$$

to this degree of approximation, then,

$$n = \frac{t}{t - e}.$$

The thickness, t, may be measured with calipers, and the apparent elevation, e, of the object by the plate is measured by noting the difference between the two positions of the microscope body when it is focused on the object uncovered and on the object covered by the plate. For measuring this distance a scale may be fastened with wax to the stand of the microscope, and a needle for a pointer to the body tube; or the fine focusing screw, if of known pitch, may be used. If the pitch is unknown, both the thickness of the plate and the elevation may be determined in terms of turns of the screw, giving the index of refraction as before. The thickness may be measured, in the first case, by focusing upon the uncovered object and then

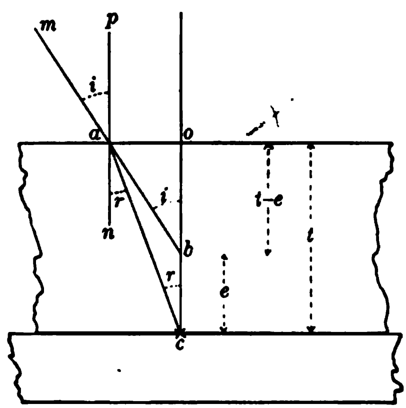


FIG. 97. INDEX OF REFRACTION WITH A MICROSCOPE

upon the upper surface of the plate when it is in position, in contact with the object.

If the plate has scratches or other marks on both surfaces, and the thickness t has been measured with calipers or otherwise, it is only necessary to focus upon the top surface and upon the lower surface through the plate, noting the depression d of the microscope. Then the index of refraction of the plate is

$$n = \frac{t}{d}.$$

In determining the index of refraction of liquids, a crystallizing dish is convenient for the receptacle. Make a fine mark on the inside of the bottom of the dish. Focus upon this, and let the scale reading be r_1; pour in the liquid to a depth as great as will allow the mark to be seen through the liquid, focus, and let r_2 be the reading; focus upon the upper surface of the liquid, which may be indicated by some floating particle, and let r_3 be the scale reading. Then the index of refraction is, as before,

$$n = \frac{r_1 - r_3}{r_2 - r_3}.$$

CX. INDEX OF REFRACTION BY DISPLACEMENT

Determine the index of refraction of a thick plane-parallel disk of glass.

181. Index of Refraction by Displacement. — A thick plate with two plane-parallel surfaces, such as a glass disk, may have its index of refraction determined by the following displacement method. The measurements may be made with simple or with more elaborate apparatus, according to circumstances and the precision required.

Let a telescope be set to view a divided scale, S, in the direction VS (Fig. 98). Set the plate whose index is desired between the telescope and scale, with its plane faces perpendicular to the line of sight. This condition is secured with sufficient accuracy by observing with the telescope that there is no apparent displacement of the scale seen through the plate. Let S be the

point of the scale under the cross wires. Rotate the plate about an axis perpendicular to the plane containing the line of sight and the scale; the scale will appear displaced, the point x now being seen under the cross wires. Let d be the amount of this displacement, t the thickness of the plate, and i the angle through which the plate has been turned. The angle of incidence of the light on the plate is i, and the angle of refraction, r, is to be found. Without detailed explanation it is evident that the following equations are true.

$$\tan r = \frac{eb}{t} = \frac{cb - ce}{t} = \frac{t \tan i - \dfrac{fe}{\cos i}}{t}$$

$$= \tan i - \frac{d}{t \cos i}.$$

The index of refraction of the plate is

$$n = \frac{\sin i}{\sin r}.$$

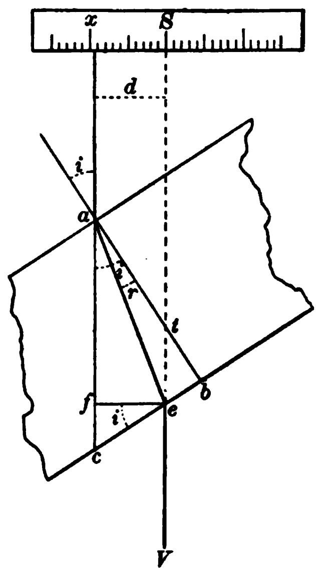


Fig. 98. Displacement by Refraction

The rotation of the plate may be measured with a protractor, or by placing the plate on an engineer's transit, or on the table of a goniometer or spectrometer.

CXI. INDEX OF REFRACTION BY TOTAL REFLECTION

Determine by Kohlrausch's method the index of refraction of a crown glass plate immersed in carbon bisulphide, of turpentine in a glass box, of a drop of Canada balsam, and of water by the immersion of an air-filled box.

182. Total Reflection. — Light moving in one medium whose index of refraction is n to the surface of another medium of

greater refractive index, N, will be totally reflected when the angle of incidence is such that its sine is greater than $\frac{n}{N}$. If the limiting value, ϕ, of the angle of total reflection is observed, and the index of refraction of one substance is known, the other index may be found from the relation

$$\sin \phi = \frac{n}{N}.$$

Various instruments using this principle have been devised, which permit the determination to the fourth or fifth decimal place of the index of refraction of any liquid, and of such solids as have indices less than the index of a liquid used in the apparatus. It is especially useful for liquids in small quantities, for solids in thin plates or in powdered form, for crystals, for bodies with only one small reflecting face, and for opaque bodies. When the face of the body is of small reflecting power, or is only approximately flat, the method is still applicable.

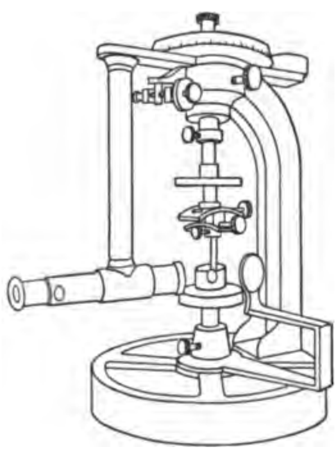


FIG. 99. TOTAL-REFLECTOMETER

183. Total-Reflectometer (Kohlrausch). *Solids.* — The body to be investigated is supported in a small flask filled with a liquid of large and known refractive index. Such liquids are carbon bisulphide (1.63), α-bromonaphthalene (1.66), and methyl iodide (1.74). The index of the liquid may be determined from tables or with a hollow prism and the spectrometer, as described in Art. 178. The support of the body passes through the center of a divided circle and is provided with centering devices for placing the face of the body in the axis of rotation, as shown in Fig. 99.

The front of the flask is a plane-parallel plate through which a small telescope may receive light reflected from the body.

The telescope should be perpendicular to the axis of rotation, and the face of the body and of the flask parallel to this axis. A Gauss eyepiece or a collimator is provided for facilitating these adjustments, which may be made according to principles explained in Art. 172.

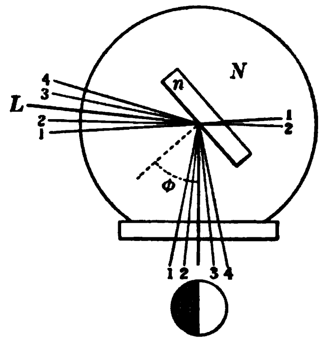


FIG. 100. KOHLRAUSCH TOTAL-REFLECTOMETER

The flask is illuminated by sodium light, L (Fig. 100), and by the interposition of a piece of oiled tissue paper, diffused light from many directions will fall upon the body. All rays having a smaller angle of incidence than the angle of total reflection (rays 1 and 2) will be partially refracted and partially reflected, while all rays having a greater angle of incidence (3 and 4) will be totally reflected. By turning the support of the body, the dividing line corresponding to the angle of total reflection may be brought to the center of the field of view of the telescope. When this position has been found, a condensing lens may be employed to increase the illumination. Read the position of the body as indicated by the divided circle. Illuminate the opposite side of the flask, and rotate the body until the limiting angle for the opposite side is observed. The difference between the two positions is twice the limiting angle, ϕ, of total reflection, and the index of refraction of the body is

$$n = N \sin \phi,$$

N being the index of the liquid.

The index of refraction of carbon bisulphide is 1.6277 at 20°, which changes 0.00080 per degree, being less for higher temperatures. The light may be placed forty or more centimeters from the flask, and a screen with a glass-covered aperture may be used to protect the liquid from the heat of the flame.

Liquids. — If a solid plane plate having a known index of refraction n is suspended in the liquid of unknown but greater

index N, and the angle of total reflection is observed, the index of refraction of the liquid is

$$N = \frac{n}{\sin \phi}.$$

If the liquid has an index less than that of the solid plane-parallel plate, the plate is to be so immersed that there is a film of air behind it. This is secured by making the plate one side of a small box. The index of the plate should be large, but it need not be known. Making the observations as before, the index of refraction of the liquid in which the box is immersed is

$$N = \frac{1}{\sin \phi}.$$

If the glass box is immersed in a liquid of known index, less than that of its plane-parallel side but greater than that of an unknown liquid, the index of the latter may be determined by filling the box with it and proceeding exactly as for the index of a solid. If only a small quantity of the unknown liquid is available, instead of filling the box with it, a single small drop placed on the inner surface of the plane-parallel plate suffices for the experiment.

Crystals. — The refraction of a crystal can only be described with the aid of two, or sometimes three, indices of refraction. In order to determine them the direction of the axis of the crystal must be known, and then the limiting angles of reflection corresponding to the various indices may be distinguished with the aid of an analyzing Nicol prism. The method of procedure is described in special treatises and in the references.

References. — *Kohlrausch*, Physical Measurements, p. 161; *Leiss*, Die optischen Instrumente, pp. 52–61; *Drude*, Theory of Optics, pp. 339–344.

184. Refractometer (Pulfrich). — This instrument uses not total reflection but refraction at grazing incidence. A glass prism of known index of refraction, greater than that of any substance to be investigated, has attached to its horizontal face a glass cylinder to contain the liquid to be tested (Fig. 101).

Behind the instrument in the plane of the prism is placed a sodium light, which is condensed upon the surface of the prism through the cylinder of liquid. The rays at grazing incidence will be refracted at the angle of total reflection, while all other rays will be partially refracted at less angles and partially reflected. A telescope capable of motion in a vertical plane about the axis of a divided circle measures the angle of emergence a; it is first set normal to the face of the prism (Art. 172), and then so that the field of view is half bright and half dark. If N is the index of the glass prism, the index of refraction of the liquid in the cylinder is

$$n = \sqrt{N^2 - \sin^2 a}.$$

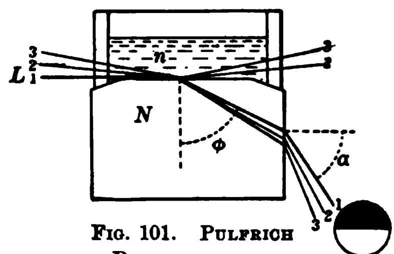


FIG. 101. PULFRICH REFRACTOMETER

Tables are usually provided which for a given prism show the values of n for given values of a. The index of refraction of water may be measured to show that the circle and prism are properly set. For water at 20°, $n = 1.3332$.

The index of refraction of liquids varies considerably with the temperature, hence the temperature of the observations must be noted; usually this should be made 20°. If the room temperature is higher than this, the liquid may be cooled a small amount, and the observation made when its temperature passes through 20°; and vice versa.

The cylinder may be surrounded by a hollow case through which hot water or steam may be passed, or a coil of wire carrying a current of electricity may be used, and the variations of the index of refraction with temperature, or the index of a substance which is transparent only when melted, may be thus investigated. In these cases a correction to the index of the prism is required.

A partition of black glass may be cemented in the cylinder, dividing it into two cells, permitting the convenient measurement of the differential refraction of two liquids.

A solid substance may be investigated if it has two surfaces approximately at right angles, one of which is plane and well polished, and the other polished sufficiently to transmit light. The edge in which the surfaces intersect must be perfect. The plane face is joined to the surface of the prism with a liquid of high refractive index, the cylinder being removed if necessary. The index is then determined as described above.

If the solid is in the form of powder, its index of refraction may be determined with moderate precision, provided its index is intermediate in value between that of two available liquids (alcohol, ether, or acetone, 1.36; benzene, 1.50; α-bromonaphthalene, 1.66) which when mixed will not dissolve the powder. The cylinder of the refractometer is filled with the powder and the mixed liquids; the proportions of the latter are so altered that the index of the mixture is the same as that of the powder. If the index of the liquid is too great, a bright band appears at the boundary between the light and dark parts of the field of view; while if the index of the liquid is too small, the boundary is indistinct. When the mixture of the liquids is correct the dividing line indicating the limiting angle is quite sharp, and then the index computed is that of both solid and liquid.

REFERENCE. — *Pulfrich*, Astrophysical Journal, Vol. 3, p. 259, 1896.

185. Crystal-Refractometer (Abbe). — In this method a hemisphere of glass is substituted for the flask of liquid of the Kohlrausch method (Fig. 102). It requires but a small quantity of liquid to join the crystal to the flat surface of the hemisphere, and by rotating the hemisphere on an axis perpendicular to the surface of the crystal it permits a convenient observation of the relations of the several limiting curves of total reflection. It is not possible to adjust the surface of the plate as accurately as in Kohlrausch's method, and the construction of the hemisphere and its adjustment introduce difficulties.

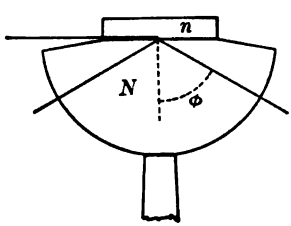


FIG. 102. CRYSTAL-REFRACTOMETER

Furthermore, it is not so well adapted to the investigation of small and imperfectly reflecting surfaces. It is used according to the principles explained for Kohlrausch's method, and is applicable for a solid whose index is less than that of the hemisphere and of the liquid with which it is joined to the hemisphere, and for any liquid with an index less than that of the hemisphere. It may be used with light at grazing incidence, or by total reflection. The reference may be consulted for the method of use.

REFERENCE. — *Leiss*, Die optischen Instrumente, pp. 38–49.

CHAPTER XVII

WAVE LENGTH OF LIGHT

CXII. WAVE LENGTH OF LIGHT BY FRESNEL'S INTERFERENCE METHOD

Determine the wave length of sodium light.

186. Interference of Light. — If two series of exactly similar waves of homogeneous light, originating in two near sources, arrive at the same point in space, the disturbance at this point is the resultant of the two waves. The loci of all points at which the two waves simultaneously arrive in the same phase will be brilliantly illuminated; the loci of points at which the waves arrive in opposite phases will be dark. These loci are hyperboloids of revolution, having the two origins for foci. If the two waves fall upon a screen, there will be seen the intersections of the screen and the hyperboloid loci; these appear as a series of alternately bright and dark *interference bands* or *fringes*. The distance from one bright band to the next is a function of the distance between the two sources, the distance from the screen and the wave length of the light. Such interference phenomena may then be used in determining the wave length of light.

187. Wave Length of Light with the Bi-Prism. — Since it is impossible to make two exactly similar sources of light, it is necessary for producing interference bands to divide one single beam into two parts which shall have the same effect as the two sources. Fresnel used for this purpose a *bi-prism*, which is equivalent to two prisms, *BP* (Fig. 103), of very small angle (one-half degree, for instance), placed base to base. If light from a slit, *S*, set parallel to the refracting edge of the prism, falls

upon the prism, it will be refracted into two beams which will appear to come from two sources, S_1 and S_2, very near to each other. These beams are each divergent, and where they overlap interference bands may be produced. These bands may be received upon a screen at M; but for purposes of measurement it is convenient to mount the slit and prism on an optical bench, which also carries a micrometer eyepiece with which the bands are to be observed.

Place the center of the slit, bi-prism, and eyepiece in the same straight line, the plane face of the prism being perpendicular to this line. Set the prism about twenty centimeters from the slit, and the eyepiece fifty centimeters from the prism. It will be well to place a bright light behind the slit, and first to find the

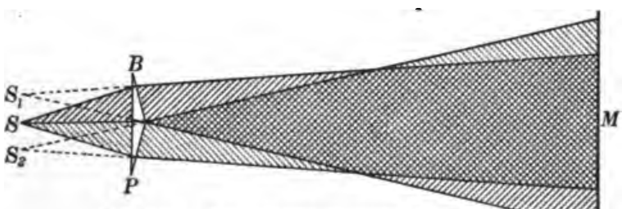


FIG. 103. INTERFERENCE WITH BI-PRISM

interference bands on a screen placed against the eyepiece. If they are not at once visible, slightly alter the position of the various parts, rotate the prism about the line of sight to make its edge more nearly parallel to the slit, narrow the slit, and correct the alignment, making any or all of these adjustments till the bands are seen with satisfactory distinctness through the eyepiece.

Illuminate the slit with the light whose wave length is required, and carefully determine with the micrometer the average distance between successive bright bands in the following manner. Read the position of ten successive bright bands; subtract the first reading from the sixth, the second from the seventh, etc.; average these differences and divide by five; the result is the difference between consecutive bands. Make three sets of readings, the mean of which is the quantity x of

the formula. If d is the distance of the slit from the cross wires, and c the distance between the two apparent sources of light, then the wave length is

$$\lambda = \frac{cx}{d}.$$

The quantity c may be determined by either of the following methods.

Select a convex lens of focal length less than one fourth the distance d; place it in such a position between the eyepiece and bi-prism that it focuses the two images of the slit in the field of view of the eyepiece. With the micrometer measure the distance, c_1, between the centers of these two images. A second position of the lens may be found in which the images are also focused, but of a different size; measure the distance, c_2, between them. Then

$$c = \sqrt{c_1 c_2}.$$

If no position of the lens focusing the images can be found, the eyepiece may be moved farther from the prism to permit the convenient observation of the images as required. Do not move the prism or slit.

The second method for measuring c requires a spectrometer. Completely adjust the spectrometer and point the telescope toward the collimator. Having measured the distance a from the slit of the interference apparatus to the bi-prism, remove the bi-prism and place it on the spectrometer table with its plane face towards the telescope. Adjust the prism until the two images of the slit which are now seen in the telescope are as distinct and as near together as possible. Measure the angle D, in seconds of arc, between these two images. The distance between the two apparent images of the slit in the interference experiment is then a multiplied by D in radians, or

$$c = \frac{aD}{206265}.$$

REFERENCE. — *Glazebrook*, Physical Optics, pp. 105–119.

188. Fresnel's Mirrors. — Instead of the bi-prism, two black-glass mirrors lying nearly in the same plane may be used to produce the two virtual light sources. While the interference bands so obtained may be sharper than those given by the bi-prism, they are not so brilliant, and the adjustments required are more complicated and troublesome. The quantities to be measured and the formulæ for determining a wave length, are the same as given for a bi-prism.

REFERENCES. — *Glazebrook*, Physical Optics, p. 119; *Preston*, Theory of Light, p. 141; *Drude*, Theory of Optics, p. 130.

CXIII. WAVE LENGTH OF LIGHT WITH THE PLANE DIFFRACTION GRATING

Determine the wave lengths of the two lines, D_1 and D_2, of the sodium or solar spectrum.

189. Wave Length of Light by Diffraction. — A diffraction grating has for its essential feature a series of parallel, equidistant apertures or surfaces from which light may come by transmission or reflection. It is usually made by ruling on glass or polished speculum metal a series of fine lines of from three to ten centimeters in length, the series sometimes consisting of one hundred thousand lines ruled at the rate of eight thousand per centimeter.

When light from a slit with a collimating lens falls upon a grating, part of it is transmitted or reflected in regular manner, and the image may be viewed with a telescope. If the spaces of the grating are parallel to the slit, in addition to this direct image, on either side of it will be found a series of diffracted spectral images of great regularity and purity. The spectra lying nearest to the direct image, on either side, are called the first-order spectra; while those successively more distant are of the second, third, etc., orders. The relations between the angle of incidence i, the angle of diffraction d, the order of the

spectrum n, the wave length of the light λ, and the distance s between the centers of successive grating spaces, is

$$\lambda = \frac{s}{n}(\sin i + \sin d).$$

The value of the grating space s is usually considered as known from the constants of the ruling machine with which the grating was made; otherwise it might be determined by comparison under a microscope, with a standard scale. The angles i and d may be measured with a spectrometer.

The spectrometer must be in perfect adjustment (Art. 172). Mount the grating in a suitable support with its lines vertical and with its face approximately in the center of the circle of the spectrometer table. If the grating support rests on three leveling screws, the grating should be placed with its face perpendicular to the line joining two of the screws, that it may be possible to adjust the inclination of the lines of the grating without disturbing the perpendicularity of its face. Make the grating perpendicular to the axis of the telescope as described for a prism face in Art. 174. Before beginning the measurements it is desirable to observe the spectrum, and to adjust the width of the slit and the inclination of the lines of the grating to secure the maximum distinctness of the spectrum lines.

For a reflection grating the necessary measures are most conveniently made in the following manner. Clamp the telescope and collimator so that the angle between them shall remain constant; it may be about 90°. Set the grating perpendicular to the axis of the telescope and read the position of the divided circle. Turn the spectrometer table carrying the grating till the direct image of the slit is reflected into the telescope; let a be the angle through which it has been turned. Illuminate the slit with the light to be investigated, and turn the grating still farther till the nth order spectrum is visible in the telescope, setting on the line whose wave length is desired; let this second angle be β. The wave length of the light is

$$\lambda = \frac{2s}{n}\cos a \sin \beta.$$

This formula is equivalent to the one previously given, for (Fig. 104),

$$2 \cos a \sin \beta = \sin (a + \beta) + \sin (a - \beta) = \sin i + \sin d.$$

A transmission grating may be used as a reflection grating, but it is equally convenient to pass the light through the grating and to use the first formula. If the grating is set perpendicular to the incident light from the collimator (by sighting the telescope upon the slit and placing the grating normal to the telescope), the angle of incidence i becomes 0°, while the angle of diffraction is at once measured by turning the telescope to the desired spectrum line. The formula then becomes

$$\lambda = \frac{s}{n} \sin d.$$

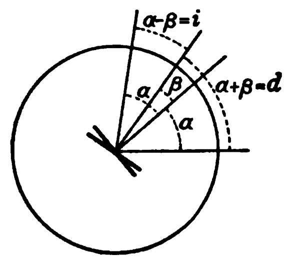


FIG. 104. RELATIONS OF ANGLES FOR GRATING MEASURES

A transmission grating may also be used in a manner analogous to that for a prism (Art. 178) by setting it in the position of minimum deviation. The deviation D is directly measured, and since $D = i + d$, and $i = d$, the wave length is

$$\lambda = \frac{2\,s}{n} \sin \tfrac{1}{2} D.$$

CXIV. WAVE LENGTH WITH THE CONCAVE-GRATING SPECTROSCOPE

(a) Assuming the wave lengths of the solar spectrum lines D_1 and F, find the wave lengths of the lines B, C, E, G, and H (second order).
(b) Construct a wave length scale for the spectroscope.
(c) Photograph the bands of the spectrum of carbon (second order).
(d) Find the wave lengths of the principal lines of any assigned spectrum by photographic comparison with the solar spectrum.

190. The Concave-Grating Spectroscope.—If a diffraction grating the lines of which are ruled on a spherical concave reflecting surface is used for spectroscopic observations, the collimating

and view telescopes of the plane-grating spectroscope are not required. The absorption of the lenses is thus avoided, which is an important consideration in some kinds of investigations, and various other advantages are secured.

A concave grating is usually mounted at one end of a rigid bar, at the other end of which, exactly at the center of curvature of the concave surface, is an eyepiece or photographic plate. An L-shaped track (Fig. 105) is provided with a slit, *S*, mounted at the angle. The bar is pivoted to wheel supports resting on the tracks, so that the grating *G* is always over one arm of the L, and the eyepiece *E* is always over the other arm. As the bar is moved along the tracks in the various possible positions, the grating, eyepiece, and slit will always lie on the circumference of a circle whose diameter is the radius of curvature of the grating. As the bar is moved to various positions the several spectra of the light illuminating the slit, which are produced by the grating, are all brought to focus on the circumference of this circle. As the eyepiece is moved along the track the spectra may be observed successively without any other adjustment.

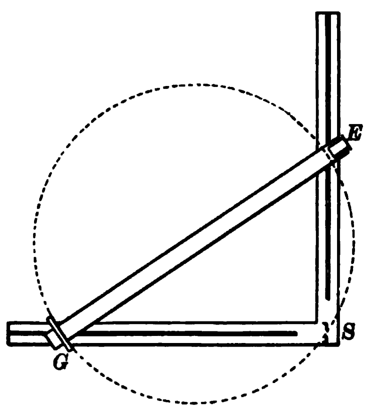


FIG. 105. CONCAVE-GRATING SPECTROSCOPE

This arrangement is especially convenient for photographic investigation, since a sensitized plate may be bent to coincide with the circumference of the circle mentioned, and a considerable portion of the spectrum may be distinctly photographed at one time. Glass plates about 5 by 1½ inches are used for work with a grating of 6 feet radius; the camera is arranged to hold the plate bent to the proper radius.

Adjustment. — It is to be determined that the two tracks are at right angles, level, and in the same plane, and that the slit

is directly over their intersection and at the height of the grating center. The instrument having been properly constructed, and once mounted and adjusted, will presumably not require alteration of these relations.

Place the grating, with the lines vertical, in its support at one end of the bar, and, with a collimating eyepiece at the other end, adjust the inclination of the grating and the length of the bar until the cross wires of the eyepiece are exactly at the center of curvature of the mirror. This is shown by the coincidence of the direct and reflected images of the cross wires and by the absence of parallax.

The slit being illuminated, as the diagonal bar is moved along the tracks the several spectra should be seen in the center of the field. If they rise or fall in the field of view, it indicates imperfect mechanical construction as regards the surfaces of the tracks, or the pivots which join the bar to the wheels. It may be necessary to sacrifice the perfection of the previous adjustment by altering the inclination of the grating until the spectra may be seen throughout the required motion of the diagonal bar.

The slit being narrow, it is to be made accurately parallel to the lines of the grating by rotating it until the definition of the spectral lines is the best possible.

It should be ascertained, when the camera is substituted for the eyepiece, that the plate coincides exactly with the previous position of the cross wires, and that it is perpendicular to the radius of curvature of the grating.

191. Normal Diffraction Spectrum. — Diffraction spectra possess the great advantage over prismatic spectra that the deviation is proportional to the wave length and to the order of the spectrum. For different gratings the deviation varies inversely as the grating space. Thus all the spectra of any given substance produced by different gratings are exactly similar, and may differ only in relative size. This is not true of spectra produced by other means. A disadvantage of diffraction spectra is the small intensity of illumination.

The deviation being proportional to the wave length, the comparison of wave lengths, with moderate precision, is reduced

to a comparison of lengths. By measuring the distance between two known lines, the value of the scale in wave lengths is determined; the wave length of an unknown line is then found by measuring its distance from one of the standard lines. The value of the scale is inversely as the order of the spectrum; for instance, the distance between two given lines is twice as great in the second-order spectrum as in the first.

The first-order spectrum is separate from the others, but the red of the second order overlaps the violet of the third, and in the higher orders the overlapping increases. Sometimes a screen of colored glass or gelatine may be convenient for separating the different spectra; for instance, if the overlapped second- and third-order spectra be observed through a red glass, the violet light is absorbed while the second-order red is seen distinctly.

References. — *Preston*, Theory of Light, pp. 226–242; *Scheiner*, Astronomical Spectroscopy, p. 52.

CHAPTER XVIII

THE INTERFEROMETER

CXV. SMALL LENGTHS WITH THE INTERFEROMETER

(a) Determine the distance between the two planes of an optical standard of length.
(b) Determine the length of a division of a scale which is nominally one tenth of a millimeter.

192. The Interferometer. — Perhaps the most precise method for measuring small distances and displacements is by means of the interferometer devised by Michelson. The measurements are made in terms of the wave length of light as a unit, and displacements as small as $\frac{1}{200}$ of a wave length may be measured. The interferometer consists essentially of a half-silvered mirror, H (Fig. 106), which divides incident light from a source, S, into two beams, one of which is reflected to a fixed mirror, M_1, and back, the other is transmitted to a movable mirror, M_2, and back.

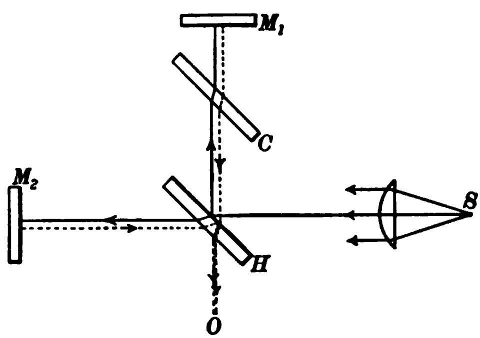


FIG. 106. PLAN OF INTERFEROMETER

The two beams which have returned to the half-silvered glass are each partially transmitted or reflected to the observer in the direction O. The beam to M_2 passes twice through the glass plate supporting the half-silvered film, and to render the paths of the two beams optically equal the beam to M_1 is caused to

pass twice through a parallel clear glass plate, C, of the same thickness as H; this is called the compensating glass, and it may have any convenient position between H and M_1.

Let the light falling on H be parallel, and the mirrors M_1 and M_2 perpendicular to the incident rays. If, then, the paths of the two beams between separation and reunion differ by any odd number of half wave lengths (including the phase difference introduced by one internal and one external reflection), upon reunion at H there will result interference, and no light will be sent to O. There is evidently an indefinite series of positions of M_2, with half wave-length intervals, which produce interference; while whenever M_2 is midway between any two positions of this series, there will be reënforcement of the two beams, with illumination in the direction O. If M_2 moves slowly and continuously, one may count the number of times it passes through an interference position, and thus determine the distance moved.

Instead of adjusting the mirror M_2 so that its entire surface is in one interference plane of the series mentioned, it is usual to give it a slight inclination, about 30″ of arc, so that it cuts across several planes, as shown in vertical projection in Fig. 107.

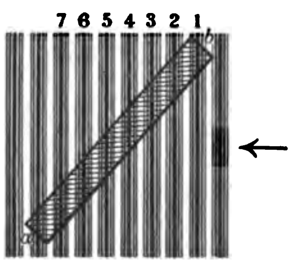


Fig. 107. Mirror intersecting Interference Planes

If the plate is observed in the direction of the arrow, as is done in the interferometer, it is seen (as represented in Fig. 108) crossed by a series of parallel bands, which represent the intersection of the plate with the series of interference planes. Further, if the plate is moved parallel to itself in the direction of the arrow, the band representing intersection with any one plane will move across the plate from the edge a to the edge b. Let band No. 4, for instance, be over an index mark (a cross scratched in the silvering) on the surface of the plate; as the plate is moved this band will move toward b, and when band No. 5 has moved to the index, the plate has evidently been moved

one half a wave length ; if the motion has caused a displacement of the bands of 0.7 of the distance between bands, the plate has been moved $0.7\frac{\lambda}{2}$, λ being the wave length of the light employed (Art. 196). Or, in general, if during the motion a number of bands pass the index (whole and fractions) equal to n, the distance moved, in centimeters, is

$$x = n\frac{\lambda}{2}.$$

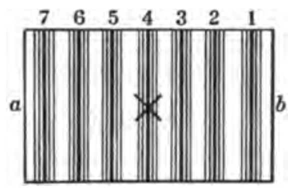


FIG. 108. FIELD OF VIEW OF INTERFEROMETER

If the inclination of the plate is slight, it cuts few planes, and the bands appear to be far apart. The distance between them may be such that it is possible to estimate a hundredth part of this distance, and thus to measure a displacement of M_2 of $\frac{1}{200}$ of a wave length. The measurement of a fraction of a band depends upon the fact that a slight change in the inclination of the compensating glass displaces the system of bands. This glass is mounted on a flexible support so that a micrometer screw, F (Fig. 109), pressing against a spring, alters this inclination. By trial the amount by which the screw must be turned to displace the system by the width of one band is determined, thus facilitating the measurement of fractions.

The influence of the inclination of the mirror M_2 upon the simple theory of interference outlined need not be considered here, since it does not alter the practical conclusions.

If, instead of the linear change in the path of either beam described, there is a variation of any optical property of a medium which the light traverses between the separation and reunion, such as an alteration in density, the result is a displacement of the bands. To measurements of this kind, and others which cannot be described, the interferometer is especially adapted; its range of application to physical investigations is very wide.

REFERENCES. — *Michelson*, Philosophical Magazine, Vol. **13**, pp. 236–242, 1882 ; *Morley*, Physical Review, Vol. **4**, pp. 3–22, 1897 ; *Wadsworth*, Physical Review, Vol. **4**, pp. 480–491, 1897 ; *Shedd*, Physical Review, Vol. **11**, pp. 304–315, 1900 ; *Michelson*, Light Waves and Their Uses.

193. Finding the Interference Bands. *Monochromatic Light.*— It is assumed that the interferometer has been properly constructed so that the diagonal glasses H and C (Fig. 109) are held vertical, parallel, and in the direction of the bisector of the angle between the two mirrors M_1 and M_2, to the degree of precision ordinarily attained in machine-shop practice. All the glasses may be provided with adjusting screws, though it is usual to have only one mirror, M_1, so arranged.

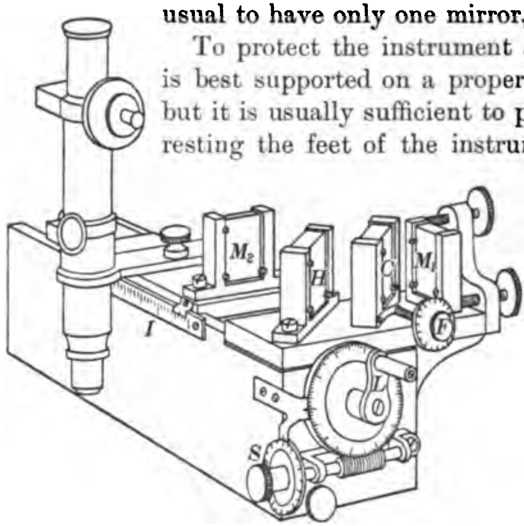


FIG. 109. INTERFEROMETER

To protect the instrument against vibrations it is best supported on a properly constructed pier; but it is usually sufficient to place it on any table, resting the feet of the instrument on soft rubber corks.

Notice that the half-silvered side of H is toward the compensator. Turn the lead screw by the handle L to make the distance of M_2 from the silver film H the same as that of M_1. A scale with index, I, may indicate this position; if not, measure with a compass or scale from any point on the edge of the silvered surface of H to M_1, and set M_2 at an equal distance from the same point of H.

Place a sodium or other monochromatic light near the focus of a lens so that an approximately parallel beam of light shall illuminate the whole of the half-silvered film, as shown in Fig. 106. Since the interferometer is a symmetrical device, the light may enter in the direction OM_1, and the bands be observed in the direction SM_2, without in any way altering the phenomena; either arrangement may be adopted according to convenience or chance. Place fine wires or threads over the surface of the lens to mark its horizontal and vertical diameters.

Look towards the interferometer in the direction OM_1, and alter the position of the light or lens, and also of the diagonal mirror H if necessary, to cause the entire surface of the mirror to be evenly illuminated. Usually there will also be visible three images of the cross wires, the third image being due to the reflection from the unsilvered side of the glass H. By slightly altering the inclination of the adjustable mirror (M_1 or M_2) in its frame, either one cross will move and two will be stationary, or two will move and one will be stationary. The odd image must be brought into exact coincidence with one of the pair; which one might be determined by simple theory, but usually trial gives the quickest solution. Having produced coincidence between two of the images, and after the vibration of the apparatus caused by touching the screws has subsided, the interference bands may be visible. Search for them by moving the eye farther from or nearer to the mirrors, and by moving it crossways. If they are not readily found, adjust the mirror to bring the odd image of the cross into coincidence with the second one of the pair. If the bands cannot be found now, the setting of M_2 at a distance from H equal to the distance of M_1 was not sufficiently exact. Turn the screw to move M_2 a fifth of a millimeter in one direction or the other, and repeat the search for the bands as described.

Having found the bands, they may be given any direction and any width by further careful adjustment of the inclination of the mirror. Usually they should be made vertical and of such a width that from five to ten bands are visible at one time. Trial will best indicate what particular movements to give to the mirror.

Bands in White Light. — Two beams of light of mixed wave lengths (white light) will ordinarily produce interference phenomena only when the relative retardation is a small number of wave lengths, not over thirty. To obtain bands in the interferometer with white light, the distance of M_2 from H may not differ from that of M_1 by more than fifteen wave lengths, a quantity less than a hundredth of a millimeter. Having found the bands with sodium light, as explained above, by means

of the slow-motion screw S move the mirror M_2 so slowly that the bands may be distinctly seen as they pass across the field. Examination will show that the distinctness of the bands suffers periodic changes, there being about a thousand bands in each group. These changes result from the fact that sodium light consists of two wave lengths which differ from each other about one part in a thousand. The combination produces fluctuations analogous to beats in sound. Further, the bands in the middle of one particular group will be found more distinct than those in any other group on either side. Set the mirror in this position of greatest maximum distinctness by quick estimation; it is then nearly in the position giving equality of paths to the two beams. Place a luminous gas flame, or other source of white light, just beyond the sodium flame without disturbing the latter; regulate the intensity of the white light until the sodium bands are seen very faintly, perhaps in one portion of the field only. Observing the mirror carefully, slowly move M_2 one way or the other till the group of brilliantly colored bands due to white light is found. Care must be taken that the motion is not so rapid that the bands pass through the field unobserved; the faintly visible sodium bands permit the control of the speed.

If the colored bands are not readily found, it may be that the wrong group has been selected; therefore search in the middle of the next group on either side of the one first tried.

With experience one may find the bands in white light certainly and more expeditiously by using the handle L, instead of the slow-motion screw, for making these settings.

Another criterion for finding the colored bands is as follows. Usually the bands with monochromatic light appear more or less curved. A motion of the screw L in one direction increases this curvature, while an opposite motion decreases it till the bands appear straight, and a continued movement gives them a curvature in the opposite direction. The position where the curvature changes — that is, where the bands are straight — is the one near which the colored bands will be found. Search for them with white light as described.

The middle band of the colored group is black. It corresponds to light which has moved over paths of equal lengths, but for which interference and not reënforcement takes place because of the phase difference introduced by the reflection of one part internally and the other part externally at the silver film H. This centѣr black band is the only one that can be identified; it is therefore taken as a starting point in making measurements.

194. Circular Bands. — If the movement of the mirror M_2 required in any measurement is small, — a millimeter or less, for instance, — the straight bands described above are most convenient for observation. But when a much greater difference of path of the two beams is necessary, circular fringes are more suitable. They are produced when the mirrors M_1 and M_2 are both exactly perpendicular to the incident light. The bands are concentric circles; a motion of M_2 causes them to enlarge and disappear, while new ones continually form at the center, or else they contract and vanish at the center. The center therefore appears alternately light and dark, in accordance with the first simple explanation. As the path to M_2 becomes more nearly equal to that to M_1, the circles enlarge in size, and finally the center of the system covers the entire plate. This is therefore the position near which colored fringes may be found.

In finding the circular bands it is convenient to begin by making the path to M_2 differ from that to M_1 by a thousand or more wave lengths, as then the circles will be smaller and several may be seen at once. The inclination of the adjustable mirror is then to be changed so as to increase the curvature of the bands until they become circles, with the center in the center of the field of view. These circles may be conveniently observed with a telescope focused for distant objects.

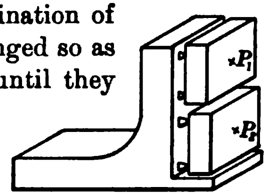


FIG. 110. INTERFERENTIAL STANDARD OF LENGTH

195. Counting Fringes. — For the purpose of laboratory exercises an optical standard of length of about 0.1 mm is desirable. This consists of a solid metal support carrying two plane-parallel plates of equal thickness, P_1 and P_2 (Fig. 110). Each plate rests against

three projecting points, with which it is kept in contact by suitable springs. The points against which P_1 rests are shorter than those for P_2 by about a tenth of a millimeter, and they are adjusted by careful grinding so that the front surfaces of the plates are accurately parallel. A cross scratched through the silvering in the center of each plate serves as an index.

The mirror M_2 of the interferometer is removed, and the standard is substituted for it. With sodium light the adjustments are made so that the fringes are seen from both plates at once. The fringes of the two sets should be of about the same width, and in the same general direction, preferably vertical.

With white light find the colored system of fringes on one plate, as P_1, and bring the black fringe exactly to the cross mark.

Illuminate with sodium light and give attention to the fringes from plate P_2. Having previously determined the direction in which the bands will be displaced when the plate P_2 is brought toward the plane in which P_1 is now set, turn the fraction-screw F to displace the bands in this direction until the center of the first dark band to come to the cross is exactly on the mark. Thus the first fraction of a band is measured. Then from this position as zero the slow-motion screw S is slowly turned, and the bands are counted as they pass the mark. It is convenient to count ten bands, then to take a new hold on the screw, to count ten more, and thus to continue until a number has been counted which is perhaps five less than the supposed number in the length being measured.

Substitute white light. If the system of colored fringes is not in sight, count five more bands with sodium light, and again look for the bands in white light. Continue the count until the colored system is found and the central black band is on the cross mark.

If the colored system is not found, either too many bands were counted before search was made with white light, or the screw may have been turned in the wrong direction.

The number of whole fringes plus the first fraction is the length required expressed in half wave lengths of the monochromatic light employed.

To give definite direction to the line of sight a telescope is convenient; but the cross wires in the eyepiece must not be used as the index for counting,—the marks on the surfaces of the plates serve better. The slow-motion screw may be extended to the telescope by means of a glass tube, connected to the screw by a short piece of rubber tube to prevent the communication of vibrations to the instrument. The count may be interrupted to rest the eye. Stopcocks for controlling the gas may be placed within reach of the observer, so that either monochromatic or white light may be used at will.

196. Monochromatic Light. *Sodium Light.*—The ordinary laboratory requirements for monochromatic light are fulfilled by placing some common salt held in a platinum wire loop or gauze basket in the edge of a Bunsen flame, where it is vaporized and rendered incandescent. The sodium light which results is not of great intensity, but it may be made of large surface. The light is made steady, and somewhat more intense, by placing an iron chimney over the flame, as indicated in Fig. 111, which shows a convenient form of sodium lamp. Instead of salt, borax may be fused into the platinum wire loop.

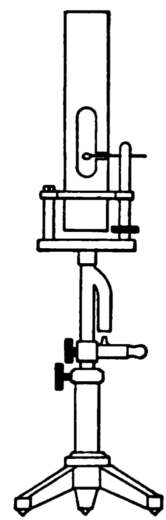


FIG. 111
SODIUM LAMP

Sodium light consists of two principal wave lengths, Rowland's values for which are

$$D_1 = 0.00005896154 \text{ cm},$$

and $$D_2 = 0.00005890182 \text{ cm}.$$

It is not possible to separate these two components of sodium light in large surface illumination, and the wave length may be taken as the mean, 0.00005893 cm.

Mercury Light.—A light which is more nearly monochromatic and at the same time of greater intensity is obtained from the mercury arc lamp. The essential part of the lamp is a ∩-shaped vacuum chamber (Fig. 112) containing mercury. By means of platinum wires the mercury in the interior is connected to two mercury cups, which receive the

terminals of an electric circuit. The quantity of mercury in the branches of the chamber may be so adjusted that the two portions are separated by a small space. By gentle shaking the two portions may be made to touch, and as they separate an electric arc is formed which vaporizes the mercury and emits a brilliant greenish-white light. The current required is of from 6 to 10 amperes, and from 24 to 32 volts. By immersing the lamp in a water bath, it can be operated for hours without interruption.

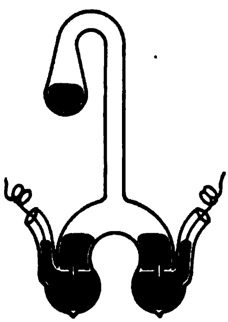


FIG. 112. MERCURY ARC LAMP

The mercury light consists of several wave lengths which, however, may be readily separated by a prism. The green portion is the most useful, and its wave length according to Fabry and Perot is 0.0000546074 cm.

Cadmium Light.—Michelson describes in *Travaux et Mémoires du Bureau International des Poids et Mesures*, Tome XI, a lamp in which metallic cadmium is vaporized with a Bunsen burner, and then rendered incandescent by the electric spark. By prismatic analysis a red light of great purity is obtained, the wave length of which is perhaps determined with greater precision than that of any other light. The wave length of this red cadmium light is 0.00006438472 cm.

CXVI. PLANE SURFACES BY OPTICAL METHODS

Test the surfaces of a prism for flatness, and the two surfaces of a glass plate for plane parallelism.

197. Plane and Plane-Parallel Surfaces.—The surfaces of prisms, of the plates used in interference experiments, some surfaces of lenses and of other optical plates are required to be optically plane. The skill of the optician is such that the surface of a first-quality plate will not deviate from a true plane by more than a tenth or a twentieth of a wave length of light. A

plane-parallel plate is one the two surfaces of which are accurately plane and parallel to each other. When two plane-parallel plates of equal thickness are required they are constructed in one piece, which is afterwards cut.

Plane Surfaces are conveniently tested only by optical comparison with a test true plane. This true plane is made by the optician by grinding three planes together until they fit, two and two, in all positions.

Such a true plane being available, it is placed upon the surface to be tested; the surface of contact is illuminated with sodium light by transmission through either plate as is convenient. Before the plates are brought into contact the two surfaces should be freed from dust and dirt, both to avoid injury to the surfaces and to permit proper contact. To light the entire surface a sodium flame is placed in the focus of a large lens, so arranged that the bundle of parallel rays produced falls on the surfaces and is then reflected to the eye. Fig. 113 shows the arrangement for testing a prism. Interference bands should appear over the entire surface. If both surfaces are plane, these bands are straight, parallel, and equidistant. If the plates are pressed closely together, the bands will be far apart and a very small departure from planeness may be discovered. A width of bands of from five to ten millimeters is most useful. After the plates have been placed in contact they must be left till they have come to a uniform temperature throughout before the testing can be relied upon.

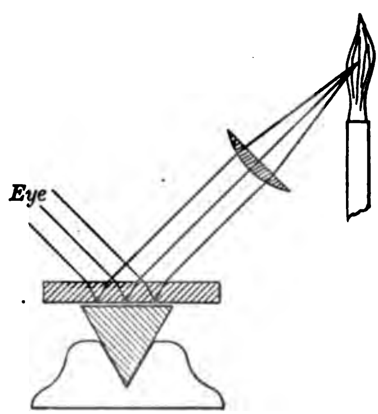


FIG. 113. TESTING A PLANE SURFACE

The form of the surface being tested may be inferred from the interference pattern exhibited, since a band traces the locus of all points in the unknown surface which are at a constant

distance from the true plane. The bands are analogous to the contour lines on a topographic map.

Plane-Parallel Surfaces. — The parallelism of the plane surfaces of a glass plate may be roughly tested by observing any well-defined distant object as seen reflected from the plate at an angle of about 45°. If the two surfaces are inclined, two images will be seen, one of which revolves about the other when the plate is turned in its own plane. If the surfaces are parallel there will be but one image, which should be sharply defined.

If the reflected image is observed with a telescope, the test becomes much more delicate because of the magnification, and it is then sufficiently sensitive for the usual requirements.

A spectrometer is most convenient for this test. Set the telescope at right angles to the collimator, and place the plate to be tested so that the illuminated slit is reflected in the telescope. A more suitable object is obtained by opening the slit wide and crossing fine wires over the opening. Then, as the plate is rotated in its own plane, the image should remain single and sharply defined.

CXVII. SILVERING GLASS BY CHEMICAL METHODS

(a) Make a thick-film mirror by Brashear's process.
(b) Make a half-silvered mirror by the Rochelle-Salts process.

198. Brashear's Process for Silvering Glass. — Mirrors for optical experiments are often made by preparing a glass surface of the desired form, upon which is then deposited a uniform film of silver. The following method, by which a hard, brilliant film is deposited from an ammoniacal solution of nitrate of silver by an organic reducing agent, is usually the most convenient and satisfactory. Two solutions are required: one, the reducing solution, should be prepared at least a week before it is used, and it may be made in large quantity and kept in stock with advantage; the other solution is to be prepared when used.

REDUCING SOLUTION

Distilled water	700 ccm
Pure sugar (loaf, granulated, or rock candy) . . .	80 g
When dissolved add	
Alcohol	175 ccm
Strong nitric acid (sp. gr. 1.42)	3 ccm
Add water to make	1000 ccm

For silvering, the mirror may rest face up on the bottom of a suitable dish; it may stand on edge, or be supported in any manner, face downward, dipping into the upper part of the solution. In the latter case the mirror may be fastened with wax to a stick laid across the dish, or it may be supported on glass feet or on paraffined wood wedges. Dr. Brashear recommends that the mirror, if round, form the bottom of the silvering dish, which is completed by wrapping a strip of paraffined paper around the edge of the mirror, this being held in place by rubber bands or fastened with several wrappings of cord.

Having selected a dish and support for the mirror, measure with water the quantity of solution that will be required to make a layer a centimeter or two thick over the surface to be silvered. For each 150 ccm of final solution, 1 g of silver nitrate and 0.5 g of caustic potash (purified by alcohol) will be required. Dissolve the silver and potash separately, using quantities of water of the proportion of 100 ccm to 1 g of the solution. Ordinary graduates or flasks are the most convenient form of vessel in which to mix the solutions. Into the silver nitrate solution pour a few drops of dilute aqua ammonia. The solution will turn to a dark brown color; add ammonia little by little till the precipitate is nearly but not quite redissolved. Now add the potash solution, when a precipitate will again be formed. This is to be nearly, but not entirely, redissolved by the addition of more ammonia, a few drops being sufficient this time. After the ammonia has been added, shake or stir the solution well and wait a minute or two to be certain that it does not entirely clear. If by chance too much ammonia has been used, a little silver nitrate is to be dissolved and added, a few drops at a time, till a permanent precipitate is formed. *This excess of silver must*

be present, the solution showing a decided brown tint. The solution may be filtered, though usually this is not necessary.

A quantity of reducing solution equal to about a twenty-fifth part of the solution just prepared is measured out. The mirror, having been properly cleaned and rinsed with distilled water, is placed in position. The reducing solution is poured into the silver and potash solution, and mixed by a quick shaking of the graduate or stirring with a glass rod; the whole is then poured into the dish. If the mirror is immersed face down, care is necessary to remove air bubbles; the mirror may well be immersed after the solution is in, being dipped in at one side first. If the mirror is at the bottom of the dish, after cleaning it is covered with a thin layer of water, and the prepared solutions are poured into the dish without further trouble. In the latter case the dish must be rocked during the time of deposition.

The solution soon turns to a black color, which in a few minutes will turn to a brown; and when it becomes a light gray and the precipitate is flocculent, which may be in ten or fifteen minutes, the operation is at an end. If the mirror is allowed to remain in the solution too long, the surface will have a bleached appearance, which polishing will hardly remove. Remove the mirror, rinse with water, and carefully wipe off the sediment with a tuft of absorbent cotton. It is then set on edge to dry; a rinsing with alcohol will facilitate the drying, or all water may be safely taken up by pressing clean blotting paper over the surface.

When dry the surface may be polished, if necessary, with a small pad of chamois leather stuffed with cotton, on which is spread a little rouge. Small, circular strokes of the pad, with light pressure, will soon bring out the deep luster of the silver.

A uniform temperature of the bath and the glass, of about 20° is essential to success.

Since fulminating silver is liable to be produced by the action of ammonia on silver oxide, especially in a warm room, all solutions should be thrown away as soon as the silvering operation is completed. The used solutions may be poured into a large jar, in which is thrown some common salt; this causes the silver

to be precipitated as the chloride, and about ninety per cent. of the original silver may be recovered.

199. Rochelle-Salts Process for Silvering Glass. — For depositing the uniform thin film of silver required on the half-silvered glass of the interferometer, the following method is more suitable than the one described above, as the silver is deposited more slowly. If a thick film is desired, two or more successive deposits may be made, each of which may require an hour's time.

Dissolve 5 g of silver nitrate in 300 ccm of distilled water, and add dilute aqua ammonia until the precipitate formed is nearly, *but not entirely*, redissolved in the manner explained in the preceding method. Filter the solution and add water to make 500 ccm.

Dissolve 1 g of silver nitrate in a small quantity of water, and pour into about half a liter of boiling water; dissolve 0.83 g of Rochelle salts in a small quantity of water, and add to the boiling solution. Continue the boiling for half an hour, till the gray precipitate collects as a powder in the bottom of the flask. Filter hot, and add water to make 500 ccm.

These solutions may be kept in the dark for a month or two.

For silvering, equal volumes of the two solutions are mixed, and the glass is supported in the mixture in whatever fashion is convenient. Various methods are mentioned in the preceding article. The thickest possible deposit may require an hour's time. A second deposit may be made upon the first if necessary to secure the desired thickness. The drying and polishing may be carried out as described above.

A half-silvered film will be produced in about a minute; only experience can determine when the proper thickness has been secured. The glass appears as though it were very lightly smoked. A film that reflects a little more than half the light incident at 45° is desirable for interferometer use. A simple method of testing is to look at two similar gas flames, one seen through the film and the other seen reflected by it. It is well to silver at once all four surfaces of the two plane-parallel plates of the interferometer and to select for use that film which is of the proper and most uniform thickness.

200. Cleaning Optical Surfaces for Silvering. — Probably the most important part of the silvering process is the proper cleaning of the surface to be silvered.

The surface is thoroughly cleaned of grease or other organic matter by the usual methods (Art. 91), using alcohol or chromic acid. Then it should be carefully cleaned with strong nitric acid, the whole surface being firmly rubbed with pure cotton tied to a rod of wood or glass. Care should be taken not to injure the surface. Rinse with water, and then wash the surface thoroughly with a strong solution of caustic potash, rubbing with a cotton brush as before. Finally rinse with distilled water, and keep the surface wet until it is placed in the silvering solution. If the distilled water wets the whole surface uniformly, the cleaning may be sufficient; if it does not wet uniformly, the operations must be repeated. The fingers should not touch the edges of the glass during the latter cleaning operations, as a layer of organic matter is apt to spread over the surface and render the silvering uneven.

Dr. Brashear recommends that the surface, after the washings described above, be rubbed with prepared chalk on a cotton wad until it is thoroughly dry and clean. It may then be put into the silvering solution at one's convenience.

The following method is sometimes convenient for holding a mirror while cleaning and immersing it. A funnel having a ground edge is placed against the back of the mirror with a ring of sheet rubber between them. Connect the stem of the funnel to an air pump with rubber tubing. A few strokes of the pump will firmly attach the funnel, which will serve as a safe and clean handle.

CHAPTER XIX

THE SPECTROSCOPE AND POLARIMETER

CXVIII. CHEMICAL COMPOSITION WITH THE SPECTROSCOPE

(a) Map the principal lines in the solar spectrum, and calibrate the scale of the spectroscope. Find the wave lengths of the lines, and draw a map of the spectrum of a solid and of a gas.
(b) Identify several unknown substances by comparing their spectra with the spectra of known elements.

201. Spectrum Analysis.—A spectroscope usually differs from the spectrometer (Art. 171) in that it has fewer adjustments, and instead of a divided circle it has a linear scale made visible in the field of the view telescope. A spectrometer may always be used as a spectroscope. The dispersion part of a spectroscope may be one or several prisms, or a diffraction grating. Fig. 114 shows the relations of the essential parts of a two-prism spectroscope.

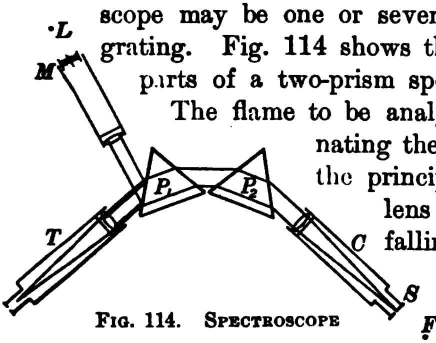


FIG. 114. SPECTROSCOPE

The flame to be analyzed is placed at F, illuminating the slit S. The slit must be in the principal focus of the collimating lens C, in order that the rays falling on the prism P_2 shall be parallel. It may have been permanently placed by the maker, or a mark on the draw tube may indicate its correct position; otherwise it must be adjusted as follows. Remove the telescope T and focus it on a distant object; replace it so as to view the slit through the collimator, the prisms being removed; focus the slit by means of its draw tube till it is seen distinctly in the telescope. (See Art. 172.)

Adjust the prisms, P_1, P_2, etc., to the position of minimum deviation, as explained in Art. 179, placing first one and then another till all are properly arranged.

Illuminate the scale *M* by a light at *L*, just bright enough to render it visible. Cause the scale image to be reflected from the prism face nearest the telescope so that it appears in the field of view crossing the whole spectrum. Focus it for distinctness, and so that there is no parallax displacement with regard to the slit image, when the eye is moved sideways.

The relative positions of the lines of the spectrum of a given substance produced by a prism vary with the composition of the material of the prism. In order that one prismatic spectrum may be compared with that from a different prism, an absolute scale of reference must be adopted. The most useful of such scales is one of actual wave lengths. To find the wave-length values of the arbitrary spectroscope scale a standard spectrum must be observed, and the relation between the positions and known wave lengths of various lines may be plotted on coördinate paper, from which the wave length of any observed line may be found. The solar spectrum is a convenient standard.

If the spectrum is formed by a diffraction grating, it is a *normal* spectrum; that is, the dispersion is proportional to the wave length. The relative positions of the lines in the spectra of any given substance formed by different gratings are always identical (Art. 191).

If a spectrometer is employed, all the adjustments described for determining index of refraction should be made. The spectral lines may be located by reference to the divided circle, the wave-length values being found by observing a standard spectrum, as described above.

Arrange a heliostat (Art. 202) to throw sunlight upon the slit of the spectroscope. If this is not convenient, the light from a bright sky will serve. Alter the width of the slit until the transverse (Fraunhofer) lines are distinct. If longitudinal lines are visible, they are due to dust or irregularities in the slit. To remove dust draw a fine sliver of soft wood through the slit.

If this does not remove the lines, widening the slit may be advantageous, though it may render the spectrum too bright or the spectral lines too wide.

Identify and locate on the scale the principal Fraunhofer lines, the *B*, *C*, *D*, *E*, *b*, *F*, *G*, *H*, and *K* lines, referring to a map of the spectrum if necessary. Using the known wave lengths of the lines (Table 25), plot a curve showing the wave-length values of the scale.

The spectra of all the obtainable elements have been observed, and the wave lengths of the principal lines have been tabulated. Spectrum analysis in its simplest form consists in observations of the lines given by the unknown substance, which by reference to the tables enable the identification of the substance. Besides position, features which aid in the identification are brightness, width, color, and sharpness. Full descriptions of the methods, which may become very complex, will be found in the references.

To study the spectrum of a solid, such as the chlorides of barium, calcium, lithium, sodium, strontium, thallium, etc., the substance may be vaporized and rendered incandescent in the Bunsen flame. Place a little of the material on a platinum wire, and hold it just within the surface of the flame, near the middle of its length. The temperature of the flame, and the length of time the substance has been in it, influence the appearance of certain lines.

Less volatile substances may be rendered incandescent by being placed in the crater of an electric arc between carbons, or an arc may be formed between electrodes of the substance being studied.

The Bunsen flame and the carbon arc both give spectral lines, which must be identified and differentiated from the lines of the material being analyzed. The sodium line is almost universally present in flame spectra.

Many lines are faint, and to observe them all outside light must be excluded, by a screen behind the source, by covering the prisms, and by covering the observer's head if necessary. The slit must be widened when searching for faint lines.

To observe the spectrum of a gas, the gas is usually placed in a glass tube having platinum electrodes, between which the discharge of an induction coil takes place, rendering the gas incandescent.

REFERENCES. — *Kohlrausch*, Physical Measurements, pp. 169–173; *Schuster and Lees*, Practical Physics, pp. 178–193; *Scheiner*, Astronomical Spectroscopy; *Landauer*, Spectrum Analysis.

202. The Heliostat. — In experimental work it is often desired to project a beam of sunlight in a direction which shall remain fixed throughout the day. This may be accomplished by means of a heliostat, the essential part of which is a mirror provided with various adjustments, moved by clockwork in such a manner as to neutralize the effects of the earth's rotation on its axis. In some forms of heliostat the beam of light is thrown in a direction parallel to the earth's axis, the beam being received on a second mirror, which then reflects it in any desired direction. In other forms the beam is sent in any given direction at the first reflection. The first form is the simpler and more certain in action, its disadvantage being one additional reflection.

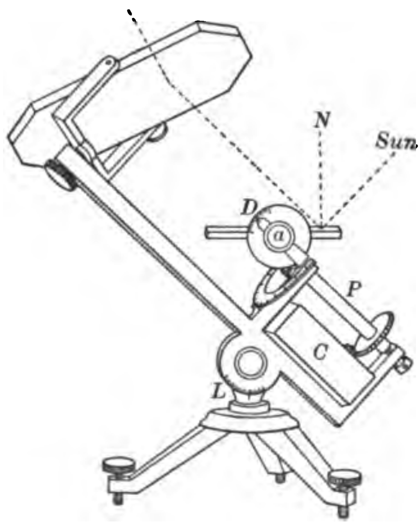


FIG. 115. HELIOSTAT

A two-mirror heliostat is shown in Fig. 115. The polar axis P is to be placed in the plane of the meridian and elevated at an angle equal to the latitude of the station. The graduated arc L indicates the proper latitude setting. The axis is thus made parallel to the earth's axis. A clock, C, is arranged to rotate this axis from east to west at a rate corresponding to the diurnal motion of the sun.

At the upper end of the polar axis the mirror is held so as to be movable about a line through *a*, which constitutes the declination axis. The normal to the surface of the mirror, *N*, should bisect the angle between the direction of the sun and the polar axis. When the sun is on the equator the normal makes an angle of 45° with the polar axis, and the declination circle *D* reads 0°. As the sun moves north or south the mirror must be inclined from this position by an amount equal to half the sun's declination. The declination circle is usually graduated with half-degree spaces numbered as whole degrees, and the setting is therefore equal to the sun's declination.

Attached to the polar axis is an hour circle, which reads 0 when the declination axis is horizontal. In starting the instrument this circle is to be set to the apparent solar time (standard time reduced to local mean time, and then corrected for the equation of time).

Approximate values of the sun's declination and the equation of time, sufficient for heliostat settings, are given in Table 22, in the Appendix. More complete tables will be found in the Nautical Almanac.

Other forms of heliostats require similar settings. The manner of making these will usually be evident from inspection.

REFERENCE. — *Leiss*, Die optischen Instrumente, pp. 284–305.

CXIX. OPTICAL ACTIVITY WITH THE POLARIMETER

Determine the specific rotation of cane sugar, and find the percentage of cane sugar in an unknown solution.

203. Rotatory Polarization. — Certain solids, liquids, vapors, and solutions have the property, when plane-polarized light is passed through them, of rotating the plane of polarization. This phenomenon is called optical activity. Some substances rotate the plane to the right and others to the left; that rotation which is related to the propagation of light, as the rotation and translation in a right-handed screw, is said to be right-handed. The amount of rotation for any active substance varies

directly as the thickness of the layer through which the light passes, and varies with the wave length of the light. For a liquid it also varies with the density or the concentration of the solution and with the temperature. The rotatory power of fluids is much less than that of solids. About thirty crystals are known to possess optical activity, while over seven hundred liquids and solutions belonging to all groups of organic compounds also have this property. The specific rotation of a liquid or substance in solution is the rotation in degrees of arc, per unit length of path of the light, per unit density of the substance. If r is the rotation produced when the light passes through l centimeters of length of the fluid, and d is its density, the specific rotation is

$$[a] = \frac{r}{ld}.$$

If the active substance is mixed with an inactive liquid, d in the above formula represents the number of grams of the substance in one cubic centimeter of the solution. A statement of the specific rotation of a substance should include the temperature, the kind of light used, as the sodium D line, and the direction of rotation, + for right-handed; as

$$\text{turpentine } [a]_D^{20^\circ} = -3^\circ.701.$$

The specific rotation multiplied by the molecular weight gives the molecular rotation.

Founded upon these principles there are methods of analysis useful for distinguishing one substance from another, or for determining the nature or strength of a solution. Rotatory polarization is employed as an aid in the analysis of various carbon compounds, such as gum resins, camphor, essential oils, alkaloids, dyestuffs, and many others; and it is extensively used in sugar analysis to determine the purity of various samples and the proper concentration of syrups in manufacturing.

204. The Polarimeter. — An instrument designed to measure the rotatory polarization of a substance is a polarimeter (Fig. 116). For general uses the rotation is measured in degrees

of arc. Often the instrument is arranged for sugar analysis, with a scale showing the percentage of sugar in solution as compared with a standard solution; it is then a *saccharimeter.*

The field of view of a polariscope consisting of two Nicol prisms is dark when the prisms are crossed. An optically active substance placed in the path of the light between the prisms rotates the plane of the polarized light and causes the field to appear brighter. That rotation of either prism necessary to restore darkness measures the rotation of the substance. To determine the exact position of the prism producing this effect is difficult, and various devices have been made to secure increased sensitiveness. Some of the more common forms are mentioned below.

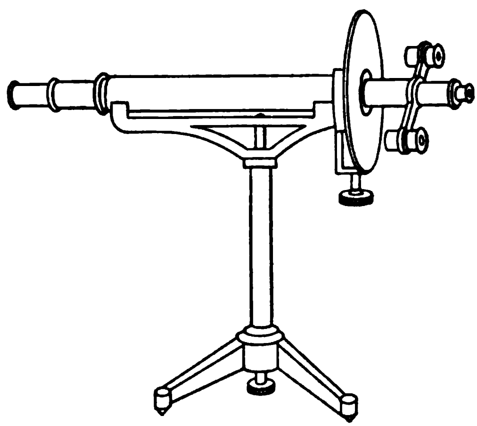


FIG. 116. POLARIMETER

Half Shade (Laurent). — The field of view embraces a plate one half of which is glass while the other is quartz cut parallel to the axis and of such thickness as to introduce a retardation of half a wave length of the particular color of light to be used (commonly sodium) between the ordinary and extraordinary rays. This causes the two halves of the field to appear unequally illuminated, except when the principal plane of the analyzing nicol is parallel to the axis of the quartz; then the two halves appear of uniform brightness. This brightness of the field and therefore the sensitiveness may be regulated by changing the position of the polarizer with respect to the quartz plate. The sensitive position is that of the eyepiece nicol producing uniformity of field illumination. Monochromatic (sodium) light is necessary.

Interference Bands (Wild). — Two plates of calcite or quartz are cut at an angle of 45° to the axis, and placed together with their principal sections at right angles. This plate is inserted in the path of the rays near the eyepiece nicol, and gives in the field of view a series of bands alternately light and dark. The polarizing nicol is rotated until these bands disappear in the center of the field, a sensitive condition. The sensitiveness,

FIG. 117. WILD'S POLARISTROBOMETER

depending upon the brightness of the light used, may be adjusted by changing the position of the eyepiece nicol with respect to the interference plate. Either monochromatic or white light may be used, though the first is better, as the entire field is then covered by the bands.

Fig. 117 represents the arrangement of the optical parts of Wild's polaristrobometer. *PN* is the polarizing nicol, *S* the active substance, *I* the interference plate, *AN* the analyzing nicol, *X* the cross wires, and *O* and *E* the observing lenses.

Sensitive Tint (Soleil). — Two quartz plates of equal thickness but of opposite rotations are placed side by side in front of the polarizing nicol. The sensitive condition is secured by rotating the eyepiece nicol until it is parallel to the polarizing nicol, in which case the two halves of the field appear equally bright, or of the same color, a grayish violet, called the tint of passage or sensitive tint. A slight displacement of the nicol either way causes one half to appear red and the other blue, with white light, or to appear unequally bright with monochromatic light.

With each of these devices the critical condition can be secured by rotating one of the Nicol prisms, and the rotation produced by an active substance put in the path of the light between the nicols may be determined by the rotation of the nicol required to compensate that of the substance. There are always two positions of the nicol, 180° apart, satisfying this

condition, and sometimes there are four positions 90° apart. To determine whether the rotation of an active substance is less or greater than 90° or 180°, or whether it is right- or left-handed, it may be necessary to use two different lengths of the substance, or two different strengths of solution. For example, an observed rotation of 30° to the right may actually be one of 60° to the left. A second observation with half the length of substance would in the latter case show the sensitive condition at a point 30° to the left and at 60° to the right; while if the first observation corresponded to an actual rotation to the right of 30°, the second observation would show only 15° to the right and 75° to the left.

Often, instead of measuring the angle of rotation produced by the active substance by rotating one of the Nicol prisms an equal amount in the opposite direction, as described, the rotation of the plane of polarization by the substance is neutralized by the opposite rotation produced by a quartz compensator. This consists of two quartz wedges having left-hand rotation, one of which can slide over the other, varying their combined thickness, together with a right-handed quartz plate. Any desired rotation either right or left can be introduced, according as the wedges combined are thicker or thinner than the single plate. The thickness of wedge required to secure the sensitive condition is measured by an arbitrary scale, which usually indicates percentage of pure sugar in a solution made

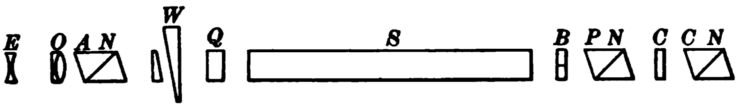


FIG. 118. QUARTZ WEDGE SACCHARIMETER

according to rule. In instruments of German manufacture one division of the scale equals 0°.346, and for sugar analysis 26.05 g of the sample is dissolved in 100 ccm of solution. The scale reading then gives at once the percentage of pure sugar. In instruments of French manufacture one scale division equals 0°.217, and the standard solution contains 16.35 g per 100 ccm of solution.

Fig. 118 shows the optical parts of a saccharimeter. PN is the polarizing nicol, B the quartz plate, S the active substance, Q the right-handed quartz plate, W the left-handed quartz wedges, AN the analyzing nicol, and O and E are the observing lenses, focused on B. To neutralize the effect of colored solutions, or colored light, or to secure any desired tint of field, a color regulator consisting of a nicol, CN, and a quartz plate, C, are often provided. The rotation of these alters the color of the field.

205. Saccharimetry. — Granulated sugar may be considered pure cane sugar, and its specific rotation is determined in accordance with the definition of Art. 203. For cane sugar the specific rotation is nearly independent of temperature.

Dissolve 20 g of dry granulated sugar in about 80 ccm of warm water, contained conveniently in a 100 ccm flask. When the sugar is thoroughly dissolved, cool the mixture to 20° and add water to make 100 ccm of solution.

Fill a 20 cm solution tube with distilled water, place it in the saccharimeter, and make the critical adjustments as described above, either for equal tint, equal shade, or for the disappearance of the interference bands. Make a number of settings, ten for example, and use the average as the zero reading of the instrument. Fill the tube with the prepared solution of pure sugar, and make the critical adjustments again, taking the average of the ten readings. Then if there are n g of sugar in v ccm of solution, and the tube has a length of l centimeters, and the observed rotation in degrees of arc is r, the specific rotation is

$$[a] = \frac{rv}{nl}.$$

Using a tube 10 cm long, repeat the observations and find the specific rotation.

Dilute the solution still remaining to half strength, by adding an equal volume of water, and with the 20 cm tube again determine the specific rotation.

With the value of the specific rotation thus found, determine the amount of cane sugar in an unknown solution provided for testing.

(When the work is finished, wash all tubes and dishes used for the sugar solutions.)

The method above described is of general application in the measurement of the rotatory power of any active substance. The treatment for clarifying colored solutions, and for the determination of sugar in the presence of invert sugar, or other active substances, may be found in treatises on chemical analysis.

References. — *Landolt*, Optical Rotating Power; *Kohlrausch*, Physical Measurements, pp. 190–198; *Preston*, Theory of Light, pp. 449–457.

Part VI — Electricity and Magnetism

CHAPTER XX

RESISTANCE

CXXV. RESISTANCE BY THE WHEATSTONE'S BRIDGE

Determine the resistance of a high- and of a low-voltage incandescent lamp (cold); also the resistances of two wire coils, separately, in series, and in parallel. Make three measures of each.

206. Electrical Measurements and Ohm's Law. — Electrical measurements consist of the direct or indirect determinations of resistances, current strengths, electromotive forces, quantities of electricity, and magnetic quantities. Of these classes of electrical measurements the first three are much the more common in technical work. The principles underlying many of the various methods are contained in what is known as Ohm's Law, and though this law is stated in very simple fashion, its wide application renders it worthy of the student's careful study.

If a specified conductor is carrying a steady current, there are three factors to which attention must be given in order that the electrical conditions may be understood. These are the difference of potential, E, between the ends of the conductor, the strength of the current, I, and the resistance, R. Ohm's Law expresses the relations between these quantities, that the product of resistance and current strength is equal to the

difference of potential. It is also often stated by the following equation:

$$I = \frac{E}{R}.$$

The units in which these quantities are measured, the ohm, the ampere, and the volt, are defined in Arts. 212, 244, and 249 respectively. Other electrical and magnetic units are defined in Arts. 262, 269, and 272.

207. Wheatstone's Bridge. — Wheatstone's bridge is a device by which the principles involved in Ohm's Law may be applied to a variety of electrical measurements. Before describing the methods specifically, a general explanation will be given.

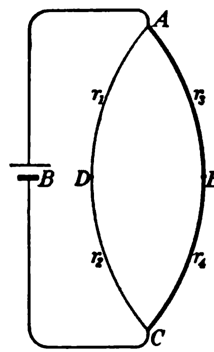


FIG. 119. BRANCHED CIRCUIT

Let A and C (Fig. 119) be two points connected by two conductors, ADC and AEC, and B a source of electromotive force producing a difference of potential between A and C, that at A being the higher. There will result a steady flow of current from A to C and through the battery; a part of the current, I_D, will go from A to C through D; and a part, I_E, from A to C through E. Represent the resistances from A to D, D to C, A to E, and E to C, by r_1, r_2, r_3, and r_4; and the potential difference between two points by V with proper subscripts. The same current flowing from A to D must also flow from D to C; therefore by Ohm's Law,

$$I_D = \frac{V_{AD}}{r_1} = \frac{V_{DC}}{r_2},$$

and similarly,

$$I_E = \frac{V_{AE}}{r_3} = \frac{V_{EC}}{r_4};$$

from which $V_{AD} : V_{DC} = r_1 : r_2$, and $V_{AE} : V_{EC} = r_3 : r_4$. The potential of any point on the branch D is less than the potential of A and greater than the potential of C, and likewise for any

point on the branch E. Therefore, whatever may be the potential of a point on the D branch, there is *always some* point on the E branch having the *same* potential. Let D and E be two points having the same potential. Then

$$V_{AD} = V_{AE}, \text{ and } V_{DC} = V_{EC};$$

and
$$r_1 : r_2 = r_3 : r_4,$$
or
$$r_1 : r_3 = r_2 : r_4,$$
and
$$r_1 r_4 = r_2 r_3.$$

The last equation, stated in words, is that the products of opposite, or alternate, resistances are equal.

From these equations, if the values of any three of the resistances, r_1, r_2, r_3, and r_4, are known, the fourth is determined; or, if only the ratio of any two adjacent resistances, together with the value of one of the other resistances, is known, the fourth is determined.

There are two distinct methods of adjustment for locating the point E, whose potential is the same as that of D, as required in the above relations. The two forms of apparatus for this purpose, the slide-wire-meter bridge and the post-office-box bridge, are described in Arts. 208 and 214.

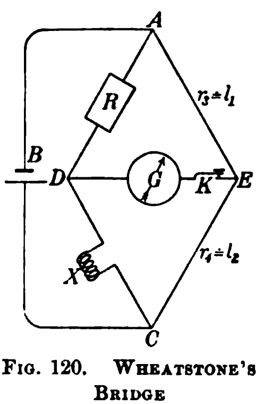


Fig. 120. Wheatstone's Bridge

208. The Slide-Wire-Meter Bridge. — The simplest form of the Wheatstone's bridge is the slide-wire-meter bridge. The conductor represented by AEC (Fig. 120) is made of a uniform wire one meter long, and the point E is determined by a sliding contact which moves over a millimeter scale. The lengths of the portions of the wire, AE and EC, can be read directly. A resistance box, R, of known value, corresponds to r_1 (Art. 207), while the unknown resistance, X, replaces r_2. Any convenient value being given to R, the potential of D becomes fixed. To find the point E, having the same potential, a "bridge" conductor containing a galvanometer is connected to D and to the slider.

If the bridge joins two points whose potentials are equal, evidently no current will flow through the galvanometer. Usually, however, when the key, K, of the bridge is closed, there will be a deflection of the galvanometer needle caused by a flow of current from D to E, or in the opposite direction. But it will always be possible to find a point of contact by moving the slider nearer to A or to C, such that there shall be no deflection of the galvanometer; then the bridge is said to be balanced, and the potential at E is the same as at D. The resistances of the two portions of the meter wire, AE and EC, are considered as proportional to their lengths, l_1 and l_2. Therefore

$$R : X = l_1 : l_2.$$

With this form of apparatus it is desirable to have R and X approximately equal, since in this case the contact will be near the middle of the meter wire, and errors of reading and of the wire will have a minimum effect.

Often the index on the slider is not over the contact, or the resistances of the connections are unsymmetrical, or the wire is of uneven size; the result determined as above described may then be slightly in error. A second determination should therefore be made after interchanging R and X; the mean of the two values will be nearly free from these errors.

209. The Galvanometer. — The most common instrument for measuring or detecting electrical currents is the galvanometer, which is founded on Oersted's discovery of the mutual action between a current and a magnet. In the galvanometer a circuit is arranged with its plane parallel to the force of a magnet; when a current passes, there is a tendency for the plane of the coil and the lines of magnetic force to set perpendicular to each other. Either one of these parts, the circuit or the magnet, may be fixed in position, the other being delicately suspended so as to move under the force mentioned. Thus there are two types of galvanometers, while there are many varieties of each.

In one type a small magnet is freely suspended and allowed to come to rest in the magnetic meridian; coils of wire are placed

bell or ∩-shaped magnet has its poles at N and S, inside the copper mass A; C_1 and C_2 are the coils of wire, and M is the mirror attached to the hook by which the magnet is suspended.

A galvanometer with a suspended coil is made dead-beat by winding the wire on a metal frame, or by surrounding it with a metal tube in which the induced currents are generated by the motion in the magnetic field. Fig. 124 shows the arrangement of the parts of this type, C being the coil suspended by the ribbons of metal, R_1 and R_2; N and S are the poles of the permanent magnets, and M the mirror for scale reading.

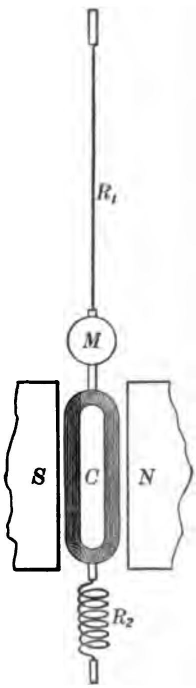


FIG. 124. PARTS OF D'ARSONVAL GALVANOMETER

The suspended part of a galvanometer sometimes consists in effect of two magnets rigidly fastened together with their poles oppositely directed. Thus the earth's effect upon the system is nearly neutralized. The magnets are suspended inside the coils in such a manner that the effect of the current is twice as great as upon a single magnet. This arrangement constitutes an *astatic* needle, and makes the most sensitive galvanometer that has been devised. This extreme sensitiveness renders the instrument somewhat troublesome to use. Fig. 125 shows the form of an astatic galvanometer with two sets of coils.

210. Adjusting a Galvanometer. — Before a galvanometer is moved it should be ascertained that the weight of the needle is taken from the suspension, to prevent breaking the latter. The manner of doing this will be evident from the construction. When the galvanometer is in position, on a pier or other suitable support, if it is of the suspended magnet type, the whole instrument should be turned on its base so that the plane of the coils is approximately in the magnetic meridian. Simple inspection will determine this sufficiently, unless the deflections of the instrument are to be used for measurement.

When greater care is necessary in any of the exercises the methods are mentioned (Arts. 216 and 253).

The instrument should be leveled. A level on the base may indicate this condition, but it often happens that the level is itself out of adjustment. In any case the needle usually serves as a better indicator of the proper position of the galvanometer. When the needle is released it should swing freely and without rubbing, and after a slight deflection it should return to the starting point.

After the needle has taken its position in the meridian it may be found that the mirror faces in an inconvenient direction. In some instruments it is possible, by carefully holding the needle, to turn the mirror so that it faces in any direction desired. It should be so turned that when a scale is placed in front of it the scale will be well illuminated.

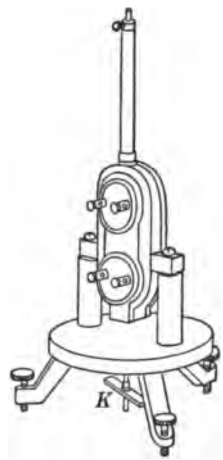


FIG. 125. ASTATIC BALLISTIC GALVANOMETER

A *control magnet* may be advantageous in adjusting a galvanometer. This may be any permanent magnet, placed near the instrument, or preferably attached to it in some convenient manner, as shown at *K*, Figs. 121 and 125. When a control magnet is used it is not necessary to place the coils in the magnetic meridian, as the magnet may be so placed as to bring the axis of the needle into the plane of the coils, in whatever direction this may be. The control magnet may increase the sensitiveness of the instrument by partially neutralizing the earth's field. A consideration of the sensitiveness of a galvanometer is given in Art. 247.

When the galvanometer is of the suspended coil type its direction with respect to the meridian is not important, and it may be set facing in the most convenient direction, and leveled.

The replacing of a broken galvanometer suspension is such a troublesome operation that especial care should be taken to avoid breaking one.

211. Telescope, or Lamp, and Scale. — A galvanometer deflection may be observed by means of a pointer attached to the needle; but with a sensitive instrument it is usual to attach a light mirror to the needle and to observe the deflections with a telescope and scale, or with a lamp and scale. The adjustment and use of a telescope and scale are sufficiently described in Arts. 33, 253, and 264.

A light illuminating a slit or a wire may replace the telescope. A lens is placed between the slit and mirror, and the various parts are adjusted so that the image of the slit, reflected by the mirror, is focused on the scale. The movements of this image indicate the needle deflections.

212. Standard International Ohm; Resistances and Rheostats. — The international ohm is defined as the resistance of a column of mercury, at 0°, 106.3 cm long, and 1 square mm area of cross section. The mass of this quantity of mercury is 14.4521 g.

Standard resistances for electrical measurements are commonly made of wire. Various alloys have been devised such that the specific resistance is high while the temperature coefficient and thermo-electric power are low, and such that the resistance shall not change with time and use. *Manganin* is the most suitable alloy; it consists of 0.84 copper, 0.12 manganese, and 0.04 nickel. Other alloys are constantan, platinoid, and German silver.

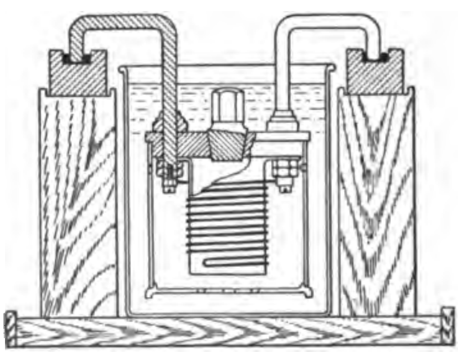


Fig. 126. Standard Resistance in Oil Bath

To avoid the self-induction of a simple helix, the wire for a resistance coil is first doubled at its middle, and the two wires are wound side by side in such fashion that the current will flow through the two in opposite directions. In coils of many turns this method gives

considerable capacity. This, as well as the self-induction, may be avoided by winding a single wire, with frequent changes in the direction of the winding, so that in the end there are as many turns in one direction as in the other. This method is necessary only for coils of more than 400 ohms resistance.

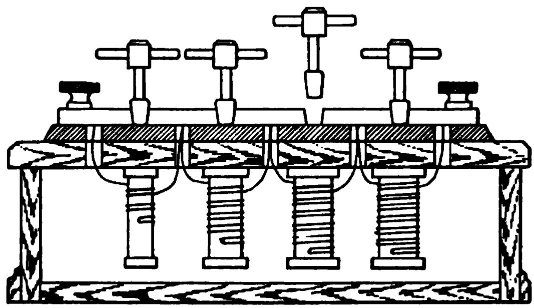


FIG. 127. RESISTANCE BOX

Standards of single resistance are usually so constructed that they may be placed in an oil bath to secure a definite and known temperature. Connections are best made by means of mercury cups, into which the terminals dip. Fig. 126 shows the construction of such a standard.

Sets of resistances, mounted to permit various combinations, are usually inclosed in wooden boxes, and are called *resistance boxes*. On the top of the box are rows of metal blocks, and between these are fitted taper plugs (Fig. 127). The coils are so joined to the blocks that when the plugs are all removed a current connected to the terminals of the box must pass through all the coils in succession, and the resistance is equal to their sum. By the insertion of a plug any coil may be shunted by a resistance so small that, for ordinary measurements, the resistance of the plugged part may be considered 0. If all the plugs are in place, the total resistance is 0. With this arrangement any combination of resistances may be secured.

The actual resistance through a plug, when it is in good order, with clean and well-fitting surfaces, is from 0.00005 to 0.0001 ohm.

The plugs are to be inserted with a firm pressure and a slight twisting movement, but care must be taken that they are not forced into the taper holes so tightly as to injure the apparatus. Plugs belonging to different boxes should never be interchanged. Sometimes the insertion of a plug warmed by contact with the hand may produce thermo-electric currents. This is avoided by the use of a nonconducting handle.

Resistance boxes are adjusted for a temperature of 20°; if the temperature differs much from this, a temperature correction may be necessary. The temperature coefficient of manganin wire may vary from $+0.00001$ to $+0.00004$.

Resistance boxes should never be used to control large currents; the heat developed may ruin the insulation or even melt the wire.

A variable resistance, not standardized, is often useful for regulating a current. Such a *rheostat* may consist of coils of wire with suitable switches, or wire arranged so that various lengths of it may be used. A pile of short carbon rods, through which the current passes, has a resistance which varies with the contact pressure of the rods against each other. By a screw press this pressure may be varied, and thus the current strength may be continuously altered through a small range. If such a rheostat is inserted in addition to a coil rheostat, it will be possible to adjust a current exactly to a desired value.

213. Keys. — A great variety of keys and connectors are required in arranging electrical apparatus. Single keys and switches need not be described. For work with the Wheatstone's bridge, and other instruments, a *double successive contact key* is useful. There are four contact points, b_1, b_2, g_1, and g_2 (Fig. 128), which are connected with the binding posts B_1, B_2, G_1, and G_2 respectively. Between b_2 and g_1 is an insulating button.

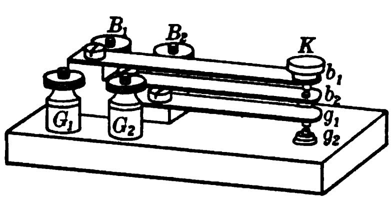


Fig. 128. Double Successive Contact Key

The battery circuit is connected to B_1 and B_2, and the galvanometer circuit to G_1 and G_2. When the knob K is pressed the

battery circuit is first closed, and then the galvanometer circuit is closed by a further depression of the knob. Thus the current is started in the apparatus and assumes a steady condition before the galvanometer is applied. The bridge contact often serves as one key, in which case only a single key is required; or if the battery is a constant cell, it may be put in a closed circuit, when no key will be required.

Various forms of commutating keys are described in Art. 218, a condenser key is shown in Art. 263, and other forms are mentioned in Arts. 229, 230, 239, 254, and 256.

CXXVI. RESISTANCE WITH THE BOX BRIDGE

Determine the resistances of the magnet coils of a sounder, of a relay, and of the primary and secondary windings of an induction coil. Use several ratios for each resistance.

214. The Box Bridge. — The slide-wire form of Wheatstone's bridge (Art. 208) may be uncertain in its results because of irregularities in the wire or in the contact. In it the sum of the ratio resistances is constant, being equal to the resistance of the meter wire, while it is often advantageous to change this sum (Art. 215); the ratio is usually a fraction, precluding mental computation, and the apparatus lacks portability. The box bridge obviates these difficulties somewhat; it is an apparatus in which the ratio resistances (r_3 and r_4 of the previous form), as well as the standard resistance, are all wire coils contained in one box. This form of bridge was introduced in connection with the telegraph work of the post-office department in England; hence it is often called the post-office resistance box.

The sets of ratio coils, R_3 and R_4 (Fig. 129), take the place of the meter wire and slider, and each usually contains resistances of 1, 10, 100, and 1000 ohms. One resistance only from each set is to be used at a time; these may be selected so that the ratio shall be either 1, 10, 100, 1000, 0.1, 0.01, or 0.001. If the reference resistance has a total value of 10000 ohms, the

range of measurement is from 0.001 ohm to 10 000000 ohms, there being from one to four significant figures in the result. Often more significant figures may be obtained by interpolation from the galvanometer readings.

Sometimes the three sets of resistances are permanently connected, and sometimes the connections have to be made as required. In the latter case arrange the apparatus in accordance with the scheme of Fig. 129. Having selected a ratio, the known resistance R is varied to secure a balance between the points D and E, as indicated by zero deflection of the galvanometer in the bridge when the key K is closed. Then, since (Art. 207)

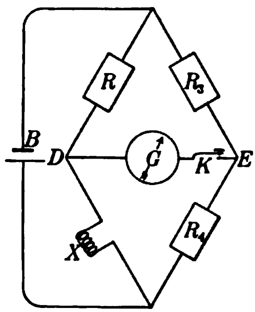


FIG. 129. BOX BRIDGE

$$R : X = R_3 : R_4,$$

the unknown resistance is obtained by the very simple calculation of multiplying R by some power of ten.

When X is wholly unknown it is well to select a ratio of equality, as 10 : 10; and with this to find an approximate value for X. Then change the ratio so that R shall have the largest possible value, and make a final determination of X. For instance, if with a ratio of 10 : 10, X is found to lie between 7 and 8 ohms, make the ratio 1000 : 1, when the balance may be found with R equal to 7341 ohms, giving for X the final value 7.341 ohms.

Sometimes, in addition to the ratio coils and the resistance coils, a key, galvanometer, and battery are included in the case of the apparatus. The collection is then called a testing set, and it may be exceedingly compact and convenient. It is also evident that the method may be employed when the apparatus is made up with three separate, simple, resistance boxes.

215. Maxwell's Rule for Wheatstone's Bridge. — An inspection of Fig. 129 indicates that the resistances R, X, R_3, and R_4 are represented by the four sides of a parallelogram, in one diagonal of which the galvanometer is placed, and in the other the battery. From the symmetry of the arrangement it does

not matter, so far as the relations between the resistances are concerned, which diagonal contains the battery. But to secure the greatest sensitiveness Maxwell gives the following rule. "Of the two resistances, that of the battery and that of the galvanometer, connect the greater resistance so as to join the two greatest to the two least of the four other resistances."

CXXVII. RESISTANCE WITH A TANGENT GALVANOMETER

Measure several unknown resistances, both large and small, by the method of three observations with a tangent galvanometer; also find the resistance of the battery employed and the electromotive force.

216. The Tangent Galvanometer. — A simple form of galvanometer consists of a number of turns of wire wound on a circular frame of large diameter, usually twenty centimeters or more, supported with its plane in the plane of the magnetic meridian, and having a short permanent-magnet needle suspended in the center of the coil (Fig. 130). The needle carries a light pointer, and a divided circle is provided for measuring the angular deflection of the needle. If various steady currents be sent through such a galvanometer, they will produce deflections of the needle such that the current strengths are proportional to the tangents of the angles of deflection. Therefore these current strengths may be expressed in any desired units by multiplying the tangents of the angles of deflection by a

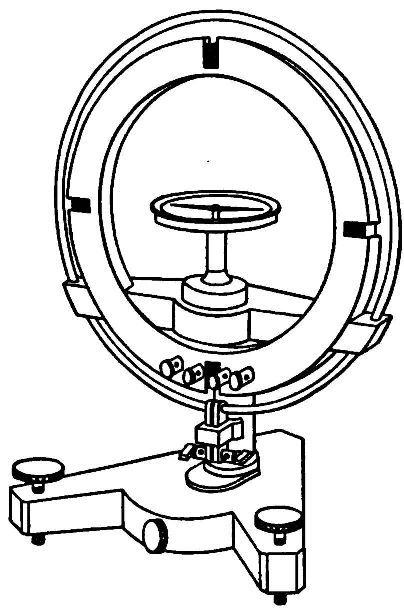


FIG. 130. TANGENT GALVANOMETER

properly determined constant. If K is the constant found as described in Art. 242, and θ is the deflection observed, the current strength in amperes is

$$I = K \tan \theta.$$

Before a tangent galvanometer is used for measurement it must be carefully adjusted to the magnetic meridian. Its own needle will serve as a compass in making an approximate adjustment. Connect a cell of battery, in series with a rheostat if necessary, through a reversing key (Art. 218) to the galvanometer. Observe the deflection produced, by averaging the readings of the two ends of the pointer; reverse the current and read again. If these opposite deflections are equal, the instrument is properly placed; if they are not equal, rotate the plane of the coil and repeat the observations; continue until the opposite deflections are equal.

REFERENCES. — For an extended account of the tangent galvanometer, and of work to be done with it, consult *Stewart and Gee*, Physical Measurements, Vol. II, pp. 225–274; *Gray*, Absolute Measurements in Electricity and Magnetism, Vol. II, pp. 347 et seq.

217. Resistance by Three Observations with a Tangent Galvanometer, and Battery Resistance by Ohm's Method. — Connect the unknown resistance, X, a rheostat, R, and a constant battery, B, in series, through a reversing key, K, with the galvanometer, G (Fig. 131). By changing the number of cells of battery used, or by altering the resistance R, cause the deflection of the galvanometer to be about 45°. The battery should be so arranged that R may be as small as possible, preferably 0. Observe the deflection of the galvanometer, measured, as should always be the case, by the average of the four readings given by the two ends of the needle when the current flows first in one direction and then in the reversed direction.

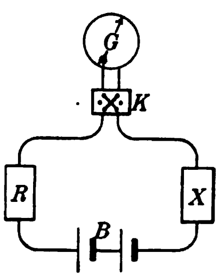


FIG. 131. RESISTANCE WITH TANGENT GALVANOMETER

If E represents the electromotive force of the battery, B its resistance, G the resistance of the galvanometer, R_1 the value

of R, X the unknown resistance, θ_1 the galvanometer deflection, and K the galvanometer constant, then, by Ohm's Law, from the first observation,

$$\frac{E}{B+G+R_1+X}=K\tan\theta_1.$$

Leaving the battery unchanged, remove X, and alter R till a deflection, θ_2, of about 30° is obtained, and let R_2 be the value of R giving this result; then, from the second observation,

$$\frac{E}{B+G+R_2}=K\tan\theta_2.$$

Further, alter R to give a deflection, θ_3, of about 60°; if R_3 is the value of the resistance required,

$$\frac{E}{B+G+R_3}=K\tan\theta_3.$$

Dividing the second equation by the third,

$$\frac{B+G+R_3}{B+G+R_2}=\frac{\tan\theta_2}{\tan\theta_3},$$

and
$$B+G=\frac{R_2\tan\theta_2-R_3\tan\theta_3}{\tan\theta_3-\tan\theta_2}.$$

Calculate the value of $B+G$, which is required in finding X. If the galvanometer resistance is known, the resistance of the battery is determined by making the second and third observations only.

Dividing the third of the above equations by the first,

$$\frac{B+G+R_1+X}{B+G+R_3}=\frac{\tan\theta_3}{\tan\theta_1},$$

from which
$$X=\frac{\tan\theta_3}{\tan\theta_1}(B+G+R_3)-(B+G+R_1).$$

By eliminating $B+G$ between the second and third equations given above, the electromotive force of the battery may be obtained if the galvanometer constant K is known, as expressed in the following relation.

$$E=K\tan\theta_2\tan\theta_3\frac{R_2-R_3}{\tan\theta_3-\tan\theta_2}.$$

The above methods for battery resistance and electromotive force are known as Ohm's methods; they may be used with any nonsensitive galvanometer, for constant current cells.

218. Commutators, Reversing Keys. — When frequent changes in the direction of a current through part of an apparatus are required, some form of commutator should be used. An efficient and easily constructed one may consist of a block of wood in which are four holes, bored only part way through, these holes being filled with mercury and connected as follows. One terminal of the circuit is connected to hole 2 (Fig. 132), and the other to hole 3; while the apparatus through which the current is to be reversed is connected to holes 1 and 4. Two wire links are placed to join 1 and 2, and 3 and 4, as shown by the heavy lines. If, now, the links are lifted and rotated 90°, so that they connect 1 and 3, and 2 and 4, as indicated by the dotted lines, the current through G will be reversed.

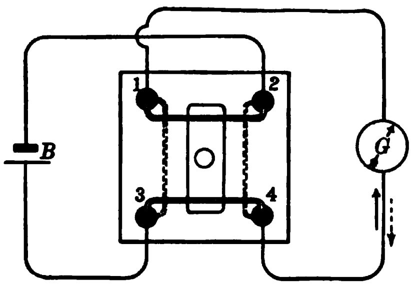


FIG. 132. ROTATING COMMUTATOR

A plug commutator is sometimes made, in which the four mercury cups are replaced by quadrants of brass, and the wire links by plugs which join the quadrants as desired.

The rocking commutator, using six mercury cups, is a most convenient form. Cups 1 and 2, or 3 and 4 (Fig. 133), are connected to the battery, and 5 and 6 to the galvanometer or other instrument; 1 and 4, and 2 and 3 are permanently connected by diagonal wires. A rocker of the form shown, consisting of two three-pronged forks, joined by an insulating handle, H,

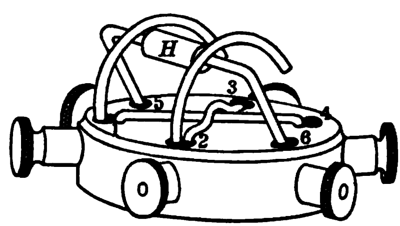


FIG. 133. ROCKING COMMUTATOR

permits 5 to be joined to either 1 or 3, and at the same time 6 is joined to either 2 or 4. The removal of the rocker opens all circuits.

If the diagonal wires of the commutator are removed, one circuit whose terminals are joined to 5 and 6 may be connected to either one of two other circuits whose terminals are 1 and 2, or 3 and 4.

There are many other forms of commutators, the operation of which will be understood from inspection.

CXXVIII. RESISTANCE BY THE DIFFERENTIAL METHOD

Compare several large and small resistances of a resistance box with approximately equal standard resistances, by means of a differential galvanometer, using the methods of single observation, substitution, and reversal.

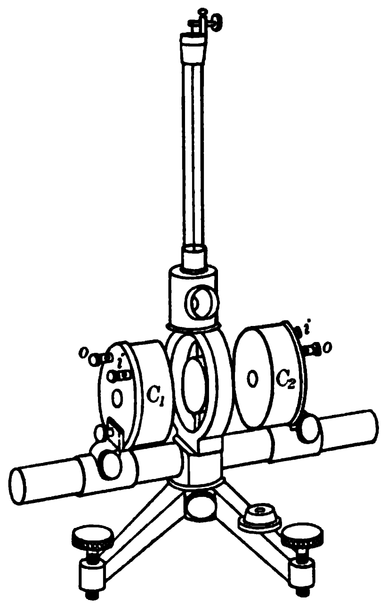


FIG. 134. DIFFERENTIAL GALVANOMETER

219. The Differential Galvanometer. — The coils of a galvanometer are often wound in two equal parts, C_1 and C_2 (Fig. 134), symmetrically placed with respect to the needle, and so connected that the same current, or equal currents, may flow through the coils in opposite directions. These currents in the two coils tend to turn the needle in opposite directions, and it can then indicate only the difference in their effects. Often the terminals of each coil are separately brought to binding posts in order that the coils may be connected to give differential effects, or by joining them so that the current flows through both in the same direction the galvanometer becomes a sensitive one of the ordinary type.

Magnetic Adjustment. — Connect the coils as indicated in Fig. 135, so that the current going in at the inner end, *i*, of the first coil enters the second by its outer end, *o*. If the terminals of the coils are not marked, a simple trial will indicate whether the coils are connected for differential or summational effects.

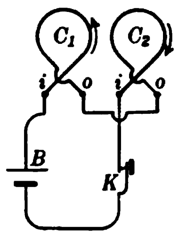


FIG. 135. MAGNETIC ADJUSTMENT

By altering the position of one coil or a part of one coil, the adjustment is to be made so that the same current flowing through the two coils in opposite directions shall produce no effect upon the needle.

Resistance Adjustment. — Connect the coils in parallel to give differential effects; that is (Fig. 136), connect one terminal of the battery to the outer end of one coil and to the inner end of the other. Make the other connections as shown; then, when the resistances of the two branch circuits are equal, equal currents will flow through the two coils, and there will be no deflection. If necessary insert external resistance in one branch circuit until this condition is fulfilled. This may be accomplished by using a rheostat, or by varying the length of one of the lead wires.

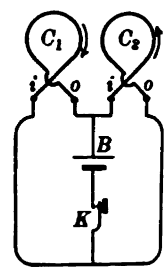


FIG. 136. RESISTANCE ADJUSTMENT

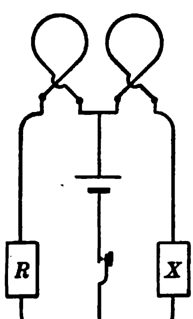


FIG. 137. METHOD OF SINGLE OBSERVATION

220. To measure a Resistance greater than that of One of the Galvanometer Coils. *By Single Observation.* — After completing the adjustments insert a standard resistance, R, and the unknown resistance, X, as indicated in Fig. 137. Adjust R until no deflection of the galvanometer is caused by closing the battery circuit. Then X is equal to R. If R cannot be adjusted to produce an exact balance, calculate the true value of X by interpolation between the nearest two values of R.

By Substitution. — In case the adjustment of the galvanometer is uncertain, after securing a balance as nearly as possible by the method described above, note the constant deflection resulting; substitute for X a standard resistance box, and adjust it to produce the same deflection. The substituted resistance is equal to X. In this case the resistance R serves merely as a rheostat, and need not be a standard.

By Reversal. — The galvanometer may be connected as indicated in Fig. 138, without any previous adjustment. S, a rheostat inserted in one branch circuit, is so adjusted that no

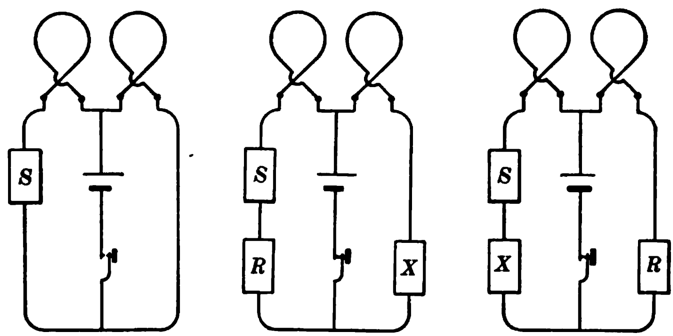


FIG. 138 FIG. 139 FIG. 140

METHOD OF REVERSAL

deflection occurs when the battery circuit is closed. Insert the standard resistance R and the unknown resistance X (Fig. 139); adjust R until a balance is again effected, and let R_1 be its value. Interchange R and X (Fig. 140), and let R_2 be the resistance required to produce a balance. If g_1 and g_2 are the entire resistances (unknown) of the two branches of the circuit,

$$R_1 : X = g_1 : g_2,$$

and

$$X : R_2 = g_1 : g_2,$$

from which

$$X = \sqrt{R_1 R_2}.$$

(Compare the above methods of using a galvanometer with the methods of using a balance described in Arts. 43 and 47.)

221. To measure a Resistance less than that of One of the Galvanometer Coils. — When the unknown resistance is less than that of one of the galvanometer coils, greater sensitiveness will be obtained by a modification of the previous methods. Connect the battery to the galvanometer so that the same current shall pass through the two coils in series in opposite directions (Fig. 141); make the standard resistance R a shunt to one coil, and the unknown resistance X a shunt to the other. Adjust R to secure a balance, when, if the galvanometer is in perfect adjustment both for magnetic effect and resistance, X is equal to R, as in the method of single observations previously described. Without previous adjustment of the galvanometer the methods of substitution and reversal may also be applied with this system of connections.

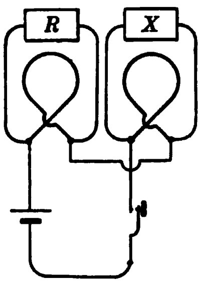


Fig. 141. Method of Shunts

CXXIX. RESISTANCE BY THE FALL OF POTENTIAL

Measure the resistance of an incandescent lamp while it is lighted; also the internal resistance of a series of secondary cells, and of the several cells separately.

222. Ammeters and Voltmeters. — The ammeter is a low-resistance galvanometer in which a pointer, moving over an empirical scale, indicates the strength of the current flowing through the instrument. It is to be connected in series with the circuit, its resistance being so low that for the currents to which it is adapted its insertion does not appreciably alter the current strength.

The voltmeter is a high-resistance galvanometer in which the empirical scale indicates the potential difference which must be applied to its terminals to cause a sufficient current to flow through it to deflect the pointer by various amounts. It is to be used on a shunt circuit, and preferably only for momentary

observations. Its resistance is so high that when applied as a shunt it does not appreciably alter the strength of the current. The voltmeter may be applied to measure the difference of potential of two points between which no current is flowing, provided that the very small current which will flow through the voltmeter does not disturb the potential difference.

Voltmeters and ammeters are commonly of the D'Arsonval galvanometer type (Art. 209), though other forms are sometimes used.

223. Resistance with Voltmeter and Ammeter. — The method of fall of potential is based upon Ohm's Law. It may be very

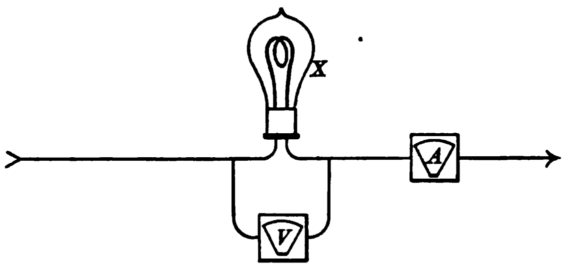


FIG. 142. RESISTANCE WITH VOLTMETER AND AMMETER

conveniently applied to the measurement of resistances of moderate size, and also of small resistances, if suitable voltmeters and ammeters are available.

Arrange to send a current of any convenient value through the resistance to be measured, and a suitable ammeter in series. Connect the voltmeter to the terminals of the unknown resistance in a shunt circuit. (See Fig. 142.) If E is the difference of potential indicated by the voltmeter while the current I is flowing, the unknown resistance is

$$X = \frac{E}{I}.$$

224. Battery Resistance with Voltmeter and Ammeter. — Connect an ammeter, a rheostat, and a key in series with the battery, and a voltmeter as a shunt to the battery, as indicated in Fig. 143. Adjust the rheostat so that when the key is closed

the current passing shall have any convenient value, I, and let E_1 be the difference of potential between the terminals of the battery, as indicated by the voltmeter, while this current is flowing; let E_2 be the voltmeter reading when the key is open and no current is flowing; then the internal resistance of the battery

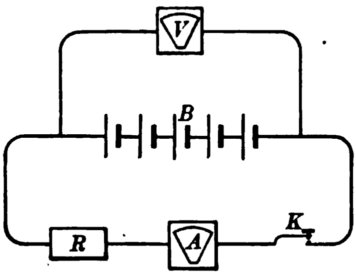


FIG. 143. INTERNAL RESISTANCE OF BATTERY

is
$$B = \frac{E_2 - E_1}{I}.$$

This method is particularly applicable to the measurement of the resistance of a secondary battery.

225. Battery Resistance with Voltmeter and Standard Resistance. — In the place of the ammeter and rheostat, in the above method, put a standard resistance. Let E_1 be the difference of potential while the current is flowing through the resistance R; and let E_2 be the difference of potential of the battery on open circuit; then the battery resistance is

$$B = R\,\frac{E_2 - E_1}{E_1}.$$

226. Battery Resistance with Condenser and Resistance. — The method last described will be better adapted to the measurement of the resistances of single cells if the voltmeter is replaced by a condenser and ballistic galvanometer. The charges received by the condenser are proportional to the potential differences. Make the connections as indicated in Fig. 144. Close the key

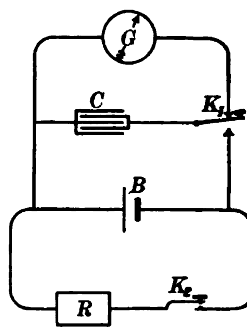


FIG. 144. BATTERY RESISTANCE WITH CONDENSER

K_2, completing the battery circuit, and at once press the key K_1 firmly and for a very short time upon the lower contact; release the key quickly. The condenser will have been charged to a potential difference equal to that of the battery terminals when

on the closed circuit, and the galvanometer will be deflected an amount proportional to this charge (Art. 253). Let d_1 be the deflection. Now, leaving the key K_2 open, again charge and discharge the condenser, and let d_2 be the deflection produced. Then, if R is the resistance in the battery circuit, the battery resistance is

$$B = R\frac{d_2 - d_1}{d_1}.$$

The observed deflections of the ballistic galvanometer, if made with a telescope and scale, must be corrected, to make them proportional to the quantities of electricity, as described in Art. 252.

The value of R should not be much greater than that of B, though if the latter is very small, too small a value of R will cause too rapid a polarization of the cell. The method is therefore adapted to constant cells and those of high resistance.

A very full description of the method is given in the reference.

REFERENCE. — *Carhart and Patterson*, Electrical Measurements, pp. 100–109.

CXXX. GALVANOMETER AND BATTERY RESISTANCE BY THE WHEATSTONE'S BRIDGE

Determine the resistance of a galvanometer by Thompson's method, and the resistance of a gravity cell by Mance's method.

227. Thompson's Method for Galvanometer Resistance.—This method is based upon the principle of Wheatstone's bridge. The arrangement of apparatus differs from that previously described (Fig. 120) in that the galvanometer is taken out of the "bridge" and is placed as the unknown resistance, as shown in Fig. 145. A current will now flow through the galvanometer continually, producing a deflection. If this deflection is too large, it may be modified by a control magnet, by a shunt, by a resistance in the battery circuit, by using the galvanometer differentially, or by using two cells of battery joined in opposition, according to circumstances.

Making the bridge connection from D to E will usually alter the current flowing from A to C, and thus cause a change in the galvanometer reading. But by trial a point, E, may be found which has the same potential as D, so that opening and closing the bridge causes no *change* in the galvanometer deflection. Then the resistances are related as follows (Art. 207):

$$r_1 : r_2 = R : G.$$

If the galvanometer is of high resistance, a box bridge is preferable to the slide-wire form, as the resistance of the ratio arms may then be made more nearly equal to that of the galvanometer, a condition securing greater sensitiveness.

228. Mance's Method for Battery Resistance. — The details of this method are those of the preceding one, except that the

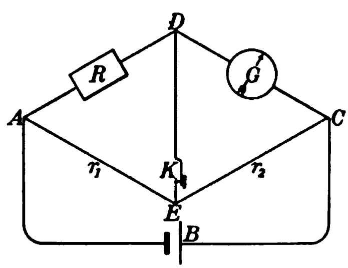


FIG. 145. THOMPSON'S METHOD

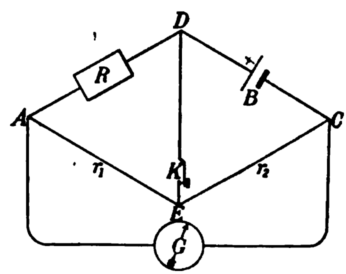


FIG. 146. MANCE'S METHOD

battery and galvanometer change places. The method is suitable only for a constant cell. The connections are shown in Fig. 146.

Some current from the battery will always flow through the galvanometer, causing a deflection. The adjustment consists in finding such a point of contact, E, that opening and closing the bridge does *not alter* the galvanometer reading. Then D and E are at the same potential, and the resistances are in the following proportion.

$$r_1 : r_2 = R : B.$$

CXXXI. BATTERY RESISTANCE BY COMPENSATION

Determine the resistances of various cells of battery.

229. Beetz's Method for Battery Resistance. — The following method for measuring the resistance of a battery has the advantage of requiring the circuit to be closed only momentarily, thus avoiding to some extent the polarization of the cell. The

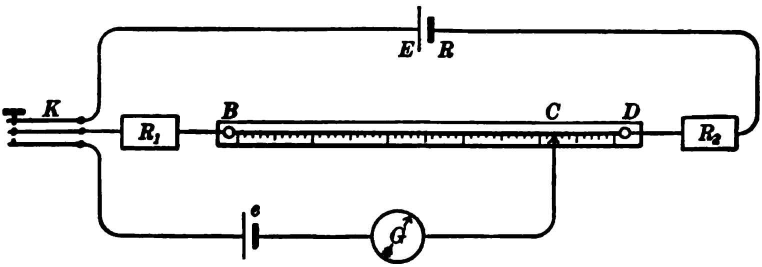


FIG. 147. BEETZ'S METHOD FOR BATTERY RESISTANCE

principles of the method may be described in connection with the apparatus as follows. Fig. 147 represents a slide-wire bridge, the resistance of the slide wire of which is known, R_1 and R_2 two small resistance boxes, K a three-point key, G a galvanometer, E the cell whose resistance, R, is to be determined, and e a cell of less electromotive force than E. The connections having been made as indicated, the like poles of both cells being connected to the key, it is evident that the resistances R_1 and R_2 and the point of contact C can be so adjusted that the difference of potential produced by e between the points K and C will be counterbalanced by the opposite potential difference between the same points produced by E. When this condition is secured the galvanometer will show no deflection. This must be determined by making only very short contacts with the key. If the key is closed even for a few seconds, the resistance of the cell changes, owing to polarization, and thus becomes uncertain. Represent the total length of the bridge wire, BD, by l_1, the length from B to C by l_2, and

the resistance of unit length of the wire by ρ (Art. 240). Neglecting the connections, the external resistance in the upper circuit is $R_1 + l_1\rho + R_2$; represent this by a_1, and let b_1 be equal to $R_1 + l_2\rho$; then by Ohm's Law the current in the upper circuit is

$$I = \frac{E}{R + a_1} = \frac{e}{b_1},$$

from which

$$\frac{E}{e} = \frac{R + a_1}{b_1}.$$

Alter the resistance R_1, or R_2, so as to obtain a new point of contact, differing considerably from the first, which will give new values, a_2 and b_2, corresponding to a_1 and b_1 of the first observation. Then, as before,

$$\frac{E}{e} = \frac{R + a_2}{b_2} = \frac{R + a_1}{b_1},$$

and

$$R = \frac{a_1 b_2 - a_2 b_1}{b_1 - b_2}.$$

Instead of the two resistance boxes, R_1 and R_2, and the slide-wire bridge, a single thin stretched wire may be used, simplifying the apparatus required.

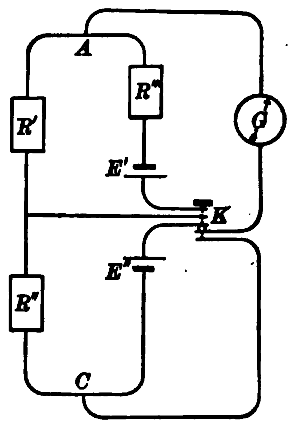


Fig. 148. Benton's Method for Battery Resistance

230. Benton's Method for Battery Resistance. — Beetz's method can be applied to the investigation of only such cells as have an electromotive force greater than the auxiliary cell. The following method is not limited in this respect, and it is better adapted for cells of low internal resistance. The apparatus required consists of three resistance boxes, an auxiliary cell, a low-resistance galvanometer, and a special key. These are connected as indicated in Fig. 148, E' being the cell investigated, and E'' the auxiliary cell, the like poles of which are connected to the key. The key has five connections, such that the first pressure brings three points into contact, closing

the two battery circuits, and then a further pressure closes the galvanometer circuit by means of an insulating contact piece.

With $R'''=0$, and R' set at any small resistance, R'' is adjusted so that there is no deflection of the galvanometer when the key is closed; then A and C are at equal potentials. The key is to be closed only for an instant, otherwise polarization may result. If R_1' and R_1'' are the values of the resistances in the boxes, and B' and B'' are the respective battery resistances, the condition for equal potentials at A and C is

$$\frac{R_1'}{R_1'+B'}=\frac{R_1''}{R_1''+B''}.$$

R' is now set at a large resistance, R_2', while R'' is left *unchanged.* R''' is adjusted until no deflection occurs upon closing the key; let R_2''' be the resistance required. The condition for equal potentials at A and C is

$$\frac{R_2'}{R_2'+R_2'''+B'}=\frac{R_1''}{R_1''+B''}.$$

Equating the first members of these equations and reducing, the internal resistance of the cell is

$$B'=\frac{R_1'R_2'''}{R_2'-R_1'}.$$

REFERENCE. — *J. R. Benton*, Physical Review, Vol. 16, p. 253, 1903.

CXXXII. ELECTROLYTE RESISTANCE BY KOHLRAUSCH'S METHOD

Find the specific resistance of a ten per cent. solution of copper sulphate and the internal resistance of various cells singly, in series, and in parallel.

231. Electrolyte Resistance with Alternating Currents. — Kohlrausch's method for electrolyte resistance avoids the influence of polarization and allows the resistance to be measured directly, just as that of a metallic conductor, by using in the measuring device currents rapidly alternating in direction and

of exactly equal strength, between electrodes of large capacity. This current is conveniently obtained from an induction coil, and the detector is usually a telephone. The principle of measurement is that of Wheatstone's bridge, though the bridge is often of special construction to facilitate the use of the induction coil and telephone, which replace the battery and galvanometer respectively. The cell whose internal resistance is required, or the vessel containing the electrolyte, is connected as the unknown resistance. A separate cell is, of course, required to operate the induction coil. A sketch of the connections is shown in Fig. 149. R_3 and R_4 represent the slide

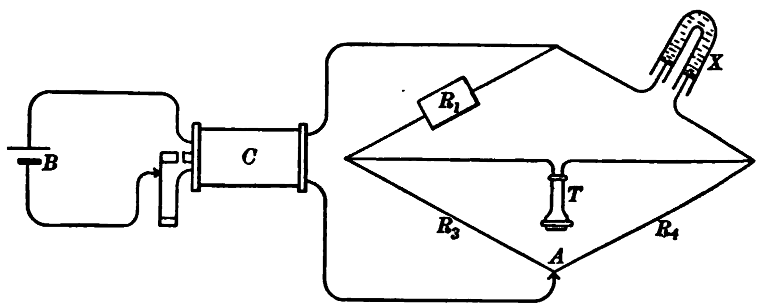


FIG. 149. TELEPHONE BRIDGE FOR ALTERNATING CURRENTS

wire, and A the sliding contact. The observation consists in adjusting the contact until the sound of the induction current in the telephone becomes a minimum. Then the ratios of the various resistances are

$$R_1 : X = R_3 : R_4.$$

The scale attached to the slide wire, instead of being a scale of equal lengths, may be divided to show directly the ratio of the lengths, R_3 to R_4, for the various positions of the sliding contact. R_1 may then be conveniently 1, 10, or 100 ohms, and the unknown resistance is found at once by multiplying the scale reading by R_1.

232. Electrolyte Resistance.—The specific resistance of an electrolyte may be calculated, as for a solid (Art. 235), from

measurements of the resistance of a column of the liquid of known length and cross section. Geometrical measurement of the space between two electrodes is seldom convenient, and the more usual method is to compare the unknown electrolyte with that of a standard solution in the same containing vessel. A useful standard solution is a saturated solution (about twenty-six per cent.) of sodium chloride. The liquid is to be shaken up with an excess of salt just before being used. The conductivity of the solution is as follows (Kohlrausch):

Temperature	15°	16°	17°	18°	19°	20°	21°
Conductivity, K	0.2015	0.2063	0.2112	0.2161	0.2210	0.2260	0.2310

Fill the electrolyte vessel with the salt solution and determine its resistance, r_1, using the Kohlrausch method described in the preceding article; substitute the unknown electrolyte and find its resistance, r_2; then the conductivity desired is, since the conductivities are inversely as the resistances,

$$K_x = \frac{Kr_1}{r_2}.$$

REFERENCES. — *Kohlrausch*, Physical Measurements, pp. 317–321; *Ostwald*, Physico-Chemical Measurements, pp. 222–229.

CXXXIII. INSULATION RESISTANCE BY DIRECT DEFLECTION

Find the resistances of glass and porcelain insulators, and of the insulation of a cable.

233. Measurement of High Resistance by Direct Deflection. — A cup-shaped insulator may be arranged for testing by pouring mercury into the interior and partially immersing the cup in a mercury bath, the edge of the cup where not immersed having been previously cleaned, dried, and coated with paraffin. Connect one terminal of the circuit to the mercury in the cup, and the other to the exterior bath. In the case of insulated wire that is waterproof, make of it a coil of a known length, and immerse it, except the ends, in a water bath. One circuit terminal is to be connected to the wire and the other to the

water bath. When the material is in sheets, portions of the surfaces may be silvered, and the electrodes connected to these silvered portions. Care should be taken to have a wide margin of clean and dry surface around the silvered portion. The connections for tests of the insulations of permanently placed wires and cables will suggest themselves.

Connect a sensitive galvanometer and the unknown resistance in series with a battery or other source of electromotive force. Usually as high an electromotive force as is available will be required, such as that of a large storage battery, a commercial lighting circuit, or a railway power circuit. Let E be the electromotive force used, measured with a voltmeter; X, G, and B the unknown, galvanometer, and battery resistances respectively; d the number of divisions of the scale at a meter distance, measuring the constant deflection produced; and K the figure of merit of the galvanometer. Then

$$dK = \frac{E}{X + B + G},$$

and

$$X = \frac{E}{dK} - (B + G).$$

The battery resistance may be neglected in comparison with the other resistances.

234. High Resistance by Direct Deflection and Double Readings. — The measurement of high resistances by the method of double readings is often more convenient than the one described above. It is not necessary to know the electromotive force and galvanometer constant, but a standard resistance of high value, 100000 ohms, for instance, is required. The electromotive force is first connected to the galvanometer and unknown resistance in series, and then to the 100000 ohms resistance and galvanometer. The deflections produced, d_1 and d_2, may be considered as inversely proportional to the resistances, from which

$$X = \frac{d_2}{d_1}\, 100000.$$

It is desirable to have the two deflections nearly equal; to secure this it will usually be necessary to shunt the galvanometer when it is used with the 100000 ohms. If this shunt is such that the galvanometer sensitiveness is reduced to $\frac{1}{n}$ of that which it had in the first trial, then

$$X = n\frac{d_2}{d_1}100000.$$

In these formulæ the resistance of the battery and galvanometer are considered as inappreciable in comparison with the high resistance being measured. This is clearly permissible for the comparison with the insulation resistance, and in comparison with 100000 ohms the resistance of the shunted galvanometer is also very small. If it is not permissible to neglect the galvanometer and battery resistance, the following complete formula may be used. Represent the high comparison resistance by R, the shunt resistance by S, and the battery and galvanometer resistances by B and G; then the unknown resistance is

$$X = \frac{d_2}{d_1}\left(B + G + R + \frac{GB + GR}{S}\right) - (B + G).$$

CXXXIV. SPECIFIC RESISTANCE BY THE COMPARISON OF POTENTIALS

Determine the resistances of short pieces of copper and German silver wire, and find their specific resistances.

235. Specific Resistance. — The resistance in ohms of a centimeter of length of a substance when the area of cross section is one square centimeter and its temperature is 0°, is its specific resistance.

If R_t is the measured resistance at the temperature t of the substance in the form of a wire of length l and sectional area s, and a is the temperature coefficient of the substance (Appendix,

Table 26), then the resistance R_0, of the piece of wire at 0°, is found from the relation $R_t = R_0(1 + at)$, and the specific resistance ρ is determined by the equation

$$R_0 = \rho \frac{l}{s}.$$

236. Resistance by Method of Comparison of Potentials.—The method of comparison of potentials is applicable to the measurement of very small resistances, such as the resistances of short lengths of conducting wires. A standard resistance is required whose value is nearly equal to that of the unknown resistance. If no single standard of a sufficiently small value is available, several standards of higher value may be used in parallel; or, if the unknown resistance is a wire, a piece of it may be selected such that its resistance is approximately equal to that of the standard.

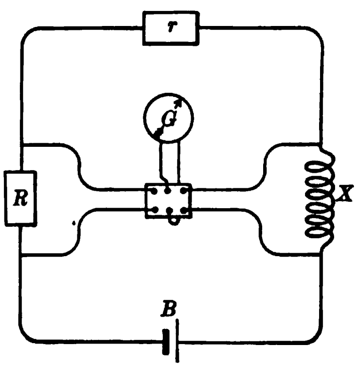


Fig. 150. Comparison of Potentials

Connect the unknown resistance X, a standard resistance, R, a rheostat, r, and a constant battery, B, in series. By means of a suitable key (Art. 218), or switch, arrange to connect a high-sensibility galvanometer, G, to the terminals of either R or X, as indicated in Fig. 150.

The readings of the galvanometer when so connected can be given a convenient value by altering r. Since the fall of potential is proportional to the resistance, the ratio of R to X will be the same as that of the currents which will flow through the galvanometer when it is connected first to R and then to X. When the apparatus is arranged as described, these currents will be very small and nearly equal, and it may be assumed that they are proportional to the observed galvanometer deflections, d_1 and d_2 respectively. Then

$$R : X = d_1 : d_2.$$

237. The Low-Resistance Bridge. — A form of apparatus often employed for measuring the resistance of wires and rods, or irregular low resistances such as armature windings, consists of a Wheatstone's bridge specially constructed so that the resistances of the connections may be eliminated by a method of double readings.

The known resistance, R_1 (Fig. 151), is a straight wire attached to a graduated scale which shows the actual resistance in ohms of different lengths of wire. The portion of this wire which is used in balancing the bridge is determined by a sliding contact, A_1. The unknown resistance, X, if a wire or rod, is stretched between two clamps, and the length of this used in balancing is limited by a movable contact, C_1, and is measured by a centimeter scale. The ratio arms are two resistances, R_3 and R_4, arranged to give various ratios.

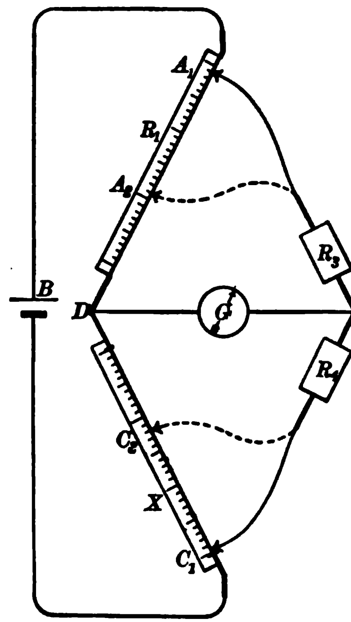


FIG. 151. LOW-RESISTANCE BRIDGE

The galvanometer and battery are connected as indicated. With the contact on the unknown resistance at C_1, and with a suitable ratio between R_3 and R_4, move the sliding contact A_1 until a balance is secured, indicated by no deflection of the galvanometer when the bridge connection is made. Then

$$A_1D : DC_1 = R_3 : R_4.$$

Move the contact on the unknown resistance to C_2. Adjust the slider to secure a balance, and if A_2 is its new position,

$$A_2D : DC_2 = R_3 : R_4,$$

from which it follows that

$$A_1A_2 : C_1C_2 = R_3 : R_4.$$

The resistance A_1A_2 is read directly from the graduated scale, and this multiplied by the ratio of R_3 to R_4, usually a power of 10, gives the resistance of the known length of the substance being investigated.

Any irregular resistance may be connected to the clamps, and the contacts C_1 and C_2 may be made to these clamps, or to other points between which the resistance is required.

A slide-meter bridge may be used for R_1 if the resistance of its wire is known, and two resistance boxes will serve for R_3 and R_4, making the method one easily applied with simple apparatus.

CXXXV. TEMPERATURE COEFFICIENT OF RESISTANCE BY FOSTER'S METHOD

Determine the constants of an ohm coil, that is, its resistance at 0° and its temperature coefficient.

238. Temperature Coefficient of Resistance. — Standard resistance coils are usually constructed, as described in Art. 212, so that they may be surrounded with ice, water, or oil baths, to give them definite known temperatures. Let the values of the resistance whose coefficient is desired be determined as described in Art. 239, when it is at two temperatures differing considerably, at 10° and 40°, for example. Care must be taken that all other parts of the apparatus remain at a constant temperature. Let R_1 and R_2 be the observed resistances at temperatures t_1 and t_2, and R_0 its resistance at 0°, and a the temperature coefficient;

then $$R_1 = R_0(1 + at_1),$$

and $$R_2 = R_0(1 + at_2);$$

whence $$a = \frac{R_2 - R_1}{R_1t_2 - R_2t_1}.$$

239. Carey Foster's Method for Comparison of Resistances. — This method is adapted to the accurate comparison of two nearly equal resistances, and it is very convenient for determining the variations in the resistance of a coil due to changes in its

temperature. The principle of the method is that of Wheatstone's bridge, modified so that, by two observations, the *difference* between two resistances is determined in terms of the slide wire, the resistance of all connections being eliminated. It is analogous to the comparison of masses by the method of double weighing, and, like this method, it is of the highest precision.

Designate the coil to be tested by X, a standard coil of the same nominal value by S, and two auxiliary coils whose resistances are each approximately equal to S, by R_1 and R_2. Connect these four coils, a galvanometer, a battery, and a suitable

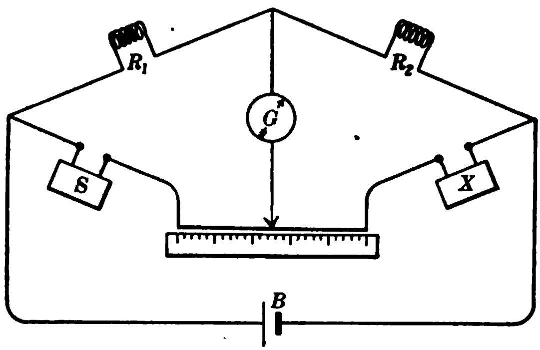


FIG. 152. THE CAREY FOSTER METHOD

bridge apparatus, as indicated in Fig. 152. Adjust the bridge contact until a perfect balance is secured. Since the four resistances are approximately of the same value, the contact will be near the middle of the wire (Art. 208). Interchange the resistances S and X. If these are exactly equal, the balance will not be disturbed; if they are not equal, it will be necessary, in order to restore the balance, to move the contact in the direction of the greater resistance just so much that the resistance of the short length of the slide wire between the two points of contact is equal to the difference in the resistances of the standard, S, and the unknown, X. Determine this length of wire by observation, and find its value in ohms by the aid of the methods of the following article.

For conveniently interchanging the resistances, and for accuracy in the measurements by this method, elaborately constructed bridges are often provided.

240. Resistance of the Slide Wire. — Usually the direct observations give the difference between the resistances compared in terms of a length of the slide wire, and it is necessary to ascertain the resistance, ρ, of the unit length of the wire, in order to express this difference in ohms.

Method I. — If a single standard resistance is available, the resistance of which is less than that of the slide wire, this standard may be balanced against a heavy copper bar of inappreciable resistance put in place of X, as described in the previous article. By interchanging and again balancing, a length of the slide wire whose resistance is equal to that of the standard is determined. From this is deduced the resistance, ρ, of unit length of the wire.

If no single standard of sufficiently small value is available, two one-ohm coils and one ten-ohm coil may be used. Place the ten- and one-ohm coils in parallel, on one side of the bridge, and the other unit coil on the opposite side, and proceed as before. The combined resistance of the pair of coils is $10 \times 1 \div 10 + 1 = 0.90909$ ohms, giving for the difference between the interchanged resistances, 0.09091 ohms. Other similar combinations will suggest themselves if they are available and more convenient.

Method II. — In case the only available standard has a resistance greater than that of the slide wire, the following method, requiring two complete observations, may be useful.

Let Q be any convenient but unknown resistance, a piece of wire for example, whose resistance is less than that of the slide wire. Balance this against the negligible copper bar resistance, and by interchanging determine the length, l_1, of the slide wire whose resistance is the same as that of Q; then

$$Q = \rho l_1.$$

Place the standard S in parallel with Q, and balance the combination, in two positions, against the negligible resistance, as

before. Then if l_2 is the length of the slide wire between the two points of contact,

$$\frac{QS}{Q+S} = \rho l_2,$$

and

$$\rho = S\frac{l_1 - l_2}{l_1 l_2}.$$

Method III. — The resistance of the slide wire may be measured directly with an independent Wheatstone's bridge. (See Arts. 208 and 237.)

Calibration of Slide Wire. — It has been assumed that the slide wire is of uniform resistance throughout, and usually the errors from such assumption are very small. But where extreme accuracy is required, it becomes necessary to calibrate the wire (Art. 241).

CXXXVI. ERRORS OF A BRIDGE WIRE BY BARUS'S METHOD

Make a ten-part calibration of the wire of a Wheatstone's bridge.

241. Calibration of Bridge Wire. — Barus's method for calibrating a bridge wire is an adaptation of the Gay-Lussac method of simple calibration, described in Art. 26. To use this method there must be provided as many separate coils of approximately equal resistance as there are parts in the proposed calibration; for instance, ten coils of one ohm each. Each coil should have terminals arranged to dip into mercury cups for making connections. For the purposes of this problem provide a series of eleven mercury cups so spaced that the ten coils may be connected in series as indicated in Fig. 153. Designate the first coil the calibrating coil, K, and the other coils by the numbers II, III, IV, etc. Connect the series of coils to the bridge wire by means of heavy copper wires or bars dipping into the two end cups; connect a cell of constant battery as indicated. A sensitive galvanometer has one terminal joined to the first mercury cup, and the other terminal to the slide contact. The current flows from A to C through the two branches of the circuit,

the series of ten coils, and the bridge wire. As explained in Art. 207, corresponding to the potential of any point in one branch, there is exactly the same potential at some point in the other branch. Theoretically, then, it ought to be possible to place the slide contact at a point, p_1, having a potential equal to that of cup 1, in which case no current will flow through the galvanometer. The construction of the slider will probably render it impossible to find this point, and it may be assumed to be at the very beginning of the wire. Take the galvanometer wire out of cup 1 and place it in cup 2, and find a contact p_2 such that there is no deflection of the galvanometer.

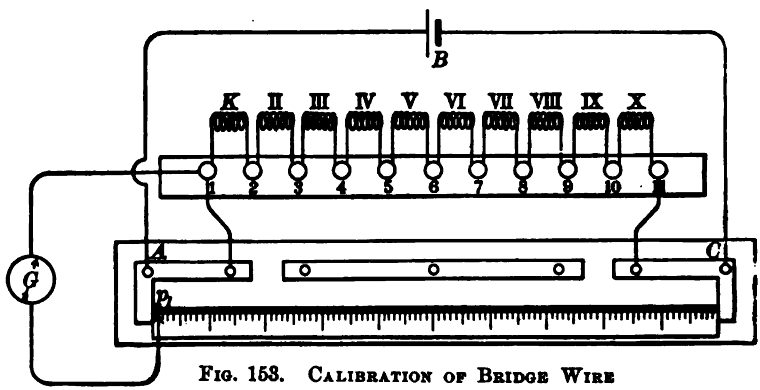


FIG. 153. CALIBRATION OF BRIDGE WIRE

The potential of p_2 is the same as that of cup 2. It then follows that the resistance of that portion of the bridge wire between p_1 and p_2 is in the same ratio to the resistance of the whole wire as the resistance of K is to the resistance of the series of ten coils. Now interchange coils K and II, and find a new contact point, p_3, giving no deflection while the galvanometer connection is in cup 2, and find a similar point, p_4, when the galvanometer is connected to cup 3. The resistance of the portion of the wire between p_3 and p_4 must be equal to that between p_1 and p_2. Point p_3 will probably be near to p_2, but will not coincide with it. Interchange coils K and III, and find new contacts, p_5 and p_6, giving no deflection when the galvanometer is connected to cups 3 and 4, and continue the operation till the coil K occupies

the place of X, and the last contact is the end of the wire. Thus there will have been found ten successive portions of the wire,

$$l_1 = p_2 - p_1,\ l_2 = p_4 - p_3,\ l_3 = p_6 - p_5, \text{ etc.,}$$

each having exactly the same resistance.

If the bridge wire were of perfectly uniform resistance throughout, these observed lengths would all be equal. Otherwise, the correction to each portion of the wire is the quantity which must be added to it to make it equal to the average of the observed lengths; and the correction at any point of the wire is the sum of the corrections to the several sections between the point and the beginning of the wire. These corrections being applied to any observed length of the wire, the length becomes strictly proportional to the resistance of the wire.

The method can be employed for a calibration in any number of equal parts other than ten.

Compare this description with that for the calibration of scales and thermometers, Arts. 26–30 and 129.

An example of the calibration of a bridge wire in ten sections is given.

CALIBRATION OF BRIDGE WIRE

BY BARUS'S METHOD

HARTMANN & BRAUN WHEATSTONE'S BRIDGE NO. 39

March 1, 1900

Position of Coil K	Readings		Section l	Section	Correction	Point	Correction
	p_n	p_{n+1}					
K_1	0.00 cm	9.89 cm	9.89 cm	0–10	+0.073 cm	0 cm	0.000 cm
K_2	9.90	19.89	9.99	10–20	−0.027	10	+0.073
K_3	20.00	20.99	9.99	20–30	−0.027	20	+0.046
K_4	30.12	40.10	9.98	30–40	−0.017	30	+0.019
K_5	40.19	50.20	10.01	40–50	−0.047	40	+0.002
K_6	49.92	59.90	9.98	50–60	−0.017	50	−0.045
K_7	59.91	69.90	9.99	60–70	−0.027	60	−0.062
K_8	70.00	79.93	9.93	70–80	+0.033	70	−0.089
K_9	80.06	90.05	9.99	80–90	−0.027	80	−0.056
K_{10}	90.12	100.00	9.88	90–100	+0.083	90	−0.083
			9.963			100	0.000

CHAPTER XXI

CURRENT STRENGTH

CXXXVII. GALVANOMETER CONSTANT WITH THE VOLTAMETER

Determine the constant of a tangent galvanometer by the electrolytic deposition of copper.

242. Constant of Tangent Galvanometer. — The constant of a tangent galvanometer (Art. 216) is the factor K, which when multiplied by the tangent of the angle of deflection, θ, produced by a current, I, gives the measure of that current; that is,

$$I = K \tan \theta.$$

To determine the constant it is only necessary to observe the deflection produced by any known current. The strength of the current can be most conveniently measured by the work it will do in a specially arranged electrolytic cell called a voltameter. The most useful forms are those in which the amounts of oxygen and hydrogen gas liberated in the water voltameter are measured, or in which the amounts of silver or copper deposited from solutions of these metals are ascertained. The silver voltameter is the most accurate and is best suited for small currents; for large currents the copper voltameter is sufficient.

243. The Copper Voltameter. — Prepare a nonsaturated solution of pure copper sulphate by dissolving 1 g of the crystals in each 4 ccm of distilled water. The solution should have a density of about 1.14. Provide an anode of pure copper and a cathode of platinum or copper. The latter must be carefully cleaned, dried, and weighed.

Adjust the galvanometer to the magnetic meridian (Art. 216), and arrange to pass a current through it and the voltameter in series. Notice that the connections are such as to make the prepared plate the cathode. A rheostat may be included in the circuit (Fig. 154) to regulate the current, which should not exceed 1 ampere for each 25 square cm of cathode area. The deflection of the galvanometer should not be less than 30° nor greater than 60°. Allow the current to pass for from a half hour to two hours, and note the exact time in seconds. The galvanometer should be read frequently during this time,— every five minutes perhaps,— to detect any unsteadiness of the current. Adjust the rheostat, if necessary, to keep the current constant. The deflection of the galvanometer is measured by the average of the four readings given by the two ends of the pointer when the current flows in one direction, and when it is reversed (through the galvanometer only) by the commutator K.

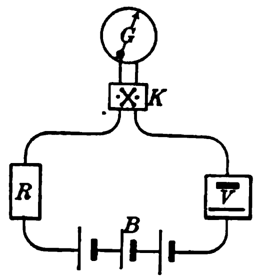


FIG. 154. CONSTANT OF GALVANOMETER

At the end of the experiment the cathode is removed and again carefully washed, dried, and weighed.

If w grams of copper are deposited in a copper voltameter in t seconds, the current flowing is, in amperes,

$$I = \frac{w}{0.0003294\, t}.$$

244. Standard International Ampere; Silver Voltameter.— The ampere is that current strength which, in a specified voltameter, deposits 0.001118 g of silver in one second. The specifications for the voltameter are given in the references. The silver voltameter is employed when the greatest accuracy is desired, and especially for currents of less than one ampere. It may be used to calibrate a standard ammeter.

REFERENCE. — *Carhart and Patterson*, Electrical Measurements, pp. 158 and 328.

CXXXVIII. JOULE'S EQUIVALENT WITH THE ELECTROCALORIMETER

Determine the mechanical equivalent of heat with the electrocalorimeter.

245. Joule's Equivalent; the Joule; the Watt. — Joule's equivalent of heat (Arts. 139 and 150) must be distinguished from the unit of energy, the *joule*. The joule is 10^7 ergs, and Joule's equivalent is 4.2 joules. The joule is defined electrically as the quantity of work done in the transference of a quantity of electricity of 1 coulomb under a pressure of 1 volt; it is also equal to the work done in 1 second by a current of 1 ampere passing through a resistance of 1 ohm.

The joule is the quantity of work equal to that defined, regardless of the time in which it is performed. The power to do this much work in one second — that is, to do a joule of work per second — is the *watt*. From the relation of the various units it follows that the power of a given current in watts is measured by the product of the current strength in amperes and the pressure in volts.

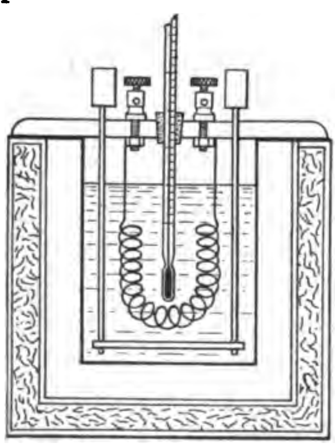


FIG. 155. ELECTROCALORIMETER

Joule's equivalent of the unit of heat may be determined with the electrocalorimeter, which consists of a coil of wire so placed in a water calorimeter that the heat developed by the passage of a current through the coil can be measured. Fig. 155 shows a simple form of such a calorimeter, while a more elaborate form is shown in Fig. 71.

Weigh the calorimeter cup and stirrer, and measure the thermometer; fill the cup with water, cooled 10° below the air temperature, to such a depth that the coil will be wholly immersed when it is in position, and find the weight of the water. Let K be the heat capacity of the calorimeter, determined as explained in Art. 140.

Measure the resistance of the coil when it is in place.

Observe the temperature of the calorimeter. Connect the current through a rheostat and an ammeter to the coil terminals, and allow it to flow until the temperature is as much above that of the room as at the beginning it was below this temperature. The current strength, which may be about five amperes, should be kept constant, and its average value, I, must be measured as accurately as possible. Note carefully the temperature and time when the current is started, and the temperature and time at the end of the experiment.

Again measure the resistance of the coil, and let R be the mean of its resistance before and after heating.

If $t_2 - t_1$ is the temperature change in the calorimeter while the current is passing, the heat developed is

$$H = K(t_2 - t_1).$$

The current, I, being expressed in amperes, the resistance, R, in ohms, and the time, T, during which the current passes, in seconds, the value of Joule's equivalent in C. G. S. units is

$$J = \frac{I^2RT}{H} \cdot 10^7.$$

By assuming the value of J, the method may be used to determine the current strength or the resistance.

CXXXIX. FIGURE OF MERIT OF A GALVANOMETER BY DIRECT DEFLECTION

Determine the figure of merit of a galvanometer, including the measurement of the battery and galvanometer resistances.

246. Figure of Merit of a Galvanometer. — The figure of merit of a galvanometer is a statement of its sensitiveness, and is usually expressed by the fraction of an ampere of current that would cause an apparent deflection, in a reflecting galvanometer, of 1 mm on a scale placed 1 m from the mirror. It is also sometimes expressed by stating the number of ohms resistance through which an electromotive force of 1 volt would have

to be passed in order that the current shall be so reduced as to produce 1 mm deflection on a scale 1 m distant from the mirror.

Connect the galvanometer in series with a cell of known electromotive force and a high resistance. Adjust the resistance to secure a readable deflection, preferably small. From the known electromotive force and the resistance of the circuit, including that of the galvanometer and battery, the current strength can be calculated. This in connection with the observed deflection enables the calculation of the figure of merit as defined.

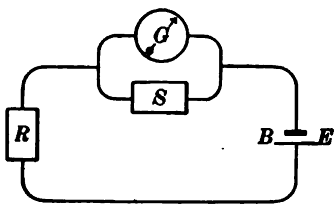


FIG. 156. FIGURE OF MERIT

If the galvanometer is a sensitive one, it will not be convenient to put a sufficiently high resistance in series with it to keep the deflection within readable limits. It will then be necessary to shunt the galvanometer (Art. 247). Connect the galvanometer, shunt, resistance, battery, and key, as indicated in Fig. 156. Let G, S, R, and B represent the resistances of the several parts of the apparatus, and E the electromotive force of the battery. Then the current through the battery is

$$I_B = \frac{E}{R + B + \dfrac{GS}{G+S}},$$

and that part of the current which flows through the galvanometer is

$$I_G = \frac{E}{R + B + \dfrac{GS}{G+S}} \cdot \frac{S}{G+S}.$$

If
$$R_V = \left(R + B + \frac{GS}{G+S}\right)\frac{G+S}{S},$$

$$I_G = \frac{E}{R_V}.$$

The quantity R_V may be called the *virtual* resistance; it is the resistance through which the given electromotive force would produce a current equal to that which actually passes through the galvanometer.

If the observed deflection of the needle is d centimeters on a scale D centimeters from the mirror, the deflection expressed in millimeters for a scale at a distance of one meter is

$$\Delta = \frac{10\,d}{\frac{D}{100}} = \frac{1000\,d}{D};$$

and finally the figure of merit is

$$F = \frac{I_G}{\Delta} = \frac{E}{R_V} \cdot \frac{D}{1000\,d}.$$

A Daniell cell is usually sufficient for this work, though a standard cell may be used. The galvanometer resistance may be determined by Thompson's method (Art. 227) or otherwise, and the battery resistance by Kohlrausch's method (Art. 231). Usually, however, the resistance of the battery and of the shunted galvanometer will be small, and may be neglected in comparison with the resistance R.

A numerical example follows.

Figure of Merit

Queen D'Arsonval Ballistic Galvanometer No. 92

May 20, 1897

Clark Standard Cell, H. & B. No. 100, at 19° .	$E =$	1.43 volts
By the telephone bridge the resistance of cell .	$B =$	20. ohms
Galvanometer resistance	$G =$	1039.5
Series resistance	$R =$	22000.
Shunt resistance	$S =$	2.

$$R_V = \left(22000 + 20 + \frac{2 \times 1039.5}{1039.5 + 2}\right)\frac{1039.5 + 2}{2} = 11\,465000 \text{ ohms.}$$

The following steady deflection readings were obtained with the scale distance $D = 100$ cm.

Zero Reading	Second Reading	Deflection
38.85 cm	56.35 cm	17.50 cm
39.30	56.60	17.30
39.33	56.60	17.27
39.33	56.68	17.35
39.20	56.40	17.20
39.30	56.40	17.10
39.30	56.43	17.13
39.30	56.40	17.10
39.30	56.40	17.10
39.35	56.45	17.10
	Mean	$d = 17.22$ cm

$$F = \frac{1.43 \times 100}{11\,465000 \times 1000 \times 17.22} = 0.000000\,00072 \text{ amperes.}$$

247. Sensitiveness of Galvanometers; Shunts. — A suspended magnet galvanometer is made more sensitive by using a longer suspension fiber, or one with less torsion. A fiber of quartz is used for the most delicate work. The sensitiveness also varies with the number of turns of wire in the coils and with their nearness to the needle. The coils are often subdivided, are made interchangeable, and adjustable as to distance, as shown in Fig. 134. The sensitiveness will be increased by diminishing the effect of the earth's field, through astatization or a control magnet (Art. 210), or by surrounding the galvanometer with an iron shield.

The sensitiveness of a suspended coil galvanometer depends upon its suspension, the number of turns of wire in the coil, and the strength of the field magnets. It is nearly independent of the earth's field. Practically the sensitiveness of this type of instrument is altered by interchanging the entire suspended system, the construction often making this a convenient operation.

Galvanometers such as are commonly used for laboratory work have figures of merit ranging from 0.000000 3 to 0.000000 0002.

If a less sensitive arrangement of the galvanometer is desired, it may be secured by using fewer turns of wire, either employing only a part of the windings already provided or substituting new windings. The two coils may be connected differentially (Art. 219) so that one portion only partially neutralizes the other. Often the most convenient method is to use a *shunt*, — that is, to connect the two galvanometer terminals by a small resistance.

A shunt does not reduce the sensitiveness proper of a galvanometer, but it enables the instrument to be used for measuring larger currents, since the shunt carries the greater part of the current, and only the smaller part passes through the galvanometer. If the ratio between the two parts is known, and that part through the galvanometer is measured, the total current may be determined. The current divides between the galvanometer and shunt in the inverse ratio of their resistances. If I_S and I_G represent the currents through the shunt and galvanometer, and S and G are their respective resistances, then

$$\frac{I_S}{I_G} = \frac{G}{S},$$

and

$$\frac{I_S + I_G}{I_G} = \frac{S + G}{S};$$

but $I_S + I_G = I$, the total current, and hence

$$I = I_G \cdot \frac{S + G}{S}.$$

If $S = \frac{1}{9} G$, the current through G is $\frac{1}{10}$ of the total current, and similarly for other values of S. $\frac{S + G}{G}$ is the *multiplying power* of the shunt.

The actual resistance through the galvanometer and shunt combined is found from the conductivity of the combination, which is equal to the sum of the separate conductivities. This conductivity is

$$K_{GS} = \frac{1}{G} + \frac{1}{S} = \frac{S + G}{GS}.$$

The resistance of the combination is equal to the reciprocal of the conductivity; that is,

$$R_{GS} = \frac{1}{K_{GS}} = \frac{GS}{S + G}.$$

The resistance of any number of multiple circuits is found in the same manner.

CXL. CURRENT STRENGTH WITH THE ELECTRO-DYNAMOMETER

Calibrate an electrodynamometer and measure with it the strength of an alternating current.

248. The Electrodynamometer. — An electrodynamometer is an instrument for measuring the strength of a current by means of the attractions and repulsions between a fixed and a movable conductor carrying this current. The conductors usually consist of flat rectangular coils of wire, the movable one being suspended with its plane normally at right angles to that of the fixed coil. The suspension is a silk thread and a spiral spring to give increased torsion. The support of this spring can be turned through a measurable angle to increase the torsion at will.

In use the instrument is to be set up with the plane of the fixed coil in the magnetic meridian, and with the movable coil swinging freely. The torsion head is adjusted so that the index of the movable coil is at 0, — that is, until the two coils are at right angles. The current to be measured is then sent through the coils, which will tend to rotate the movable coil. This tendency is counteracted by turning the torsion head of the spring to maintain the coil in its initial position.

If θ is the twist of the spring and K the constant of the instrument, the current strength is

$$I = K\sqrt{\theta}.$$

The instrumental constant may be determined by the copper voltameter in a manner similar to that described for the tangent galvanometer in Art. 242.

The electrodynamometer is particularly adapted to the measurement of alternating currents, since a current in either direction tends to turn the coil in one direction.

REFERENCE. — *Carhart and Patterson*, Electrical Measurements, pp. 127 and 167.

CHAPTER XXII

ELECTROMOTIVE FORCE

CXLI. ELECTROMOTIVE FORCE BY COMPENSATION

Compare the electromotive forces of several cells with that of a standard cell, using the compensation method.

249. Standard International Volt; Standard Cells. — The international volt is that potential difference which will produce a current strength of one ampere (Art. 244) through a resistance of one ohm (Art. 212). It is impossible to produce directly an electromotive force of exactly one volt; it is therefore necessary to define it as a certain fraction of the electromotive force of a standard cell. A standard cell must be capable of reproduction, and must be constant.

The Clark Standard Cell is the almost universal standard of electromotive force. It has mercury for its positive electrode, and amalgamated zinc for the negative; the electrolyte is a saturated solution of mercurous sulphate and zinc sulphate. Minute specifications and notes for the preparation of standard cells are given in the reference. The form of the Carhart-Clark portable cell is shown in Fig. 157. The materials are sealed in a small test tube, and the whole is mounted in a protecting case which often contains a thermometer.

The electromotive force of the Clark cell at $t°$ is

$$E_C = 1.4292 - 0.00123\,(t - 18) - 0.000007\,(t - 18)^2 \text{ volts.}$$

At the usual laboratory temperatures this cell has the electromotive force, in volts, as given in this table.

t	15°	18°	20°	21°	22°	23°	24°	25°
E_C	1.4328	1.4292	1.4267	1.4255	1.4242	1.4229	1.4216	1.4202

The Weston Standard Cell is similar to the Clark cell except that cadmium and cadmium sulphate are substituted for zinc and zinc sulphate. It has the advantages over the Clark cell of a temperature coefficient which is practically zero, and in having an electromotive force which is nearly one volt. If it is made with a constant solution of cadmium sulphate which is saturated at 4°, its electromotive force at the usual laboratory temperatures is

$$E_W = 1.0190 \text{ volts.}$$

That the electromotive force of a standard cell may remain constant the cell must be used with great care and only in open circuit or compensation methods. To prevent polarization and consequent deterioration, the current taken from such a cell should never exceed $\frac{1}{20000}$ of an ampere; that is, it should not be used directly in a circuit of less than 30000 ohms resistance. If the cell is of small size, a much higher safety resistance may be required.

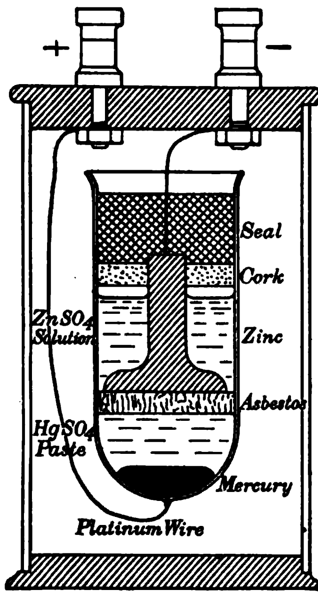


Fig. 157. Carhart-Clark Standard Cell

Reference. — *Carhart and Patterson*, Electrical Measurements, pp. 176–186, 330–335.

250. Electromotive Force by Compensation. — One of the most useful methods for the comparison of electromotive forces, the compensation method, may be described as follows. Connect two similar resistance boxes, R and R' (Fig. 158), each of 10000 ohms or more, in series with a constant battery. The battery may consist of one or more cells, such that its electromotive force shall be somewhat in excess of that of any cell to be measured. The total resistance in use in the two boxes between A and B is always to be equal to the *total resistance of one box.*

The battery B will maintain a constant potential difference, e, between these points, while the fall of potential between A and D will be proportional to that part of the total resistance which is in R. Connect one of the cells to be tested, a sensitive galvanometer, a safety high resistance (100000 ohms, for instance), and a key, in series, as a shunt to the resistance R, as indicated. The cell must be so placed that its electromotive force will oppose that of the battery B. *Keeping the sum of the resistances in use in R and R' always the same*, so vary that part of the total resistance which is in R that upon momentarily closing the key the galvanometer shows no deflection. If the two resistance boxes are of the same pattern, the adjustment is facilitated by noting that when a plug is removed from any place in one box a plug is always put into the corresponding place in the other box. If the arrangement is not sufficiently sensitive, the resistance Q may be removed after compensation has been approximately secured, and the adjustment is then perfected. The resistance Q must be replaced before beginning measurements for a second cell.

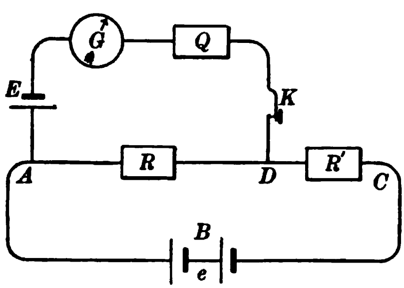


FIG. 158. COMPENSATION METHOD

When the adjustment is completed, the electromotive force tested, E_1, is compensated by the potential difference between A and D; and if R_1 is the resistance used in R, and R_1' that in R',

$$E_1 : e = R_1 : R_1 + R_1'.$$

Substitute for E_1 a second cell of electromotive force E_2, and adjust as before; if R_2 is the resistance in R necessary for compensation,

$$E_2 : e = R_2 : R_2 + R_2';$$

and since $R_1 + R_1' = R_2 + R_2'$,

$$E_1 : E_2 = R_1 : R_2.$$

251. The Potentiometer. — With an especially arranged resistance box, called a compensation apparatus or potentiometer, the method described above for comparing electromotive forces becomes very convenient, and is perhaps the most precise method known. The resistance box is constructed so that the resistance in one circuit remains constant while that in the branch circuit is varied through all possible values.

One of the most convenient forms of compensation apparatus has its parts arranged as shown in Fig. 159, its total resistance being 14999.9 ohms. The following order of adjustment may be followed. The standard cell, a Clark element for instance, is connected at S; the unknown electromotive force,

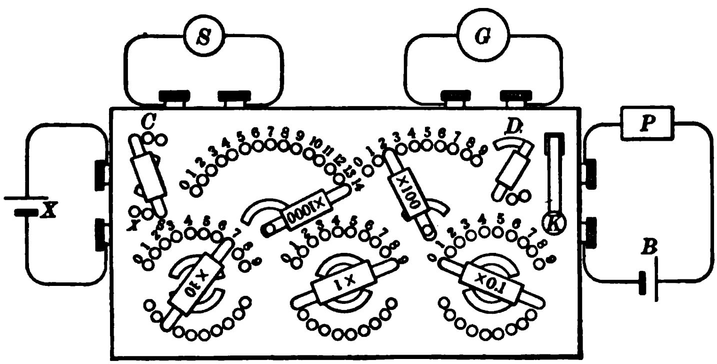


FIG. 159. THE POTENTIOMETER

which may not exceed 1.5 volts, is connected at X; a galvanometer at G; and a battery in series with a resistance of 5000 ohms or more is connected as shown at B and P. This battery must have an electromotive force somewhat greater than 1.5 volts; it may be a single accumulator cell, or two gravity or Leclanché cells. The several switches are to be set to indicate the significant figures of the electromotive force of the standard cell. If the Clark cell is at the temperature 19°, for example, its electromotive force is 1.4279. Set the switch of the thousands row at 14, the hundreds switch at 2, the tens at 7, the units at 9, and the tenths switch at 0. Then the resistance of

that part of the main circuit to which the galvanometer is connected is 14279.0 ohms. Turn the switch D to 100000, which places a safety resistance of 100000 ohms in the circuit of the standard cell corresponding to Q of Fig. 158. Now adjust the resistance P so that upon closing the key K there will be no deflection of the galvanometer. Turn D to 10000 and improve the adjustment of P if required; and finally turn D to 0, which cuts out the safety resistance, and perfect the adjustment of P till no deflection results from closing the key.

This makes the instrument *direct reading*, in that the number of ohms required for compensation is exactly ten thousand times the electromotive force of the cell attached, the standard in this case.

Turn the switch D back to 100000, and turn the switch C to X, which places the unknown cell in the compensation circuit. By adjusting the resistance levers of the potentiometer, place them so that again there is no deflection when the key is closed; turn D to 10000, and improve the adjustment, and finally turn D to 0, and perfect it. The reading of the various dials of the potentiometer is then ten thousand times E, the electromotive force of the cell X being tested.

The potentiometer may be used without the extra resistance P, but it is then not direct reading, and the proportion of the previous article is to be used in computing the unknown electromotive force. In this case also the unknown electromotive force may have any value not exceeding that of the battery B.

CXLII. ELECTROMOTIVE FORCE WITH A CONDENSER

Compare the electromotive forces of several cells with that of a standard cell by means of a condenser.

252. Comparison of Electromotive Forces with the Condenser. — If a condenser (Art. 262) is charged from several different sources, the charges received will be proportional to the electromotive forces of the sources. By measuring these charges the electromotive forces may be compared. This method has the

advantage that the cells are used only on open circuit, thus avoiding polarization.

Connect a ballistic galvanometer, G (Art. 253), a condenser, C, a charge and discharge key, K (Art. 263), and one of the cells to be tested, B, as indicated in Fig. 160. The key being in charge position, the condenser will receive a charge proportional to the electromotive force, E_1, of the cell. By pressing the key firmly, but for a very short time, upon the galvanometer contact, the condenser will discharge through the galvanometer, and will deflect the needle through an angle, θ_1, such that $\sin \frac{1}{2}\theta_1$ is proportional to the charge. By substituting for the cell first used a second one of electromotive force E_2, a deflection, θ_2, will be obtained. If the distance of the scale from the mirror is D and the two observed scale readings are d_1 and d_2,

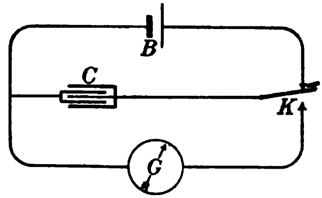


FIG. 160. CONDENSER METHOD

$$E_1 : E_2 = \sin \tfrac{1}{2}\,\theta_1 : \sin \tfrac{1}{2}\,\theta_2 = \sin \tfrac{1}{4} \tan^{-1} \frac{d}{D} : \sin \tfrac{1}{4} \tan^{-1} \frac{d_2}{D}.$$

If the angular deflections of the needle do not exceed 6°, the sines and tangents may be taken as proportional to the corresponding arcs, and then

$$E_1 : E_2 = d_1 : d_2.$$

Any number of cells being thus compared, if one of them is a standard, the electromotive forces of the others may be determined.

(CAUTION. — Do not allow the cells to be short circuited nor to be directly connected to the galvanometer.)

253. The Ballistic Galvanometer. — A ballistic galvanometer is one designed to measure the total quantity of electricity passing through it in a current of very short duration. The needle must have a period so long that the current will have ceased before it has moved appreciably from its position of rest. The needle should have the least possible damping. The quantity of

electricity thus passing through the galvanometer is proportional to the sine of half the angle of the first swing of the needle.

The deflections of the needle are usually measured by observing with a telescope a centimeter scale as seen reflected in a mirror attached to the needle. The following adjustments are essential. If the galvanometer is of the suspended magnet type (Art. 209), the plane of the coils must be in the plane of the magnetic meridian; and the suspension must be so adjusted that when the needle is at rest the torsion is zero, and so that the needle is magnetically symmetrical with respect to the plane of the magnetic meridian. If the galvanometer is of the suspended coil (D'Arsonval) type, the effect of the earth's field may be neglected; and it is sufficient that the torsion of the suspensions is zero when the needle is at rest, and that the plane of the coil is parallel to the force of the stationary field magnets. The direction of the mirror will usually indicate whether these conditions are approximately fulfilled, while a sufficient test of the accuracy of the adjustment is that equal charges sent through the galvanometer in opposite directions produce equal and opposite deflections. Fig. 161 represents a high sensibility D'Arsonval galvanometer, which is exceedingly convenient for ballistic observations.

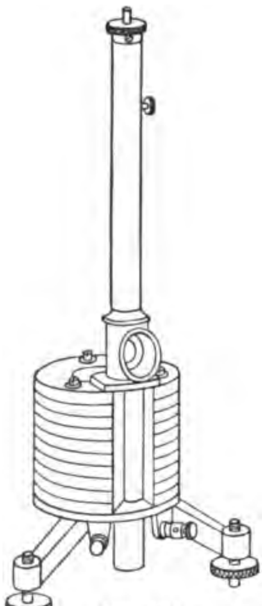


Fig. 161
D'Arsonval Galvanometer

In addition to these adjustments of the galvanometer it is necessary that the telescope and scale be so placed that the mirror when at rest reflects to the cross wires of the telescope that point of the scale which is in the same vertical plane as the axis of the telescope.

When these adjustments are made the observed displacements of the scale, when the needle is deflected, are proportional to the

tangents of twice the angles of deflection. Tables are often provided, giving corrections, which, when added to the scale readings, make these readings directly proportional to the arcs, or to some function of the arcs, whose tangents they measure.

After a ballistic galvanometer has been deflected, some special manipulation is required to bring the needle quickly to rest. If it is of the suspended coil type, a short circuiting of the galvanometer is the most efficient method. The current induced by the motion of the needle in the field of the magnet reacts upon the needle, damping its vibrations. If the galvanometer is of the suspended magnet type, the motions of the needle may be checked by the manipulation of a permanent magnet held in the hand; or by temporarily connecting to the galvanometer a small coil of wire which is arranged to slide over the end of a fixed bar magnet. By moving this coil on the magnet an induced current is produced whose direction and strength are easily controlled, and which can be made to bring the needle to rest. When this is accomplished the circuit of the auxiliary coil is broken. If the needle cannot be brought entirely to rest, the condenser may be discharged precisely when the needle is at a turning point in its vibrations, and the deflection is to be calculated from this turning point and not from the point of equilibrium.

For further consideration of the ballistic galvanometer, see Arts. 267 and 268.

CXLIII. ELECTROMOTIVE FORCE WITH THE QUADRANT ELECTROMETER

Compare the electromotive forces of various cells.

254. The Quadrant Electrometer. — The quadrant electrometer has four metallic *quadrants*, which together form a short cylindrical box, the quadrants being supported within a metal case by insulating posts and separated from one another by small spaces; the alternate quadrants are connected by wires, and each pair is connected to an insulated binding post outside the case; within the quadrants a flat aluminum needle is

suspended either by a bifilar silk suspension or by an insulated metallic wire suspension. In Fig. 162 the electrometer is shown with the case open. The quadrants are at Q, one of them being removed to show the needle N. In the bottom of the case is a glass dish containing pure concentrated sulphuric acid. The needle connects with the acid either by a platinum wire or by a piece of mica. The acid may serve three purposes: it keeps the interior of the electrometer dry, and it checks the vibration of the needle; it may also serve as the inner coating of a Leyden jar, whose outer coating is the metal case, giving to the needle an increased capacity. This last use of the acid is not necessary, and if the connection between needle and acid is mica, the condenser effect is not employed.

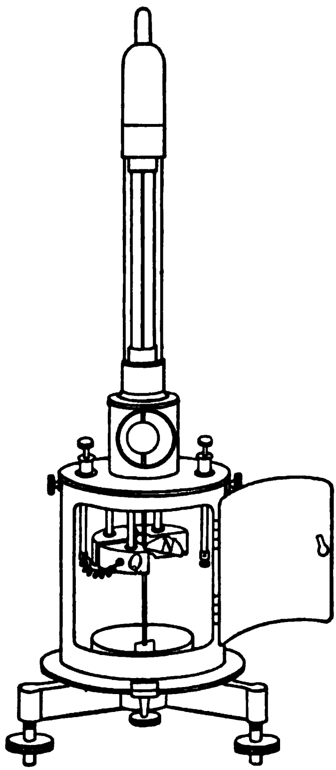


FIG. 162. QUADRANT ELECTROMETER

By means of sliding rings on the quadrant binding posts, connect the quadrants to the metal case, and connect the case to the ground. The needle being discharged, so adjust its support that it hangs symmetrically over one of the diametral spaces separating the quadrants. The position of the mirror will indicate this adjustment with sufficient accuracy. Give the needle a static charge from any convenient source, as an electrophorus, through the acid or through the metallic suspension, according to the construction of the instrument. If the needle is permanently deflected by this charge, it must be brought back to its zero position by adjusting one of the quadrants provided with a movement for this purpose, or by

means of the leveling screws, or by a slight adjustment of the torsion head.

Now insulate the quadrants and connect one pole of the cell to be tested to one quadrant binding post, and the other pole to the second post. The potential difference given to adjacent quadrants will turn the needle until the electrostatic actions are balanced by the couple generated in the twisted suspension. The angle of deflection is proportional to the potential difference. It is desirable to deflect the needle first in one direction and then in the other by reversing the cell connections; this is conveniently accomplished by means of a special reversing key devised by Lord Kelvin. The mean of the two deflections is free from errors of torsion.

If the average scale deflection produced by an electromotive force, E_1, is d_1, and d_2 is that due to an electromotive force, E_2, the scale being distant from the mirror D, then

$$E_1 : E_2 = \tan^{-1}\frac{d_1}{D} : \tan^{-1}\frac{d_2}{D}.$$

To determine the potential of an insulated conductor, connect one pair of quadrants to the ground, and the second pair to the conductor in question.

The electrometer may also be used by connecting the quadrants to the poles of a large series battery, the middle of the series being connected to the ground. This maintains the adjacent quadrants at equal opposite potentials. The conductor whose potential is to be determined, as referred to the zero potential of the earth, is connected to the needle.

255. Electrometer for measuring Large Potential Differences. — If the electrometer is too sensitive when arranged as above explained, it may be made less sensitive by connecting the needle and one pair of quadrants to one pole, and the other pair of quadrants to the second pole. In this case the deflection of the needle is proportional to the square of the potential difference.

Arranged in this manner the quadrant electrometer is applicable to the measurement of alternating potentials.

REFERENCE. — *Stewart and Gee*, Practical Physics, Vol. II, pp. 431–438.

CXLIV. ELECTROMOTIVE FORCE WITH THE CAPILLARY ELECTROMETER

Determine the variation of contact difference of potential between copper and copper sulphate solution, as the concentration of the solution varies.

256. The Capillary Electrometer. — Lippmann has devised a simple electrometer based upon the fact that the surface tension of mercury in contact with an electrolyte varies with the difference of potential at the surface of contact. When a current passes from the acid to the mercury the area of the surface of separation tends to decrease, owing to an increase of surface tension. A large mercury surface and a small capillary surface are connected by dilute sulphuric acid (one part of acid to five of water) in a suitably shaped glass vessel (Fig. 163). At C is a capillary surface in a tube of about 0.5 mm bore, and at S is the large surface. Platinum wires connect with the two portions of mercury.

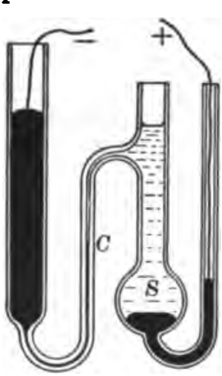


FIG. 163. CAPILLARY ELECTROMETER

When a difference of potential is applied to the two wires the capillary meniscus changes its position by amounts approximately proportional to this difference when it does not exceed a tenth of a volt. A small microscope with an eyepiece micrometer is convenient for observing the meniscus. The deflection may be from five to ten scale divisions for 0.001 volt.

A more sensitive form of electrometer, giving certain indications to 0.0001 volt, employs a much finer capillary, and has its parts arranged as shown in Fig. 164. The upper mercury tube is 50 cm or more long; connected to it by a flexible tube is a reservoir, permitting the height of the mercury to be varied. The capillary dips into dilute sulphuric acid which rests on mercury in the lowest vessel. A microscope is provided to observe the capillary surface of separation.

The sensitiveness of the electrometer depends upon the fineness of the capillary; if the point is conical, the mercury may be made to assume a desired position by varying the height of the upper column. The displacement of the meniscus measured with the micrometer eyepiece may serve to determine the potential difference; or when the surface has been displaced it may be restored to its former position by changing the pressure above, the potential difference being considered proportional to the change in pressure. The electrometer is standardized by comparison with a standard cell; but as the relation between pressure and potential changes and has therefore to be frequently redetermined, the instrument is satisfactory only for measuring very small differences of potentials and for indicating the absence of potential difference in compensation methods (Art. 250).

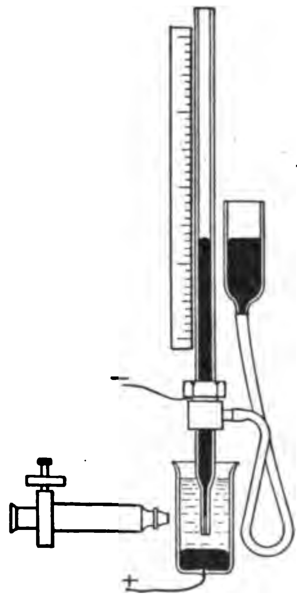


FIG. 164. CAPILLARY ELECTROMETER

The mercury in the capillary should always be connected to the negative (zinc) pole, otherwise the mercury becomes corroded. If the difference of potential exceeds one volt, hydrogen is evolved, which interferes with proper action. If either of these effects has been produced, the capillary meniscus is cleaned by forcing a drop of mercury entirely through the capillary.

Occasionally the mercury should be drawn up out of the capillary, allowing the acid to wet the small tube, as this is a condition of sensitiveness.

The electrometer remains in condition for use only when it is kept on closed circuit. Hence a three-point key (Fig. 170) is desirable, arranged to short circuit the electrometer when the lever is in its normal position, the short circuit being broken

and the external electromotive force being applied when the key is pressed.

Instructions for making three forms of capillary electrometers are given in the reference.

REFERENCE. — *Ostwald*, Physico-Chemical Measurements, pp. 202–208.

257. Contact Difference of Potential. — Any two dissimilar substances in contact are at different potentials. In the cases of two solids or two metals this difference is usually very small, while for a metal and an electrolyte it may be about a volt. The difference depends, for a given metal and electrolyte, upon the concentration of the ions of this metal in the electrolyte.

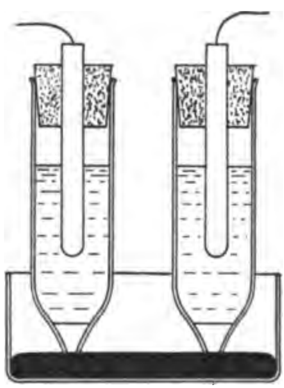


FIG. 165. CONTACT CELL

The variation with concentration may be observed in a concentration cell, as suggested by Ostwald, arranged as shown in Fig. 165. Two short glass tubes each have one end conically reduced; in the small end is placed a plug of macerated filter paper. These tubes are filled with the electrolytes, either of different salts or of different concentrations of the same salt. The electrodes are supported by corks, and the bottoms of the tubes are placed in a suitable dish which contains a conducting liquid, — mercury, for instance.

The compensation method is the best for making the measurements. The connections are to be made as shown in Fig. 166. R and R' are two resistance boxes which for ordinary measurements may be of 1000 ohms, or for more refined work may be of 10000 ohms each, as described in Art. 250. B is a cell which furnishes a constant difference of potential greater than that of the cell being investigated. It may be a small secondary cell or a gravity cell. C is the concentration cell to be measured, E is the capillary electrometer, and K is a three-point short-circuiting key. The resistances in R and R' are to be varied, their sum always being 1000 (or 10000) ohms, till

compensation is secured. The value of the difference of potential at C is determined by substituting for C a standard cell (Art. 249).

The difference of potential measured is the difference of the two contact differences of potential of the two electrodes of C, since these are oppositely directed. For the laboratory exercise first fill the two tubes with equal solutions of 1 : 200 gram-molecular solution of anhydrous copper sulphate. The proportions for this are copper sulphate crystals ($CuSO_4 + 5\ H_2O$), 250 g, and water (3600 – 90), 3510 g. One tube still containing the first solution, the second is to be filled with a 1 : 150 solution (250 g of crystals to 2610 g of water); and again with a 1 : 100 solution (250 g of crystals to 1710 g of water); the change of potential being measured in each case.

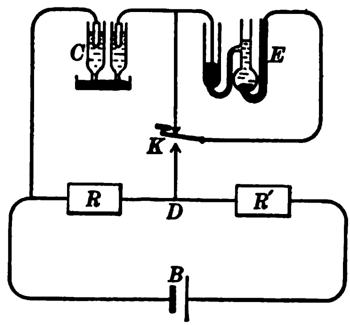


FIG. 166. COMPENSATION METHOD WITH ELECTROMETER

REFERENCE. — *Ostwald*, Physico-Chemical Measurements, pp. 209–216.

CXLV. ELECTROMOTIVE FORCE WITH AN AMMETER

Determine the electromotive forces and resistances of several cells, using a milliammeter or tangent galvanometer.

258. Electromotive Force and Resistance of Cells with a Milliammeter and a Resistance. — The following methods for investigating constant cells are based upon Ohm's Law, and they require very simple apparatus. A sensitive ammeter, a tangent galvanometer, or other galvanometer whose law is known, and a resistance box, are all that is needed. For three of the methods it is necessary to know the resistance of the galvanometer, and for one its constant is required. The formulæ will be given for the ammeter, but if another instrument is used, it is only necessary, except in the first formula, to substitute for

the current value I, the function which is proportional to the current, as $\tan\theta$ for a tangent galvanometer, etc.

In the same manner as is developed in Art. 217, if I_1 and I_2 are the current strengths flowing through resistances R_1 and R_2 in series with the ammeter, produced by a cell whose electromotive force is required, the value of the electromotive force, independent of the resistances of the battery and ammeter, is

$$E = \frac{I_1 I_2}{I_2 - I_1}(R_1 - R_2);$$

and if G is the resistance of the ammeter, the battery resistance is

$$B = \frac{I_1 R_1 - I_2 R_2}{I_2 - I_1} - G.$$

Electromotive forces may be compared, using a milliammeter and a resistance box, in four ways, as follows.

Let G be the ammeter resistance, and B the resistance of the cell which produces a current I_1, through a resistance R_1 in series with the ammeter; and let B_2, I_2, and R_2 be the corresponding quantities for a second cell. Representing the electromotive forces of the cells by E_1 and E_2 respectively,

$$E_1 : E_2 = (B_1 + G + R_1)\, I_1 : (B_2 + G + R_2)\, I_2.$$

If $B_1 + R_1$ be made equal to $B_2 + R_2$, then

$$E_1 : E_2 = I_1 : I_2.$$

If R_1 and R_2 be so adjusted that I_1 equals I_2, then

$$E_1 : E_2 = G + B_1 + R_1 : G + B_2 + R_2.$$

Connect the two cells to be compared in series and let I_1 be the current flowing through any convenient resistance; interchange the poles of one cell so that the electromotive forces are opposed, and let I_2 be the current flowing, the resistance remaining unchanged; then

$$E_1 : E_2 = I_1 + I_2 : I_1 - I_2.$$

CXLVI. THERMO-ELECTROMOTIVE FORCE BY DIRECT MEASURE

(a) Determine the thermo-electromotive force of lead-copper, lead-iron, and copper-iron thermo-elements at various temperatures between 0° and 100°. Plot the results. Make a thermo-electric diagram for copper and iron, and find their neutral temperature.
(b) Calibrate a thermo-electric pyrometer.

259. Thermo-Electromotive Force. — If one junction of a thermo-couple is at a constant temperature, 0°, while the other junction is raised to different temperatures, since the contact difference of potential in general varies with the temperature, there will result an electric current the electromotive force of which is to be measured.

To the ends of a wire of one of the metals to be studied, solder wires of the second material. Place the two junctions in narrow test tubes in which are thermometers, and place the tubes, one in an ice bath and the other in a water or oil bath.

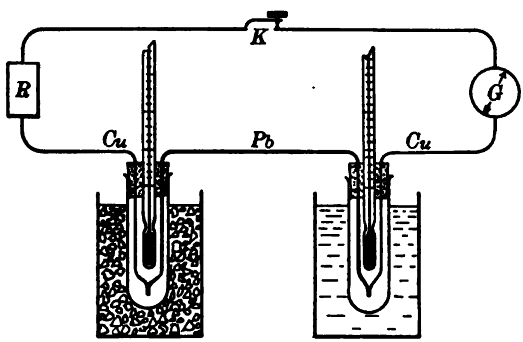


FIG. 167. THERMO-ELECTROMOTIVE FORCE

For better heat conduction the tubes may be partially filled with petroleum. Connect the two ends of the thermo-couples, through a resistance, to a galvanometer, as indicated in Fig. 167.

By varying the temperature of one junction as much as is convenient, the varying electromotive force is measured with the galvanometer. This electromotive force may be taken as

proportional to the deflection. The resistance is included in the circuit, to regulate the current so that the deflection may be readable. It is preferable that this should not be altered during the measures, in which case, if the total resistance in the circuit, including the galvanometer resistance, is over fifty ohms, the value of the resistance need not be considered. If the circuit resistance is low, the variation of resistance of the thermo-couple with temperature must be taken into account.

In the several experiments the direction of the electromotive forces must be noted.

Instead of measuring the deflection only, it is better to measure the actual electromotive force by the compensation method of Art. 250. Or if the galvanometer constant is known (Art. 246) the electromotive force may be computed from the resistance of the circuit, and the deflection.

260. Thermo-Electric Power and Thermo-Electric Diagram. — The ratio of the electromotive force of a thermo-element to the difference in temperature of the junctions is the thermo-electric power of the couple at the mean temperature. A thermo-electric diagram is one showing these relations. Such diagrams are usually constructed with reference to lead as the standard, since hot lead and cold lead in contact show no thermo-electromotive force, while with other metals there is a difference of potential between the metal hot and the same metal cold.

The neutral point for two metals may be found from the diagram; it is the temperature corresponding to the point of intersection of their thermo-electric power lines.

CXLVII. ERRORS OF A VOLTMETER BY COMPENSATION

Calibrate a voltmeter at ten or more equidistant points of its scale.

261. Calibration of Voltmeter. — The compensation method of Art. 250 may be adapted to the calibration of a voltmeter, as follows. A storage or other battery of such size as to give the various voltages corresponding to the points at which it is desired to calibrate the voltmeter must be provided. Pass the

current from this battery through a standard resistance, R_1, of large value, 100000 ohms for example, and through a standard of variable resistance, R_2, connected in series. Connect the voltmeter V (Fig. 168), to indicate the total difference of potential; and to the terminals of R_2 connect one or more standard cells, S, a galvanometer, G, and a key, K, in series, so that the electromotive force of the cells shall oppose that of the battery B. It will now be possible so to adjust the resistance R_2 that the difference of potential between its terminals shall be exactly equal to that of the standard cells. That this condition has been secured will be indicated by there being no deflection of the galvanometer when the key K is closed. Since the fall of potential is proportional to the resistance, the total potential, E, measured by the voltmeter, is to that of the n standard cells, ne, as $R_1 + R_2$ is to R_2;

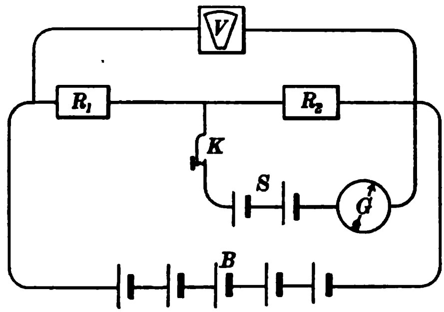


FIG. 168. VOLTMETER CALIBRATION

$$E = ne \frac{R_1 + R_2}{R_2}.$$

The difference between this computed value of E and the voltmeter reading is the correction to the scale at this point.

Alter the battery potential and repeat the process for each point to be calibrated.

CHAPTER XXIII

CAPACITY

CXLVIII. RELATIVE CAPACITY WITH A BALLISTIC GALVANOMETER

Compare the capacities of the various parts of a divided condenser. Determine the capacity of a condenser by comparison with a standard condenser.

262. Standard International Coulomb and Farad; Standard Condensers. — The coulomb is that quantity of electricity which is transferred by one ampere (Art. 244) in one second. The farad is the capacity of a conductor such that its potential is increased by one volt (Art. 249) when it receives a charge of one coulomb. The farad is too large a capacity for laboratory uses, and the one millionth part of the farad, called the *microfarad*, is the working unit.

Standard condensers are usually constructed of tin-foil sheets with mica insulation, and have capacities of $\frac{1}{3}$ or 1 microfarad. They are sometimes subdivided into five parts of 0.5, 0.2, 0.2,

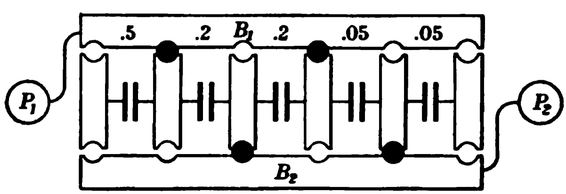


FIG. 169. PLAN OF DIVIDED CONDENSER

0.05, and 0.05 microfarads, which by means of the blocks and plugs shown in Fig. 169, can be connected to give a great variety of capacities. The first section of the condenser has one terminal

connected to the first block and the other to the second block; the second section of the condenser is connected between the second and third blocks, etc. The charging current is to be connected to the posts P_1 and P_2, which connect respectively to the bars B_1 and B_2. If any two *consecutive* blocks are connected to *opposite* bars, it makes no difference which, that portion of the condenser between the blocks will receive a charge. In Fig. 169 the portion of the condenser connected is 0.45.

The method of connection described is that in which the several parts are in parallel. They might be connected in series, in which case the capacity of the series is the reciprocal of the sum of the reciprocals of the several parts.

Care must be taken not to insert plugs at the two ends of one block, for this would short circuit the charging battery.

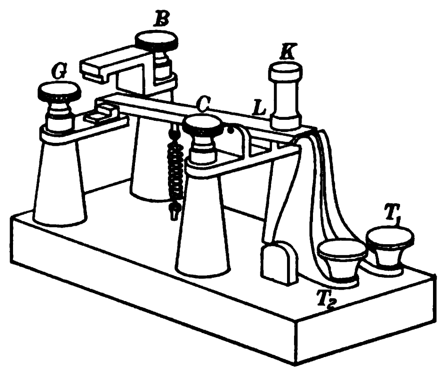


FIG. 170. CONDENSER KEY

263. Condenser Keys.— For facilitating the charging and discharging of condensers, keys are made in many forms, the essential features of which may be explained with the aid of Fig. 170. One terminal of the galvanometer is to be connected to G, one pole of the battery to B, and one side of the condenser to C, the other connections being as shown in Fig. 171. A spiral spring draws the lever L downward in the position shown. By pressing the knob K, the front of the lever is brought downward till it catches under the two triggers, T_1 and T_2. This brings the end of L into contact with B, which is the *charge* position. By pressing T_1 the spring will pull the lever downward, but the trigger T_2 will catch it and hold it horizontal so that L does not touch either B or G; this is the *insulate* position. By pressing T_2 the lever takes the position shown, which is the *discharge* position.

In another form of condenser key, a flexible lever is normally in contact with the upper point (the charge position), while it may be pressed away from this point into contact with the lower one (the discharge position).

264. Comparison of Condenser Capacities. — If different condensers, or different parts of one condenser, are charged to the same potential, the quantities of electricity in the charges will be proportional to the capacities of the condensers. These quantities may be compared by discharging the condensers through a ballistic galvanometer (Art. 253).

Connect a ballistic galvanometer, G, a condenser, C, a charge and discharge key, K, and a cell, B, as indicated in Fig. 171.

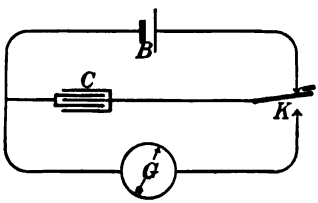


FIG. 171. COMPARISON OF CAPACITIES

After charging the condenser, by changing the key to the galvanometer contact, the condenser will be discharged and the galvanometer needle will be deflected through an angle, θ_1, such that $\sin \frac{1}{2} \theta_1$ is proportional to the charge and therefore proportional to the capacity C_1. By using a second condenser of capacity C_2, a deflection θ_2 will be obtained. If the distance of the scale from the mirror is D, and the observed scale readings are d_1 and d_2,

$$C_1 : C_2 = \sin \tfrac{1}{2} \theta_1 : \sin \tfrac{1}{2} \theta_2 = \sin \tfrac{1}{4} \tan^{-1} \frac{d_1}{D} : \sin \tfrac{1}{4} \tan^{-1} \frac{d_2}{D}.$$

If the angular deflections of the needle do not exceed 6°, the sines and tangents may be considered proportional to the corresponding arcs, and then

$$C_1 : C_2 = d_1 : d_2.$$

(CAUTION. — Do not allow the cell to be short-circuited, nor to be directly connected to the galvanometer.)

The following numerical example illustrates the comparison of capacities. A part of these observations are used in the determination of the absolute capacity of one of the condensers, explained on page 358.

Comparison of Capacities

October 18, 1897

S. & H. Standard Condenser No. 6829; nominal capacity, 1.003 microfarad. Q Paper Condenser; nominal capacity, ¼ microfarad. Queen Ballistic Galvanometer No. 92.

The two condensers were charged successively from H. & B. Clark Cell No. 100, and the following deflections were obtained, as read in centimeters on a scale distant 100 cm.

Condenser No. 6829			Condenser Q		
Zero Reading	Second Reading	Deflection	Zero Reading	Second Reading	Deflection
39.55 cm	76.10 cm	36.55 cm	39.8 cm	48.9 cm	9.1 cm
.65	.25	.60	.7	.8	.1
.70	.20	.50	.7	.9	.2
.75	.25	.40	.7	.8	.1
.80	.20	.50	.7	.8	.1
.80	.30	.50	.8	.9	.1
.85	.40	.55	.7	.8	.1
.85	.40	.55	.7	.9	.2
.85	.40	.55	.7	.8	.1
.83	.30	.47	.7	.8	.1
		Mean $d_1 = 36.52$ cm			Mean $d_2 = 9.12$ cm

$$(\text{No. } 6829) : (Q) = \sin \tfrac{1}{4} \tan^{-1} \frac{36.52}{100} : \sin \tfrac{1}{4} \tan^{-1} \frac{9.12}{100} = 1 : 0.260.$$

Capacity of Condenser No. 6829 = 0.999 microfarad (p. 358).
Capacity of Condenser Q = 0.259 microfarad.

CXLIX. RELATIVE CAPACITY BY BRIDGE METHODS

Determine the capacities of the parts of a subdivided condenser by comparison with a standard condenser.

265. Bridge Methods for Comparison of Capacities. — The following zero methods for condenser comparisons are similar to the Wheatstone's bridge method for resistance comparisons.

De Sauty's Method. — Connect the two condensers, two resistances, a dead-beat galvanometer, a battery of from 10 to 15 volts, and a charge and discharge key, as shown in Fig. 172, the key being usually on the discharge point. Adjust the resistances until the galvanometer shows no deflection when the condensers are charged and discharged. When this condition is fulfilled

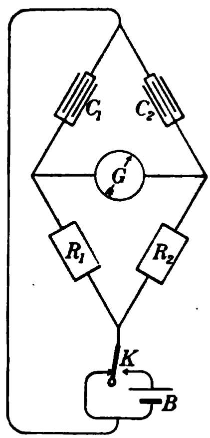


FIG. 172. DE SAUTY'S METHOD

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}.$$

The resistances, R_1 and R_2, should have values of from 1000 to 10000 ohms, and must be noninductive.

Gott's Method. — Connect two condensers, two resistances, a dead-beat galvanometer, a battery of from 10 to 15 volts, and keys, as indicated in Fig. 173. As compared with the arrangement of the previous method, the battery and galvanometer have changed places. It will now be possible so to adjust the resistances, R_1 and R_2, that upon first closing the key K_1, which remains closed, and then closing the key K_2, no deflection of the galvanometer is produced. When this condition is fulfilled, the following relation is true.

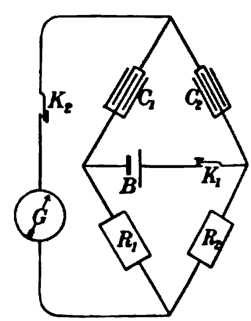


FIG. 173. GOTT'S METHOD

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}.$$

After both keys have been closed as described, and the resistances have been altered, the condensers must be discharged by closing K_2 only, before again charging through K_1. As before, the resistances should be large, between 1000 and 10000 ohms.

REFERENCE. — *Gray*, Absolute Measurements in Electricity and Magnetism, Vol. I, p. 440.

CL. RELATIVE CAPACITY BY THE METHOD OF MIXTURES

Determine the capacity of a condenser by comparison with a standard condenser.

266. Comparison of Capacities by the Method of Mixtures. — In this method two condensers are charged with opposite potentials of such values that their charges when mixed neutralize each other. Their capacities are inversely as these potentials.

Connect apparatus as indicated in Fig. 174. A battery of from 10 to 15 volts is joined in series with two large resistances, R_1 and R_2; the condensers, C_1 and C_2, are arranged to be connected to R_1 and R_2 respectively, by means of a special double key, K. A rocking commutator (Fig. 133) with the diagonal connections removed is suitable for this purpose. With the key in proper position the condensers will be charged to potentials which are proportional to R_1 and R_2. When the key is rocked to its second position the connections as shown are such that the two charges will be mixed, and the residual charge is only their difference. By closing the key k, this residual will be discharged through the galvanometer G. The resistances are to be so adjusted that the galvanometer shows no deflection, when the condensers are charged and discharged as described; then

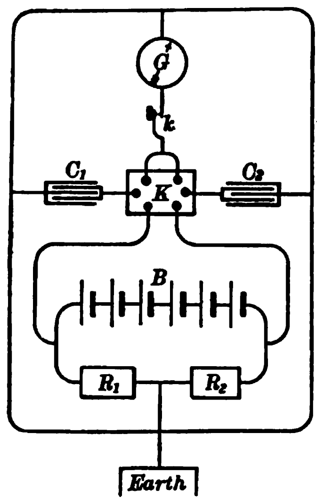


FIG. 174. CAPACITY BY METHOD OF MIXTURES

$$C_1 : C_2 = R_1 : R_2.$$

The method is suitable for cable capacity measures, the conductor of the cable being joined to K, and the sheath of the cable or the water in which it is immersed being connected to

the earth. The ground connection is necessary for cable testing, but may be omitted in other cases. Absorption by the condensers may cause an error, and often the time required for mixing the charges is appreciable; hence high insulation is essential.

CLI. ABSOLUTE CAPACITY OF A CONDENSER WITH A BALLISTIC GALVANOMETER

Determine the absolute capacity of a condenser.

267. Absolute Capacity with a Ballistic Galvanometer. — When a ballistic galvanometer (Art. 253) is used for absolute measurement, it becomes necessary to consider the period of vibration of the needle as well as the deflection, and to make corrections for the effect of the amplitude upon the period and for the effect of damping on the deflection (Art. 93).

The Period of the needle may be found from observations of transits, as explained in Art. 83, a set of ten transits being observed, and, after a lapse of fifty or more vibrations, a second set of ten.

This observed period, T, is reduced to the period in an infinitely small arc, T_0, by the approximate formula,

$$T_0 = T - \frac{T}{256} \cdot \frac{d^2}{D^2},$$

in which d is the observed scale reading (average) and D is the distance of the scale from the mirror. Complete formulæ and tables to facilitate making this correction will be found in the references.

The Logarithmic Decrement is the natural logarithm of the ratio of decrease in the arcs of successive vibrations of the needle. Let a_1 be the arc of any swing of the needle, expressed in scale divisions, and let a_{1+n} be the arc of the nth following swing; then the logarithmic decrement is

$$\lambda = \frac{2.3026}{n} (\log a_1 - \log a_{1+n}).$$

Correction of the Tangent to the Arc. — If d is the observed number of scale divisions from the center of the scale to the turning point of the swing, expressed in centimeters, and the scale distance is D, the number which is proportional to the arc of swing is

$$A = d - \frac{d^3}{3\,D^2}.$$

The Absolute Capacity of a Condenser may be found, after the corrected period of the needle, T_0, and the logarithmic decrement, λ, of the galvanometer have been determined from the results of the following deflection experiments.

Charge the condenser with a given cell and discharge it through the galvanometer, as described in Art. 264. If d_1 is the observed scale reading when the distance of the scale from the mirror is D, the corrected deflection is

$$\Delta_1 = 2 \sin \tfrac{1}{4} \tan^{-1} \frac{d_1}{D}.$$

Then connect the same cell in series with the galvanometer and a very high resistance, R, and let d_2 be the deflection produced. Usually it will be necessary to shunt the galvanometer in order to secure a readable deflection. This deflection properly corrected will be

$$\Delta_2 = \tan \tfrac{1}{2} \tan^{-1} \frac{d_2}{D}.$$

If the galvanometer resistance is G, the battery resistance B, and if the galvanometer is shunted with a resistance S, then the virtual value of the resistance through which the cell acted in producing the observed deflection d_2 is

$$R_V = \left(R + B + \frac{GS}{G + S}\right) \frac{G + S}{S}.$$

Finally the absolute capacity of the condenser C is given by the following formula.

$$C = \frac{T_0 \left(1 + \frac{1}{2}\lambda\right) \Delta_1}{\pi R_V \Delta_2}.$$

REFERENCES. — *Stewart and Gee*, Practical Physics, Vol. II, pp. 364–369, 407; *Kohlrausch*, Physical Measurements, pp. 219–225, 347, 382; *Carhart and Patterson*, pp. 207–213, 227.

A numerical example of the determination of absolute capacity is given.

ABSOLUTE CAPACITY OF SIEMENS & HALSKE CONDENSER No. 6829

NOMINAL CAPACITY, 1.003 MICROFARAD

QUEEN D'ARSONVAL BALLISTIC GALVANOMETER No. 92

October 21, 1897

Two determinations of the period of the needle were made, the reduced average of which gives

$$T_0 = 16.589 \text{ s.}$$

OBSERVATIONS AND COMPUTATION FOR THE LOGARITHMIC DECREMENT

Scale distance, 100 cm. Center of scale, 40.00

TURNING POINT p	$d = p - 40.00$	$\frac{d^3}{3D^2}$	CORRECTED TURNING POINT	ARC
64.69	+ 24.69	+ 0.50	64.19	
16.52	− 23.48	− 0.43	16.95	$47.24 = a_1$
59.13	+ 19.13	+ 0.23	58.90	$41.95 = a_2$
21.43	− 18.57	− 0.21	21.64	$37.26 = a_3$
55.09	+ 14.93	+ 0.12	54.97	$33.33 = a_4$
25.07	− 12.03	− 0.11	25.18	$29.79 = a_5$

$$\left.\begin{array}{l} \frac{1}{3}(\log a_4 - \log a_1) = 0.05049 \\ \frac{1}{3}(\log a_5 - \log a_2) = 0.04955 \end{array}\right\} \text{Mean} = 0.05002$$

$$\lambda = 2.3026 \times 0.05002 = 0.11518.$$

The observations for the deflection produced by the discharge of the condenser are given in the example on page 353.

$$d_1 = 36.52. \qquad \Delta_1 = 0.17485.$$

The observations for the steady deflection are given in the example on page 328.

$$d_2 = 17.22. \qquad \Delta_2 = 0.08257.$$

$$R_V = 11\,465000.$$

The absolute capacity of the condenser is

$$C = \frac{16.598 \times 1.05759 \times 0.17485}{\pi \times 11\,465000 \times 0.08257} = 0.9991 \text{ microfarad.}$$

268. Ballistic Constant. — In the above method the electromotive force of the cell is eliminated by using the same cell in both the condenser and deflection observations. If a standard cell of known electromotive force is used in the deflection experiment, the ballistic constant of the galvanometer may be determined and then the instrument may be used to measure capacities or other quantities in absolute measure without the repetition of this part of the above method. This constant is the factor by which an observed ballistic deflection must be multiplied to give the absolute measure of the quantity of electricity that passed through the galvanometer. If E is the electromotive force of the cell, and T_0, R_2, and Δ_2 have the values given in the preceding article, the ballistic constant of the galvanometer is

$$K = \frac{T_0 E}{\pi R_2 \Delta_2}.$$

The absolute capacity C, of a condenser charged to a potential difference e, λ and Δ_1 having the values assigned in the preceding article, is

$$C = K \frac{(1 + \frac{1}{2}\lambda)\,\Delta_1}{e}.$$

CHAPTER XXIV

INDUCTION

CLII. INDUCTANCE BY COMPARISON

Determine the self-induction of several coils by comparison with a standard of self-induction.

269. Standard Henry; Coefficient of Self-Induction. — The unit of induction is the *henry;* it is the induction due to a current whose strength is changing at the rate of one ampere per second with a resulting induced electromotive force of one volt. The *coefficient of self-induction* of a coil is the total induction through it per unit change of current producing the induction.

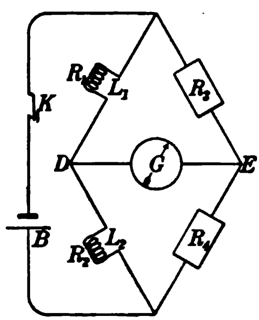


FIG. 175. SELF-INDUCTION

The self-induction of two coils may be compared with a Wheatstone's bridge by Maxwell's method; if the self-induction of one coil is known, that of the other is determined. Let R_1 and R_2 (Fig. 175) be the resistances of two arms of a Wheatstone's bridge containing the coils whose coefficients of self-induction, L_1 and L_2, are to be compared; let R_3 and R_4 be noninductive resistances. Connect these with a galvanometer, a battery, and a key, as indicated. If a steady current is flowing, then, as in Art. 207, no current will flow across the bridge from D to E when R_3 and R_4 are so adjusted that

$$R_1 : R_2 = R_3 : R_4.$$

Also it can be proved that if the current varies so that the

self-induction of the coils enters into the current relations, there will be no current through the galvanometer when

$$L_1 : L_2 = R_3 : R_4.$$

As it is not convenient to determine this latter condition alone, both conditions must be satisfied at the same time. The adjustments must be so made that no current flows through the galvanometer while the circuit is closed, and also there must be no current when the battery circuit is made or broken. This means that the resistances in the arms of the bridge apparatus which contain the self-inductances must be made to have the same ratio as the self-induction coefficients. If a standard of self-induction is used for one coil, it is usually constructed so that its self-induction may be varied. This alone may permit the obtaining of a balance; if not, noninductive resistance must be inserted in series with one of the inductive coils so as to change the value of R_1, for instance, without changing L_1. By one or both of these operations it will be possible to secure the conditions for no deflection. Then

$$L_1 : L_2 = R_3 : R_4.$$

REFERENCES. — *Carhart and Patterson*, Electrical Measurements, p. 255; *Stewart and Gee*, Practical Physics, Vol. II, p. 394.

CLIII. COEFFICIENT OF MUTUAL INDUCTION BY COMPARISON

Verify the laws of mutual induction, and compare two mutual inductances.

270. Mutual Induction. — That the induced current in the secondary coil varies directly as the current in the primary coil and inversely as the resistance in the secondary circuit, and

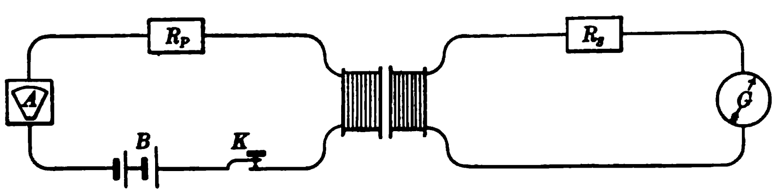


FIG. 176. MUTUAL INDUCTION

the dependence of the induction upon the relation of the primary and secondary, may be proved by comparing the currents produced under various conditions, by means of a ballistic galvanometer. If the constants of the current are known, the coefficient of mutual induction may be calculated. If the making of a current, I, in the primary (Fig. 176) induces a quantity of electricity Q in the secondary, and the entire resistance of the secondary circuit is R, the coefficient of mutual induction, M, is obtained from the following relation.

$$Q = M\frac{I}{R}.$$

REFERENCE. — *Nichols*, A Laboratory Manual of Physics, Vol. I, p. 240.

271. Comparison of Two Mutual Inductances. — Let it be required to compare the mutual induction of two coils, C_1 and C_2, with that of coils C_3 and C_4. Connect the two primary coils, one from each pair, C_1 and C_3 (Fig. 177), in series with a battery, the circuit containing a key, K. Connect the secondary coil, C_2, in series with a resistance, R_2, and a sensitive galvanometer; connect the coil C_4, and a resistance, R_4, to the same galvanometer so that the induced current in C_4 shall oppose that in C_2. By adjusting R_2 and R_4 so that the making and breaking of the primary circuit causes no deflection of the galvanometer, the coefficients of mutual induction, M_{1-2} and M_{3-4}, are in the same ratio as the resistances R_2 and R_4; that is,

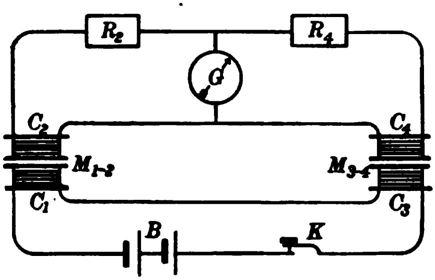


FIG. 177. COMPARISON OF MUTUAL INDUCTANCES

$$M_{1-2} : M_{3-4} = R_2 : R_4.$$

REFERENCE. — *Carhart and Patterson*, Electrical Measurements, p. 265.

CHAPTER XXV

MAGNETIC QUANTITIES

CLIV. EARTH'S HORIZONTAL MAGNETIC INTENSITY WITH THE MAGNETOMETER

Determine the horizontal intensity of the earth's magnetism, and the magnetic moment of a magnet.

272. The Earth's Magnetic Elements. — The vertical plane which contains the direction of the earth's magnetic force is the *magnetic meridian;* the angle between this plane and that of the geographic meridian is the magnetic *declination.* The direction of the force is inclined to the horizontal in the magnetic meridian at an angle called the magnetic *inclination* or *dip.* The *total intensity* of the earth's magnetism is the number of dynes of force with which a unit magnet pole would be urged along in the direction of the force. The unit of intensity is called the *gauss;* the actual intensity varies from 0.28 to 0.78 of a gauss. This total intensity may be considered as consisting of two components, — the *vertical intensity*, V, and the *horizontal intensity*, H. The intensity and direction of this force are subject to changes which may be analyzed into daily, annual, secular, and irregular variations. It is important to know the values of the horizontal intensity and its variations; if the total intensity and its direction are required, they are most readily determined from the values of the horizontal and vertical components, or from the horizontal intensity and dip.

273. The Magnetometer. — If a magnet is suspended with its axis horizontal, it will come to rest in the magnetic meridian. After displacement from the meridian it will vibrate in the manner of a torsion pendulum, with a period which varies

inversely as the square root of the restoring force. This force is the product of the horizontal intensity of the earth's magnetism and the magnetic moment of the magnet. A determination of the period furnishes one equation of condition for determining these two unknown quantities.

If the same magnet is placed at known distances from a compass needle, in a line through the center of the needle perpendicular to the plane of the magnetic meridian, the needle will be deflected to a position such that the tendency of the earth's field to restore the needle to the meridian is counterbalanced by the moment due to the magnet tending to set the needle east and west. The observed deflection furnishes a second relation between the horizontal intensity and the magnetic moment, permitting their values to be found.

The instrument for making these measurements, the *magnetometer*, may be simple or elaborate, and while the formulæ concerning the general principles are simple, as given here, many corrections may be made which greatly complicate both the observations and their reduction. Even in simple measurements care is required to avoid the effects of local magnetic disturbances and variations in the strength of the earth's field.

Vibration Observations. — Suspend the magnet with its axis horizontal, by a fiber having as little torsion as possible. The torsion may be reduced to a minimum by first suspending in place of the magnet a bar of nonmagnetic material having the same mass as the magnet, and adjusting the torsion head so that the bar would come to rest with its axis in the magnetic meridian. Displace the magnet from its position of equilibrium so that it shall vibrate with an amplitude not exceeding 5°, and carefully determine the period, T, of a complete vibration by the method of Art. 83. Then, if I is the moment of inertia of the magnet, M its magnetic moment, and H the horizontal intensity of the earth's magnetism,

$$MH = \frac{4\,\pi^2 I}{T^2}.$$

The moment of inertia, I, may be found by observing the period of the needle when free and when a load of known

moment of inertia is added to it, as explained in Art. 84; or it may be calculated from the mass and dimensions of the magnet, by the formula of Art. 85.

The value of MH thus determined may be corrected for amplitude of swing, torsion of suspension, rate of clock, and for the effects of temperature and induction upon the magnet, as explained in the references.

Deflection Observations. — Place the magnet used in the previous experiment with its axis perpendicular to the plane of the magnetic meridian and passing through the center of a needle capable of horizontal deflections which can be measured. The latter may be a form of compass, or a needle carrying a mirror for telescope and scale observations.

With the magnet at a known distance from the center of the needle, observe the deflection produced, reading both ends of the pointer if it is of the compass form; turn the magnet end for end, keeping it at the same distance, and observe the opposite deflection produced; make observations with the magnet in two positions, at an equal distance on the opposite side of the needle. Let θ_1 be the average deflection obtained from the eight readings (or four readings if made with a telescope), and let r_1 be the distance between the centers of the magnet and the needle, — that is, half the distance between the two positions of the magnet. Place the magnet at a different distance, r_2, from the needle, and determine as before the average deflection, θ_2. Then

$$\frac{M}{H} = \tfrac{1}{2}\,\frac{r_1^5 \tan\theta_1 - r_2^5 \tan\theta_2}{r_1^2 - r_2^2}.$$

Corrections may be made for the effects of temperature and induction upon the magnets.

Unless the deflections produced are too small for accurate reading, the shortest distance of the deflecting magnet from the needle should be not less than four times its length, and the greatest distance should be 1.4 times the lesser.

Results. — Using the values of MH and $\frac{M}{H}$ thus determined, M and H are found separately, which are the magnetic moment

of the magnet and the horizontal intensity of the earth's magnetism respectively.

274. The Kew Magnetometer. — A form of magnetometer is sometimes used, for which the method of observing the deflections differs slightly from that described above. The bar carrying the deflecting magnet, instead of remaining perpendicular to the plane of the magnetic meridian, is turned about the center of the needle, so that the magnet is always perpendicular to the axis of the needle. This requires the substitution of the sine function of the angle of deflection for the tangent in the formula given above; otherwise the method and formulæ are not changed.

REFERENCES. — *Kohlrausch*, Physical Measurements, pp. 240–260; *Stewart and Gee*, Practical Physics, Vol. II, pp. 285–308; *Gray*, Absolute Measurements in Electricity and Magnetism, Vol. II, p. 69 et seq.

CLV. VARIATION OF THE EARTH'S HORIZONTAL MAGNETIC INTENSITY WITH THE VARIOMETER

Determine the relative horizontal intensity and direction of the earth's magnetism at several assigned points about the laboratory, and plot the results.

275. The Magnetic Variometer. — A magnetic needle which is made to assume a position at right angles to the magnetic meridian by means of a deflecting magnet acting in opposition to the earth's magnetism is very sensitive to changes in the field strength, and it may be used to determine the variations occurring at one place; or, by placing the instrument in several locations, the relative intensities at these points may be compared.

The deflecting magnet is supported below the needle so that it may be rotated in a horizontal plane about the same axis as that on which the needle swings. Its distance below the compass may also be varied. An outline of such a variometer is shown in Fig. 178.

The instrument may have a mirror needle and a telescope and scale for observing the deflections, and the one deflecting magnet

may be replaced by four small magnets placed around the needle and in the same horizontal plane.

The adjustment and use of the variometer involve the following operations.

(a) *The Axis of Rotation* of the instrument is to be made vertical, as may be indicated by the level.

(b) *Sensitiveness.* — The magnet when in the meridian must exert a force on the needle slightly in excess of that of the earth. Turn the magnet to the zero of its divided circle and rotate the whole instrument until the *N* pole of the magnet is to the north and the *n* end of the needle is to the south. It should be possible to rotate the instrument until the reversed needle is parallel to the magnet; that is, until its *s* end indicates 0° on the compass circle. If such a position cannot be found, the magnet must be set nearer to the compass. The greater the distance between magnet and needle at which this condition is fulfilled, the greater the sensitiveness.

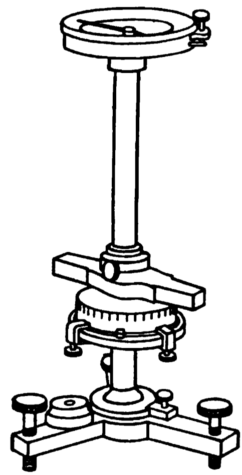


FIG. 178. THE VARIOMETER

(c) *To place the Variometer in the Magnetic Meridian.* — The operations described in the first part of the preceding paragraph place the instrument approximately in the meridian; but if the position of the magnet is altered to secure proper sensitiveness, it is necessary to make again the meridian adjustment. Set the magnet to the zero of its divided circle. Turn the whole instrument until the *N* end of the magnet is to the north and the *s* end of the needle indicates 0° on the compass circle. The needle is then in the meridian. Clamp the instrument to the base.

(d) *Angle of Rotation of the Magnet.* — Turn the magnet to one side till the needle points east and west, and fix a stop to limit this position. Turn the magnet to the other side of the meridian until the needle again points east and west, and place the second stop to determine this position.

It is not necessary for comparison that the deflection of the needle should be exactly 90°, though it simplifies the computation if this exact deflection is produced when the instrument is set in the place where the horizontal intensity is known.

Half of the angle of the rotation of the magnet between the stops is the quantity θ of the formulæ. The stops must remain unchanged in position during the entire set of comparisons.

(e) *To perfect the Meridian Adjustment.* — If the circle readings for the positions of the two steps, determined as just described, are different, it indicates that the 0° line of the magnet and compass circles are not in the same plane. To improve the meridian adjustment, set the large magnet midway between the two stops; then, as in (c), turn the whole instrument until the *s* end of the reversed needle indicates 0° on the compass circle. The needle now indicates the magnetic meridian. The change in the position of the instrument required for this is usually so small that the adjustment is not disturbed. It may be repeated if required.

In placing the instrument at a new station, the first setting is made with the magnet in the position required in this adjustment; that is, in the position which corrects the zero error of its circle. This correction is not constant because of possible displacement of the circle when it is clamped in a new position on the upright support.

276. Magnetic Declination. — The variometer being set in the meridian as described, by means of sighting pieces on the compass box the angle between the direction of the needle and any given line, as the true north and south or the lines of a building, may be found.

277. Comparison of Horizontal Intensities. — The variometer having been adjusted as described above, in the first location, preferably a place for which the absolute value of the horizontal intensity is known, the magnet is turned against one stop, and the amount by which the needle deviates from the magnetic east and west (90° and 270°) is observed by reading both ends of the needle. The magnet is turned to the second stop, and the deviations of the two ends of the needle from east and west are

again read. The average of the four readings is δ of the formulæ, and it is positive if, when the N end of the magnet is turned toward the east, the n end of the needle is deflected more than 270° (that is, the n end is north of west, as shown in Fig. 179).

If the magnet and needle have the relative position shown in Fig. 179, the forces acting on the s pole of the needle are the field of the magnet in the direction sF, and the earth's field in the direction sH. If the needle is at rest, the components of

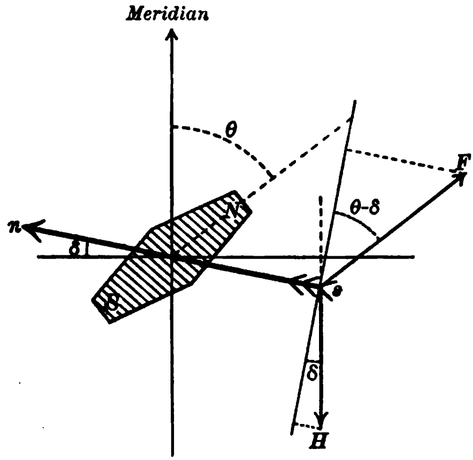


FIG. 179. FIELD OF VARIOMETER

these two forces in the directions perpendicular to the needle must be equal and opposite. If F is the strength of the magnet's field and H that of the earth, θ and δ having the meanings already explained,

$$H \cos \delta = F \cos (\theta - \delta).$$

The instrument is taken to the second station and adjustments (a) and (e) only are performed. The observations just described are then repeated.

By adopting suitable subscripts the results of the measures in the two locations may be represented as follows:

$$H_1 = F(\cos \theta + \sin \theta \tan \delta_1),$$
$$H_2 = F(\cos \theta + \sin \theta \tan \delta_2).$$

Dividing one equation by the other, the unknown field strength of the magnet is eliminated, and by reducing,

$$\frac{H_1}{H_2}=\frac{1+\tan\theta\tan\delta_1}{1+\tan\theta\tan\delta_2}.$$

If the deflection of the needle at the first station is exactly 90°, δ_1 is 0 and $$H_2=H_1(1+\tan\theta\tan\delta_2).$$

In this manner any number of positions may be compared, and if the intensity at one place is known, the intensity for all the locations may be found.

REFERENCE. — *Kohlrausch*, Physical Measurements, pp. 252–258.

278. Oscillation and Deflection Methods for Comparison of Horizontal Intensities. — If a magnet is caused to vibrate as a torsion pendulum in a location where the horizontal intensity has the value H_1, and its period is T_1, determined as explained in Art. 83; and if the same magnet has a period T_2 when oscillating in a field of horizontal intensity H_2, then

$$H_1 : H_2 = T_2^2 : T_1^2.$$

If a magnet placed in the magnetic east and west line through the center of a suspended needle which is in the location of horizontal intensity H_1, deflects the needle $\theta_1°$; and if the deflection produced in the same apparatus when it is in a location of intensity H_2, is $\theta_2°$; then

$$H_1 : H_2 = \tan\theta_2 : \tan\theta_1.$$

CLVI. EARTH'S MAGNETIC INCLINATION AND INTENSITY WITH THE EARTH INDUCTOR

Determine the magnetic inclination with the earth inductor by two methods; determine the component and total intensities.

279. The Earth Inductor. — A conductor which can be moved in a determined manner in the earth's magnetic field may have currents induced in it which are often useful for magnetic measurements and comparisons. The earth inductor consists of

a coil of wire 20 cm or more in diameter, suspended so as to be turned about an axis in its own plane; this axis is supported by a frame which rotates on a second axis at right angles to the first, enabling the axis of the coil to be given any desired inclination from horizontal to vertical. The coil may be rotated continuously in this frame by means of a crank; or stops may be provided limiting its rotation to 180°, which rotation may be produced by spiral springs. The terminals of the coil make contact through rings and brushes with an external circuit. (See Fig. 180.)

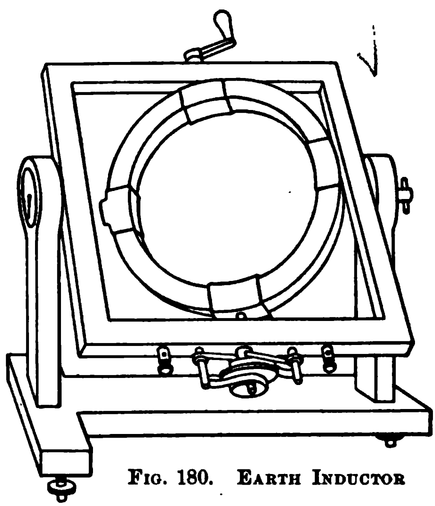


Fig. 180. Earth Inductor

If a circuit of wire be moved in a magnetic field so as to change the number of lines of force inclosed by the coil, a current is induced in the circuit which is proportional to the change in the number of lines of force. If the coil consists of n turns of wire, each of area a, with its face perpendicular to the lines of a field whose strength is F, the number of lines inclosed is naF. If the plane of the coil is rotated 180°, the same number of lines passes through it in the opposite direction, and there has been a change of $2\,naF$ lines. If the strength of the earth's magnetic field is F, and the direction of the lines of the force makes an angle of dip with the horizontal represented by θ, the horizontal component, H, of this force is $F \cos \theta$, and the vertical component, V, is $F \sin \theta$. If the plane of the coil of the earth inductor is rotated about an axis parallel to the horizontal component, from a position in which one face of the coil is vertically upward, and therefore in the position in which the maximum vertical component of the force passes through it, to a position 180° from this, the coil will cut

only the vertical component. If the entire resistance of the circuit of which the coil is a part is R ohms, the quantity of current induced by this rotation, in centimeter-gram-second units, is

$$Q_V = \frac{2\,naF\sin\theta}{R\cdot 10^8}.$$

If the plane of the coil rotates about an axis parallel to the vertical component, from a position of its face perpendicular to the horizontal component, to the position 180° from this, the coil will cut only the horizontal component of the force, and the induced current will be

$$Q_H = \frac{2\,naF\cos\theta}{R\cdot 10^8}.$$

The ratio of the first of these quantities to the second is the ratio of the vertical to the horizontal components, and this is the tangent of the angle of dip. The ratio of the two quantities can be determined without knowing any of the factors by which they are expressed above, by measuring the currents with a ballistic galvanometer. For determining the absolute value of the quantities these formulæ or those of Art. 281 may be employed.

280. Magnetic Inclination.—Adjust the earth inductor so that the two rectangular axes of rotation shall be level, and so that the axis of the coil shall lie in the plane of the magnetic meridian and the plane of the coil shall be horizontal. These conditions may be secured by the aid of a compass and a level. Connect the coil in series with a ballistic galvanometer (Art. 253) and a resistance for controlling the current. Then, by means of the spiral springs, release and stop, cause the coil to be suddenly rotated 180°, and observe the galvanometer deflection produced. Make ten or more observations, and correct the average of the readings as described in Art. 252; let d_1 be the corrected average. Now place the axis of the coil vertical, as determined by a divided circle, by a plumb line or by other convenient method, and having the plane of the coil perpendicular to the plane of the magnetic meridian, cause the coil to be suddenly rotated 180°. Let d_2 be the corrected

average value of the observed galvanometer deflections, obtained with the coil in this position. Then the magnetic inclination, or the angle of dip, θ, is obtained from the following relation.

$$\tan \theta = \frac{d_1}{d_2}.$$

Second Method for Inclination. — If the coil of the earth inductor is rotated about an axis parallel to the lines of force of the field, there will be no variation in the number of lines of force inclosed by the coil and no current will be induced. By trial such an inclination of the axis may be found, which gives at once the magnetic dip.

281. Magnetic Intensity. — The absolute value of either component of the earth's magnetic intensity may be determined from the observations of the preceding article if the constants of the earth inductor and the galvanometer are known. The dip having been determined, the total intensity may also be found.

In accordance with the explanation of Arts. 267, 268, and 279, there follow the relations:

$$Q_V = \frac{2naF\sin\theta}{R\cdot 10^8} = K\left(1 + \tfrac{1}{2}\lambda\right) d_1,$$

and $$V = F\sin\theta = \frac{K\left(1 + \frac{1}{2}\lambda\right) d_1 R\, 10^8}{2na},$$

and similarly $$H = F\cos\theta = \frac{K\left(1 + \frac{1}{2}\lambda\right) d_2 R\, 10^8}{2na};$$

from which $$F = \frac{V}{\sin\theta} = \frac{H}{\cos\theta}.$$

The maker of an earth inductor usually supplies a certificate giving the number of turns, n, of wire in the coil, and the diameters of the coils from which the average area, a, may be computed; or the wires may be exposed to view for counting and measurement. The values of all the other quantities contained in these formulæ are sufficiently explained in the articles to which reference is made.

CLVII. DISTRIBUTION OF MAGNETISM BY ROWLAND'S METHOD

Determine the distribution and total flow of induction of a permanent magnet.

282. Flow of Induction from a Magnet. — If a circuit be moved from a position in which it incloses all the lines of force of a magnet, to a position in which it incloses none of these lines, there will be induced in it a quantity of electricity which measures the total flow of induction.

Provide a test coil of wire of a known number of turns, n, which will fit around the bar magnet, but with sufficient clearance to permit the coil to slide along the bar. Place the coil over the center of the bar magnet, and connect its terminals, through a resistance, R, to a ballistic galvanometer. Suddenly draw the coil off the bar and observe the galvanometer deflection. Let this deflection, corrected as explained in Art. 252, be d, and let the constant of the galvanometer (Art. 268) be K, and its logarithmic decrement (Art. 267) be λ; the number of unit tubes of induction emanating from the magnet is

$$N = \frac{K(1 + \frac{1}{2}\lambda)\, d\, 10^8 R}{n}.$$

283. Distribution of Magnetism. — When the test coil is on the bar magnet, stops may be clamped on each side of it to limit

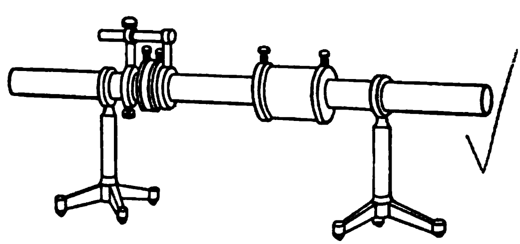


FIG. 181. DISTRIBUTION OF MAGNETISM

its motion, and if the coil is suddenly moved from one stop to the other a current will be induced which is proportional to

the average normal component of magnetization between the two points. (See Fig. 181.) By placing the two stops so as to include between them the successive portions of the length of the magnet from one end to the other, and using the test coil in each position, the galvanometer deflections will be proportional to the flow of induction from the magnet at these points. If these deflections are plotted as ordinates, with the positions as abscissæ, a curve representing the distribution of magnetism is obtained.

REFERENCES. — *Stewart and Gee*, Practical Physics, Vol. II, p. 388; *Barker*, Physics, pp. 674 and 812.

CLVIII. INTENSITY OF A MAGNETIC FIELD BY ROWLAND'S METHOD

Find the number of unit tubes of induction per square centimeter in the field between the two poles of an electromagnet.

284. Comparison of Magnetic Fields with the Earth's Field. — Provide a test coil of wire of such size as to inclose that portion of the field which it is desired to measure. Connect the coil C

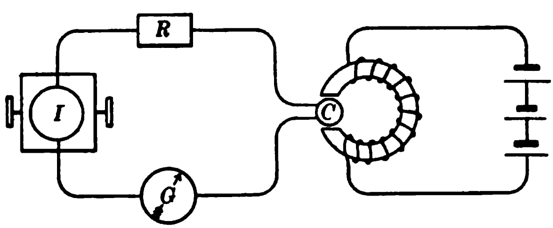


FIG. 182. INTENSITY OF MAGNETIC FIELD

(Fig. 182) in series with a resistance, R, an earth inductor, I, and a ballistic galvanometer, G. Suddenly rotate the plane of the coil 180° from a position in which the lines of force are perpendicular to this plane; let the galvanometer deflection produced be d_1. The earth inductor, being in adjustment for measuring the horizontal component, H, of the earth's field (Art. 279), is

rotated 180°; let the resulting galvanometer deflection be d_2. If the number of turns of wire in the test coil is N, each of area A, and the number of turns in the earth inductor is n, each of area a, then the strength of the field is

$$F = H\frac{nad_1}{NAd_2}.$$

If the test coil, instead of being rotated 180° in the field, is simply withdrawn from the field, the deflection d_1 will be only half as large as in the above case, and

$$F = 2\,H\frac{nad_1}{NAd_2}.$$

Reference. — *Stewart and Gee*, Practical Physics, Vol. II, pp. 382–387.

CLIX. TEMPERATURE COEFFICIENT OF A MAGNET BY DEFLECTION METHODS

Determine the temperature coefficient of a permanent magnet.

285. Temperature Coefficient of a Magnet. — The change in magnetism, per unit magnetism, per degree change of temperature, is the temperature coefficient of a magnet. It has a negative value for increasing temperatures. It may be determined by measuring the change in the effect of a magnet upon a suspended needle, when the temperature of the magnet changes. Two methods of manipulation by which the needle will be more sensitive to the control of the magnet are given.

Zero Deflection Method. — Immerse the magnet to be experimented upon in an oil bath, the temperature of which can be conveniently changed. Place the bath magnetically east or west and at a convenient distance from a suspended magnetic needle, such for instance as the needle of a dead-beat galvanometer. Let the *angle* of deflection produced (not exceeding 20° or 30°) be θ. Now, by means of a compensating magnet, reduce the deflection of the needle nearly to zero. The magnet remaining unchanged in position, let its temperature be altered

$t°$. Let the resulting change in the deflection be d, as read with a telescope and scale distant D from the mirror on the needle. Then the temperature coefficient is

$$\mu = \frac{1}{2 \tan \theta} \cdot \frac{d}{Dt}.$$

Ninety-Degree Deflection Method. — Place the magnet whose coefficient is desired, in the oil bath, in the same horizontal plane with a short magnetometer needle, and with its center in the plane of the magnetic meridian through the center of the needle. The magnet is to be adjusted accurately to the magnetic meridian by being turned so that the needle is undeflected. Now turn the magnet out of the meridian till the needle is deflected about 90°, and let θ be the angle through which the magnet has been turned. Change the temperature of the magnet $t°$ without altering its position, and, if the needle deflection *changes* δ degrees, the temperature coefficient is

$$\mu = \frac{1}{2 \tan \theta} \cdot \frac{\delta}{t}.$$

REFERENCES. — *Kohlrausch*, Physical Measurements, p. 260; *Stewart and Gee*, Practical Physics, Vol. II, p. 486.

Part VII — Appendix

CHAPTER XXVI

TABLES, CONSTANTS, AND REFERENCES

286. Explanation of Tables. — The data contained in the following tables have been selected and arranged from many sources. The results of recent investigations have been incorporated as far as possible. The tables for the density and volume of water and of mercury are based upon the determinations of density made at the International Bureau of Weights and Measures and reported to the International Congress of Physics, held at Paris in 1900. The table of wave lengths of light is founded upon Michelson's determination of the wave length of cadmium light.

The tables have been limited both as to subject-matter and range by the requirements of ordinary laboratory practice.

The values of some of the quantities tabulated, such as the elastic constants of solids, and electrical resistances, vary greatly with circumstances. Sometimes limiting values are given for these quantities, and at other times such single values as seem best to represent the quantities as ordinarily observed.

It has not seemed convenient to indicate the sources of information in general. The tables which have been most frequently consulted are: *Landolt und Börnstein*, Physikalisch-Chemische Tabellen; *Kohlrausch*, Praktische Physik; *Everett*, The C. G. S. System of Units; the *Smithsonian Physical Tables* and the *Smithsonian Meteorological Tables*.

1. Reduction to Vacuum of Weighings made with Brass Weights in Air

If a body of density s has in air the apparent weight of m grams, its weight reduced to vacuum is $m + mk$ grams. k is computed for air of density 0.0012 and for brass weights of density 8.4. (See Art. 48.)

$$k = \frac{0.0012}{s} - \frac{0.0012}{8.4}.$$

s	k	s	k	s	k	s	k
0.7	+ 0.00157	1.3	+ 0.00078	3.0	+ 0.00026	10	− 0.00002
0.8	136	1.4	71	3.5	20	12	4
0.9	119	1.6	61	4.0	16	14	6
1.0	106	1.8	52	5.0	10	16	7
1.1	095	2.0	46	6.0	06	18	8
1.2	086	2.5	34	7.0	03	20	8
1.3	078	3.0	26	8.0	01	22	9

2. Density of Various Substances

Solids					
Aluminum	2.7	Ice	0.9167	Silver	10.5
Beeswax	0.96	Iridium	21.8–22.4	Tin	7.3
Brass	8.1–8.7	Iron, Cast	7.1–7.7	Wood, Box	0.95–1.16
Calcspar	2.7	Wrought	7.8	Cork	0.2
Caoutchouc	0.95	Steel	7.8	Ebony	1.2
Copper	8.5–8.9	Ivory	1.9	Lignum Vitæ	1.2
Glass, Common	2.4–2.6	Lead	11.3	Mahogany	0.56–0.85
Flint	3.0–5.9	Nickel	8.8	Oak	0.60–0.90
Gold	19.3	Platinum	21.4	Pine	0.35–0.50
Hard Rubber	1.15	Quartz	2.65	Pitch Pine	0.84
Liquids at 20°					
Alcohol	0.789	Glycerin	1.23	Sulphuric Acid	1.832
Amyl Acetate	0.88	Hydrochloric		Turpentine	0.87
Bromoform	2.86	Acid, 40%	1.20	* Water, Pure	0.998
Carbon		Methyl Iodide	3.34	Sea	1.024
Bisulphide	1.264	Nitric Acid	1.522	* Mercury	13.546
Chloroform	1.489	Olive Oil	0.92		
Ether	0.715	Petroleum	0.88	* See Table 3	
Gases at 0° and 76 cm					
Air	0.001 293	Chlorine	0.003 091	Nitrogen	0.001 251
Carbon Dioxide	0.001 965	Hydrogen	0.000 08987	Oxygen	0.001 429

3. Density of Water and of Mercury

at the Temperature t of the Hydrogen Thermometer

(Thiesen, Guillaume, International Congress of Physics, Paris, 1900)

t	Water	t	Mercury
°		°	
0	0.999 823	0	**13.5950**
1	.999 882	1	.5925
2	.999 923	2	.5901
3	.999 947	3	.5876
4	**0.999 955**	4	.5851
5	0.999 947	5	13.5827
6	.999 923	6	.5802
7	.999 884	7	.5777
8	.999 831	8	.5753
9	.999 763	9	.5728
10	0.999 682	10	13.5703
11	.999 587	11	.5679
12	.999 480	12	.5654
13	.999 359	13	.5629
14	.999 226	14	.5605
15	0.999 081	15	13.5581
16	.998 925	16	.5556
17	.998 756	17	.5531
18	.998 577	18	.5507
19	.998 387	19	.5482
20	0.998 185	20	13.5457
21	.997 974	21	.5433
22	.997 752	22	.5408
23	.997 520	23	.5384
24	.997 278	24	.5359
25	0.997 026	25	13.5335
26	.996 765	26	.5310
27	.996 496	27	.5286
28	.996 214	28	.5261
29	.995 926	29	.5237
30	0.995 628	30	13.5212

4. Volume of a Glass Vessel at 20°

If a glass vessel apparently contains 1 g of water (or of mercury) at the temperature t, being weighed with brass weights in air of density 0.00120, the volume of the vessel at 20° is as given in the table. The coefficient of cubical expansion of glass is assumed to be 0.000025.

t	Volume-Water	t	Volume-Mercury
°	ccm	°	ccm
5	1.001 49	5	0.073 647
6	.001 49	6	.073 658
7	.001 50	7	.073 669
8	.001 53	8	.073 681
9	.001 57	9	.073 693
10	1.001 63	10	0.073 704
11	.001 70	11	.073 716
12	.001 78	12	.073 727
13	.001 88	13	.073 739
14	.001 99	14	.073 750
15	1.002 10	15	0.073 762
16	.002 23	16	.073 773
17	.002 38	17	.073 785
18	.002 53	18	.073 797
19	.002 70	19	.073 808
20	1.002 87	20	0.073 820
21	.003 06	21	.073 831
22	.003 26	22	.073 843
23	.003 47	23	.073 854
24	.003 69	24	.073 866
25	1.003 92	25	0.073 877
26	.004 15	26	.073 889
27	.004 39	27	.073 900
28	.004 65	28	.073 912
29	.004 92	29	.073 923
30	1.005 19	30	0.073 935

5. Coefficient of Static Friction

Wood on Wood	0.20–0.50	Leather on Metal, Dry	0.50 –0.60
Metal on Oak, Dry	0.50–0.60	Oily	0.15 –0.23
Wet	0.24–0.26	Leather on Oak	0.27 –0.38
Metal on Metal, Dry	0.15–0.20	Smooth Surfaces, Oiled	0.03 –0.08
Steel on Glass	0.10–0.20	Rolling Friction	0.004–0.006

6. Relative Viscosity at 19°

Cylinder Oil	191
Machine Oil	102
Wagon Oil	75
Olive Oil	22
Whale Oil	8

7. Elastic Constants of Solids

Substance	Young's Modulus	Rigidity	Elastic Limit	Velocity of Sound
	dynes per scm	dynes per scm	dynes per scm	cm per s
Aluminum	6.5×10^{11}	$2.4–3.3 \times 10^{11}$		5.10×10^5
Brass	7.7–9.3	3.1–3.6	$4.5–11 \times 10^8$	3.56
Copper	8.5–11.6	3.2–4.5	3–7	3.74
Glass	4–6	1.7–2.4	2.3	4.42
Iron, Cast	6.9–10.8	2.6–4.2	7.	4.32
Iron, Wrought	$19.3–20.9 \times 10^{11}$	$7.7–8.5 \times 10^{11}$	20×10^8	5.06×10^5
Steel	20.1–21.6	8.0–8.8	33–40	5.22
Platinum	15.	6.1–6.5	14–25	2.69
Silver	7.	2.6	3–11	2.61
Wood	0.7–1.5	0.07–0.12	1.5–2.4	3.

8. Compressibility of Water, Mercury, and Glass

Substance	Compression per Megabarye	Elasticity of Volume
Water	5.0×10^{-5}	2.03×10^{10}
Mercury	3.9×10^{-6}	2.60×10^{11}
Glass	2.6×10^{-6}	3.90×10^{11}

9. Surface Tension at 20°

Water, Pure	73
Water, Saturated with Olive Oil	41
Water, Saturated with Oleate of Soda	25
Olive Oil	35
Turpentine	28
Alcohol	22

10. Absolute Value of Gravity

Not reduced to sea level. By the United States Coast and Geodetic Survey, founded upon the provisional value for Potsdam, 1900–1903

Atlanta	979.523	Denver	979.608	San Francisco	979.964
Austin	979.282	Ithaca	980.299	St. Louis	980.000
Boston	980.395	Kansas City	979.989	Terra Haute	980.071
Cambridge	980.397	Little Rock	979.720	Washington	980.111
Charleston	979.545	Montreal	980.742	Worcester	980.323
Charlottesville	979.937	New Orleans	979.323	London	981.201
Chicago	980.277	New York	980.266	Paris	980.941
Cleveland	980.240	Philadelphia	980.195	Potsdam	981.274
Cincinnati	980.000	Princeton	980.177	Para	978.03
Colorado Springs	979.489	Salt Lake City	979.802	Spitzbergen	983.08

11. Local Geographical Data for Cleveland

Case Observatory, Latitude, 41° 30′ 14″.53; Longitude, $5^h\ 26^m\ 25^s.82$
Main Building, Altitude, 21210 cm; Bearing, West of North, 41° 15′
Reduction of Central Standard to Local Mean Time + $33^m\ 34^s.18$

12. Specific Gravity of Air

At the temperature t, and under the pressure of H cm of mercury, the specific gravity of air referred to water at 4° is

$$\frac{0.001293}{1 + 0.00367\,t} \cdot \frac{H}{76}.$$

t	Pressure H in Centimeters					
	72.0	73.0	74.0	75.0	76.0	77.0
°						
10	0.001 182	0.001 198	0.001 215	0.001 231	0.001 247	0.001 264
11	178	193	210	227	243	259
12	173	190	206	222	239	255
13	169	186	202	218	234	251
14	165	181	198	214	230	246
15	0.001 161	0.001 177	0.001 193	0.001 210	0.001 226	0.001 242
16	157	173	189	205	221	238
17	153	169	185	201	217	233
18	149	165	181	197	213	229
19	145	161	177	193	209	225
20	0.001 141	0.001 157	0.001 173	0.001 189	0.001 205	0.001 221
21	137	153	169	185	201	216
22	134	149	165	181	197	212
23	130	145	161	177	193	208
24	126	142	157	173	189	204
25	0.001 122	0.001 138	0.001 153	0.001 169	0.001 185	0.001 200
26	118	134	149	165	181	196
27	115	130	146	161	177	192
28	111	126	142	157	173	188
29	107	123	138	153	169	184
30	0.001 104	0.001 119	0.001 134	0.001 150	0.001 165	0.001 180

Proportional Parts	17	16	15
cm			
0.1	2	2	1
0.2	3	3	3
0.3	5	5	4
0.4	7	6	6
0.5	8	8	7
0.6	10	10	9
0.7	12	11	10
0.8	14	13	12
0.9	15	14	13

13. Capillary Depression of Mercury in Glass Tubes

Diameter of Tube	Height of Meniscus in Centimeters							
	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
cm	cm	cm	cm	cm	cm	cm	cm	cm
0.4	0.083	0.122	0.154	0.198	0.237			
0.5	.047	.065	.086	.119	.145	0.180		
0.6	.027	.041	.056	.078	.098	.121	0.143	
0.7	.018	.028	.040	.053	.067	.082	.097	.113
0.8		.020	.029	.038	.046	.056	.065	0.077
0.9		0.015	0.021	0.028	0.033	0.040	0.046	.052
1.0			.015	.020	.025	.029	.033	0.037
1.1			.010	.014	.018	.021	.024	.027
1.2			.007	.010	.013	.015	.018	.019
1.3			.004	.007	.010	.012	.013	.014

14. Reduction of Barometer Readings to 0°

When the height of a mercury column has been measured with a brass scale, the length of which is correct at 0°, the mercury and scale being at the temperature t, the observed height will be reduced to 0° by subtracting the quantity corresponding to the temperature and height, taken from the following table. (See Art. 54.)

t	Observed Height in Centimeters					
	72.0	73.0	74.0	75.0	76.0	77.0
°	cm	cm	cm	cm	cm	cm
10	0.12	0.12	0.12	0.12	0.12	0.12
11	.13	.13	.13	.13	.14	.14
12	.14	.14	.14	.15	.15	.15
13	.15	.15	.16	.16	.16	.16
14	.16	.17	.17	.17	.17	.17
15	0.17	0.18	0.18	0.18	0.18	0.19
16	.19	.19	.19	.19	.20	.20
17	.20	.20	.20	.21	.21	.21
18	.21	.21	.22	.22	.22	.22
19	.22	.22	.23	.23	.23	.24
20	0.23	0.24	0.24	0.24	0.25	0.25
21	.25	.25	.25	.25	.26	.26
22	.26	.26	.26	.27	.27	.27
23	.27	.27	.28	.28	.28	.29
24	.28	.28	.29	.29	.29	.30
25	0.29	0.30	0.30	0.30	0.31	0.31
26	.30	.31	.31	.32	.32	.32
27	.31	.32	.32	.33	.33	.34
28	.33	.33	.34	.34	.34	.35
29	.34	.34	.35	.35	.36	.36
30	0.35	0.35	0.36	0.36	0.37	0.37

15. Reduction of Mercury-in-Glass Thermometer Reading to the Normal Hydrogen Scale

for Jena Normal Glass, 16^{III}

(See Art. 133)

Reading	0°	10	20	30	40	50
Correction	0°.000	−0.055	−0.090	−0.109	−0.115	−0.109
Reading	**50°**	**60**	**70**	**80**	**90**	**100**
Correction	−0°.109	−0.096	−0.076	−0.053	−0.027	0.000

16. Boiling Temperature of Water *t*

at the Barometric Pressure *B*

B	*t*	*B*	*t*	*B*	*t*	*B*	*t*	*B*	*t*	*B*	*t*
cm	°	cm	°	cm	°	cm	°	cm	°	cm	°
72.0	98.49	73.0	98.88	74.0	99.26	75.0	99.63	76.0	**100.00**	77.0	100.37
.1	.53	.1	.92	.1	.29	.1	.67	.1	.04	.1	.40
.2	.57	.2	.95	.2	.33	.2	.70	.2	.07	.2	.44
.3	.61	.3	98.99	.3	.37	.3	.74	.3	.11	.3	.48
.4	.65	.4	99.03	.4	.41	.4	.78	.4	.15	.4	.51
72.5	98.69	73.5	99.07	74.5	99.44	75.5	99.82	76.5	100.18	77.5	100.55
.6	.72	.6	.10	.6	.48	.6	.85	.6	.22	.6	.58
.7	.76	.7	.14	.7	.52	.7	.89	.7	.26	.7	.62
.8	.80	.8	.18	.8	.55	.8	.93	.8	.29	.8	.66
.9	.84	.9	.22	.9	.59	.9	.96	.9	.33	.9	.69

17. Fixed Points for High Temperatures

(See Art. 138)

Substance	Boiling Point	Substance	Melting Point
	°		°
Alcohol	78.26	Tin	232
Water	**100.0**	Zinc	419
Naphthalene	218.0	Aluminum	657
Mercury	356.7	Gold	1064
Sulphur	445.2	Copper	1084
Zinc	930.	Platinum	1775

18. Heat Constants of Liquids

(See also Table 17)

Substance	Cubical Expansion	Specific Heat	Boiling Point	Heat of Vaporization
			°	cal
Alcohol	0.001 10	0.58	78.3	210.
Benzene	0.001 24	0.40	80.3	94.4
Glycerin	0.000 50	0.58	290.	
Mercury	0.000 182	0.0332	356.7	62.
Sulphur	0.000 21	0.2	445.2	362.
Turpentine	0.000 94	0.42	159.	70.
Water	0.000 18	1.	100.0	539.

19. Heat Constants of Solids

(See also Table 17)

Substance	Linear Expansion	Specific Heat	Melting Point	Heat of Fusion
			°	cal
Aluminum	0.0000 23	0.22	657	
Brass	19	.093	900	
Copper	17	.093	1084	
German Silver	18	.095	1000	
Glass	0.0000 08	0.19	1100	
Iron	12	.11	1300	30.
Nickel	13	.11	1470	4.6
Platinum	09	.032	1775	27.
Silver	0.0000 19	0.056	961	21.
Steel	11	.12	1350	
Tin	23	.054	232	13.
Zinc	29	.094	419	28.

20. Reduction of Psychrometric Observations

Values of $0.00660\,B\,(t - t_w)\,[1 + 0.00115\,(t - t_w)]$, t being the temperature of the dry-bulb thermometer, t_w that of the wet-bulb thermometer, and B the barometric pressure. (See Art. 151.)

$t - t_w$	Barometric Pressure B in Centimeters							
	70.0	71.0	72.0	73.0	74.0	75.0	76.0	77.0
°	cm	cm	cm	cm	cm	cm	cm	cm
1	0.047	0.048	0.048	0.049	0.050	0.050	0.051	0.052
2	.093	.094	.096	.097	.098	.100	.101	.103
3	.139	.141	.143	.145	.147	.149	.152	.154
4	.186	.189	.191	.194	.197	.199	.202	.204
5	0.232	0.236	0.239	0.243	0.246	0.249	0.252	0.256
6	.279	.283	.287	.291	.295	.299	.303	.307
7	.326	.331	.336	.340	.345	.350	.354	.359
8	.373	.379	.384	.389	.395	.400	.405	.411
9	.421	.427	.432	.438	.444	.450	.456	.462
10	0.468	0.474	0.481	0.488	0.494	0.501	0.508	0.515
11	.515	.522	.530	.537	.544	.551	.559	.566
12	.562	.570	.578	.586	.594	.602	.611	.619
13	.610	.618	.627	.636	.645	.653	.662	.671
14	.658	.667	.676	.686	.695	.705	.714	.723
15	0.706	0.716	0.726	0.736	0.746	0.756	0.766	0.776
16	.754	.764	.775	.786	.796	.807	.818	.829
17	.802	.813	.824	.836	.847	.859	.870	.882
18	.850	.862	.874	.886	.898	.910	.922	.935
19	.898	.911	.923	.936	.949	.962	.975	.987
20	0.946	0.960	0.973	0.987	1.000	1.014	1.027	1.041

21. Relative Humidity

The table gives the per cent. of saturation of air, t being the temperature of the air, d the dew-point, and $t-d$ the depression of the dew-point. (See Art. 151.)

Depression of Dew-Point $t-d$	Dew-Point, d						
	$-5°$	$0°$	$+5°$	$10°$	$15°$	$20°$	$25°$
°							
0	100	100	100	100	100	100	100
1	93	93	93	94	94	94	94
2	86	87	87	88	88	88	89
3	80	81	81	82	83	83	84
4	74	75	76	77	78	78	79
5	69	70	71	72	73	74	75
6	64	66	67	68	69	70	70
7	60	61	62	63	65	66	67
8	56	57	58	60	61	62	63
9	52	53	55	56	57	58	60
10	49	50	51	53	54	55	56
12	42	44	45	47	48	49	50
14	37	38	40	41	43	44	45
16	32	34	35	37	38	39	
18	28	30	31	33	34	35	
20	25	26	28	29	30	32	
25	18	19	21	22	23		
30	13	14	16	17			

22. Declination of the Sun, and Equation of Time

Date	Declination	Diff. 1 Day	Equation of Time	Date	Declination	Diff. 1 Day	Equation of Time
	°	°	m s		°	°	m s
Jan. 0	− 23.1	0.11	+ 3 15	July 9	+ 22.4	0.15	+ 4 49
10	− 22.0	0.18	+ 7 42	19	+ 20.9	0.21	+ 5 58
20	− 20.2	0.25	+ 11 13	29	+ 18.8	0.26	+ 6 13
30	− 17.7	0.30	+ 13 32	Aug. 8	+ 16.2	0.30	+ 5 27
Feb. 9	− 14.7	0.34	+ 14 27	18	+ 13.2	0.34	+ 3 44
19	− 11.3	0.37	+ 14 5	28	+ 9.8	0.36	+ 1 11
March 1	− 7.6	0.38	+ 12 36	Sept. 7	+ 6.2	0.39	− 1 59
11	− 3.8	0.40	+ 10 15	17	+ 2.3	0.39	− 5 26
21	+ 0.2	0.39	+ 7 23	27	− 1.5	0.38	− 8 55
31	+ 4.1	0.38	+ 4 19	Oct. 7	− 5.4	0.38	− 12 4
April 10	+ 7.9	0.35	+ 1 23	17	− 9.2	0.35	− 14 31
20	+ 11.4	0.33	− 1 5	27	− 12.7	0.32	− 16 0
30	+ 14.7	0.29	− 2 52	Nov. 6	− 15.9	0.26	− 16 16
May 10	+ 17.6	0.23	− 3 48	16	− 18.7	0.22	− 15 7
20	+ 19.9	0.18	− 3 45	26	− 20.9	0.16	− 12 36
30	+ 21.7	0.12	− 2 49	Dec. 6	− 22.5	0.08	− 8 54
June 9	+ 22.9	0.05	− 1 11	16	− 23.3	0.01	− 4 17
19	+ 23.4	0.01	+ 0 55	26	− 23.4	0.08	+ 0 41
29	+ 23.3	0.09	+ 3 2	Jan. 5	− 22.6		+ 5 34

23. Tension and Mass of Aqueous Vapor in Saturated Air

The table gives the tension e, and the mass f per cubic meter, of water vapor in air saturated at the temperature t. (See Art. 151.)

t	e	f	t	e	f	t	e	f
°	cm	g	°	cm	g	°	cm	g
0	0.46	4.9	10	0.92	9.4	20	1.75	17.3
1	0.49	5.2	11	0.98	10.0	21	1.86	18.3
2	0.53	5.6	12	1.05	10.7	22	1.98	19.4
3	0.57	6.0	13	1.12	11.4	23	2.11	20.6
4	0.61	6.4	14	1.20	12.1	24	2.24	21.8
5	0.65	6.8	15	1.28	12.8	25	2.38	23.1
6	0.70	7.3	16	1.36	13.7	26	2.53	24.5
7	0.75	7.8	17	1.45	14.5	27	2.68	25.8
8	0.80	8.2	18	1.55	15.4	28	2.84	27.3
9	0.86	8.8	19	1.65	16.3	29	3.01	28.8
10	0.92	9.4	20	1.75	17.3	30	3.18	30.4

24. Index of Refraction of Various Substances

for Sodium Light, D Line, $\lambda = 0.5893\ \mu$.

Air, Dry, 0°	1.0002945	Glass,		α-Bromonaphthalene, 15°	1.6518
20°	1.0002773	Light crown	1.5153	20°	1.6495
Ice	1.31	Heavy crown	1.6152	Methyl Iodide, 15°	1.7429
Water, 15°	1.33361	Light flint	1.6085	20°	1.7419
20°	1.33319	Heavy flint	1.7515	Calcite,	
Alcohol	1.3616	Heaviest flint	1.9	Ordinary Ray	1.6585
Benzene	1.5005	Carbon Bisulphide, 15°	1.6317	Extraordinary	1.4864
Canada Balsam	1.54	20°	1.6277	Quartz,	
		Phosphorus in CS_2	1.97	Ordinary Ray	1.5442
				Extraordinary	1.5533

25. Wave Length of Lines of Solar Spectrum

in Air at 20°, Pressure 76 cm; Unit, Micron = 0.001 mm

(See Art. 196)

Line	Element	Wave Length	Line	Element	Wave Length	Line	Element	Wave Length
		μ			μ			μ
A		0.7628	D_2	Na	0.58900	e	Fe	0.43836
a		0.7185	E_1	Fe, Ca	0.52703	f	H	0.43405
B	O	0.68701	b_1	Mg	0.51837	G	Fe, Ca	0.43079
C	H	0.65629	c	Fe	0.49576	h	H	0.41018
α	O	0.62781	F	H	0.48614	H	H, Ca	0.39685
D_1	Na	0.58960	d	Fe	0.46682	K	Ca	0.39337

26. Specific Resistance of Various Substances

ρ is the resistance at 0° of a conductor 1 cm long and 1 scm in section. α is the rate of increase in resistance per degree increase in temperature. The resistance of a conductor of length l and section s at the temperature t is (see Art. 235)

$$R_t = \rho \frac{l}{s}(1 + \alpha t).$$

Substance	ρ	α	Substance	ρ	α
	ohms			ohms	
Silver	0.16×10^{-5}	0.0037	Iron	1.04×10^{-5}	0.0060
Copper	0.17	43	Steel	5.	60
Gold	0.20	36	Lead	2.00	39
Aluminum	0.30	39	Manganin	4.20	003
Zinc	0.57×10^{-5}	0.0036	German Silver	2.09×10^{-5}	0.0003
Nickel	0.70	60	Brass	0.8	40
Platinum, Pure	1.08	32	Mercury	9.41	09
Commercial	1.40	32	Gas Carbon	500.	− .0005

27. Electromotive Force and Internal Resistance of Cells

Cell	E. M. F.	Resistance	Cell	E. M. F.	Resistance
	volts	ohms		volts	ohms
Edison-Lalande	0.7	0.03	Grove	1.9	0.1–0.2
Daniel	1.08	0.85	Bunsen	1.9	0.1–0.2
Gravity	1.1	1–5	Bichromate	2.0	0.08–0.40
Silver Chloride	1.1	4.	Storage	2.0	0.004–0.02
Dry Cell	1.3	0.2–1.0	* Clark Standard	1.4267	20–50
Leclanché	1.4–1.7	0.4–0.2	* Weston Standard	1.0190	20–50

* See Art. 249.

28. Vibration Frequency of Tones of the Musical Scale

Higher or lower octaves are obtained by multiplying by some power of 2.

Scientific Diatonic Scale $C_3 = 256$		Musical Equal-Tempered Chromatic Scale $A_3 = 435$			
C_3	**256.**	C_3	258.65	G_3	387.54
D_3	288.	$C^{\flat}_{\sharp 3}$	274.03	$G^{\flat}_{\sharp 3}$	410.58
E_3	320.	D_3	290.33	A_3	**435.**
F_3	341.33	$D^{\flat}_{\sharp 3}$	307.59	$A^{\flat}_{\sharp 3}$	460.87
G_3	384.	E_3	325.88	B_3	488.27
A_3	426.66	F_3	345.26	C_4	517.30
B_3	480.	$F^{\flat}_{\sharp 3}$	365.79		
C_4	512.				

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30. Miscellaneous Constants and Numbers

$\pi = 3.14159265$ $\pi^2 = 9.869604$ $\log \pi = 0.49714987$
$4\pi = 12.566$ $\frac{1}{4}\pi = 0.079577$ $\log 4\pi = 1.09921$

1 Radian $= 57^\circ.2958 = 3437'.75 = 206265''$
Logarithms 1.758123 3.536274 5.314425

Base of natural logarithms, $e = 2.7182818$; $\log e = 0.434294$.
Mean radius of the earth, 6.37106×10^8 cm.
Mechanical equivalent of heat, 4.184×10^7 ergs. (See Art. 139.)
Candle power: 1 English candle = 0.95 German candle = 1.14 Hefner units. (See Art. 155.)
Velocity of sound in dry air at 0°, 33136 cm per s; in illuminating gas at 0°, 49000 cm per s. (See Table 7, and Art. 114.)
Velocity of light in vacuum, 2.9989×10^{10} cm per s.
Specific rotation of sugar, $[\alpha]_D^{20^\circ} = 6^\circ.65$ per cm. (See Art. 203.)
Electro-chemical equivalents: silver, 0.001118; copper, 0.0003279; g per s per ampere. (See Arts. 243, 244.)
Temperature coefficient of magnetism, from 0.0003 to 0.001.

31. Metric and English Equivalents and Abbreviations

1 inch	= 2.539998 cm	1 micron	= 0.001 mm	m	meter
1 foot	= 30.47997 cm	1 meter	= 39.37011 inches	dm	decimeter
1 yard	= 91.43992 cm	1 kilometer	= 0.621371 miles	cm	centimeter
1 mile	= 160934.3 cm	1 gram	= 15.43236 grains	mm	millimeter
AVOIRDUPOIS		1 milligram	= 0.01543 grains	km	kilometer
1 grain	= 64.79892 mg	1 kilogram	= 2.204622 pounds	μ	0.001 millimeter
1 ounce	= 28.34953 g	1 liter	= 1.75980 pints	$\mu\mu$	0.000001 millimeter
1 pound	= 453.5924 g	APPROXIMATE EQUIVALENTS		tenth-meter	$= 10^{-10}$ m
TROY		1 millimeter	$= \frac{1}{25}$ inch	g	gram
1 grain	= 64.79892 mg	1 meter	= 3 ft, 3 in, $\frac{3}{8}$ in	mg	milligram
1 ounce	= 31.10348 g	1 kilometer	$= \frac{5}{8}$ mile	kg	kilogram
APOTHECARY		1 gram	$= 15\frac{3}{7}$ grains	scm	square centimeter
1 ounce	= 31.10348 g	1 kilogram	$= 2\frac{1}{5}$ pounds	ccm	cubic centimeter
1 quart	= 1.13649 l	1 liter	$= 1\frac{3}{4}$ pints	l	liter

32. Natural Trigonometrical Functions

Degree	Sine		Tangent		Cotangent		Cosine		Degree
0	0.0000	175	0.0000	175	∞		1.0000	02	90
1	.0175	174	.0175	174	57.29		0.9998	04	89
2	.0349	174	.0349	175	28.64		.9994	06	88
3	.0523	175	.0524	175	19.08		.9986	10	87
4	.0698	174	.0699	176	14.30		.9976	14	86
5	0.0872	173	0.0875	176	11.43		0.9962	17	85
6	.1045	174	.1051	177	9.514		.9945	20	84
7	.1219	173	.1228	177	8.144		.9925	22	83
8	.1392	172	.1405	179	7.115	801	.9903	26	82
9	.1564	172	.1584	179	6.314	643	.9877	29	81
10	0.1736	172	0.1763	181	5.671	526	0.9848	32	80
11	.1908	171	.1944	182	5.145	440	.9816	35	79
12	.2079	171	.2126	183	4.705	374	.9781	37	78
13	.2250	169	.2309	184	4.331	320	.9744	41	77
14	.2419	169	.2493	186	4.011	279	.9703	44	76
15	0.2588	168	0.2679	188	3.732	245	0.9659	46	75
16	.2756	168	.2867	190	3.487	216	.9613	50	74
17	.2924	166	.3057	192	3.271	193	.9563	52	73
18	.3090	166	.3249	194	3.078	174	.9511	56	72
19	.3256	164	.3443	197	2.904	157	.9455	58	71
20	0.3420	164	0.3640	199	2.747	142	0.9397	61	70
21	.3584	162	.3839	201	2.605	130	.9336	64	69
22	.3746	161	.4040	205	2.475	119	.9272	67	68
23	.3907	160	.4245	207	2.356	110	.9205	70	67
24	.4067	159	.4452	211	2.246	101	.9135	72	66
25	0.4226	158	0.4663	214	2.145	95	0.9063	75	65
26	.4384	156	.4877	218	2.050	87	.8988	78	64
27	.4540	155	.5095	222	1.963	82	.8910	81	63
28	.4695	153	.5317	226	1.881	77	.8829	83	62
29	.4848	152	.5543	231	1.804	72	.8746	86	61
30	0.5000	150	0.5774	235	1.732	68	0.8660	88	60
31	.5150	149	.6009	240	1.664	64	.8572	92	59
32	.5299	147	.6249	245	1.600	60	.8480	93	58
33	.5446	146	.6494	251	1.540	57	.8387	97	57
34	.5592	144	.6745	257	1.483	55	.8290	98	56
35	0.5736	142	0.7002	263	1.428	52	0.8192	102	55
36	.5878	140	.7265	271	1.376	49	.8090	104	54
37	.6018	139	.7536	277	1.327	47	.7986	106	53
38	.6157	136	.7813	285	1.280	45	.7880	109	52
39	.6293	135	.8098	293	1.235	43	.7771	111	51
40	0.6428	133	0.8391	302	1.192	42	0.7660	113	50
41	.6561	130	.8693	311	1.150	39	.7547	116	49
42	.6691	129	.9004	321	1.111	39	.7431	117	48
43	.6820	127	.9325	332	1.072	36	.7314	121	47
44	.6947	124	.9657	343	1.036	36	.7193	122	46
45	0.7071		1.0000		1.000		0.7071		45
Degree	Cosine		Cotangent		Tangent		Sine		Degree

33. Logarithms

N	0	1	2	3	4	5	6	7	8	9	PP 1	PP 2	PP 3	PP 4	PP 5	PP 6	PP 7	PP 8	PP 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

33. Logarithms

N	0	1	2	3	4	5	6	7	8	9	Proportional Parts 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

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G

H

M

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jarke
750 net

Miller